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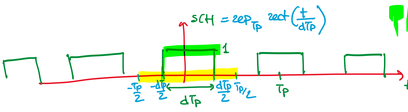
Friday, 28 March 2025 09:19

ES 1 CALCOLARE S_K PER UN'ONDA QUADRA CON DUTY CYCLE $d<1$



$$S_K = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} s(t)\, e^{-jK\omega_0 t}\, dt \qquad s(t) = \text{rect}\left(\frac{t}{dT_p}\right) \qquad \omega_0 = \frac{2\pi}{T_p}$$

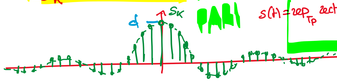
$$= \frac{1}{T_p}\int_{-\frac{dT_p}{2}}^{\frac{dT_p}{2}} \text{rect}\left(\frac{t}{dT_p}\right) e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{T_p}\int_{-\frac{dT_p}{2}}^{\frac{dT_p}{2}} e^{-jK\omega_0 t}\, dt = \begin{cases} \frac{dT_p}{T_p} & K=0 \\ \frac{1}{T_p}\,\frac{e^{-jK\omega_0 t}}{-jK\omega_0}\Big|_{-\frac{dT_p}{2}}^{\frac{dT_p}{2}} & K\neq 0 \end{cases}$$

$\omega_0 T_p = 2\pi$

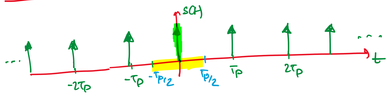
$$\frac{-e^{-jK\omega_0\frac{dT_p}{2}} + e^{jK\omega_0\frac{dT_p}{2}}}{+jK\omega_0 T_p} = \frac{e^{j\pi Kd} - e^{-j\pi Kd}}{j2\pi K} = \frac{2j\sin(\pi Kd)}{j2\pi K}\cdot d = d\,\text{sinc}(Kd)$$

$S_K = d\,\text{sinc}(Kd)$



$$s(t) = 2ep_{T_p}\,\text{rect}\left(\frac{t}{dT_p}\right) = \sum_{K=-\infty}^{+\infty} d\,\text{sinc}(Kd)\, e^{jK\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_p}$$

ES 2 CALCOLARE S_K PER $s(t) = \text{comb}_{T_p}(t) = 2ep_{T_p}\,\delta(t)$

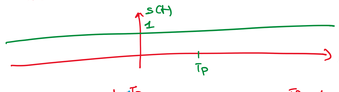


$$S_K = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} \delta(t)\, e^{-jK\omega_0 t}\, dt \qquad \omega_0 = 2\pi/T_p$$

$$= \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} \delta(t)\, e^{-jK\omega_0 t}\, dt = \frac{1}{T_p}\, e^{-jK\omega_0 t}\Big|_{t=0} = \frac{1}{T_p}$$

$$s(t) = 2ep_{T_p}\,\delta(t) = \sum_{K=-\infty}^{+\infty} \frac{1}{T_p}\, e^{jK\omega_0 t} \qquad \omega_0 = 2\pi/T_p$$

ES 3 TROVARE S_K PER $s(t) = 1$ E PERIODO T_p

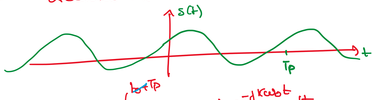


$$S_K = \frac{1}{T_p}\int_{0}^{T_p} 1\cdot e^{-jK\omega_0 t}\, dt = \begin{cases} \frac{T_p}{T_p} = 1 & K=0 \\ \frac{0}{T_p} = 0 & K\neq 0 \end{cases}$$

$= \delta(K)$

$$s(t) = \sum_{K=-\infty}^{+\infty} \delta(K)\, e^{jK\omega_0 t} = 1$$

ES 4 CALCOLARE S_K PER $s(t) = \cos(\omega_0 t + \varphi_0)$ CON $\omega_0 = 2\pi/T_p$



$$S_K = \frac{1}{T_p}\int_{0}^{T_p} \cos(\omega_0 t + \varphi_0)\, e^{-jK\omega_0 t}\, dt$$

$$\frac{1}{2}e^{j\varphi_0}e^{j\omega_0 t} + \frac{1}{2}e^{-j\varphi_0}e^{-j\omega_0 t} = s(t)$$

$$s(t) = \sum_{K=-\infty}^{+\infty} S_K\, e^{jK\omega_0 t}$$

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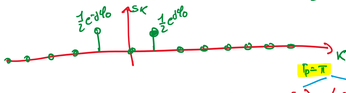
$$S_K = \begin{cases} \frac{1}{2}e^{j\varphi_0} & K=1 \\ \frac{1}{2}e^{-j\varphi_0} & K=-1 \\ 0 & \text{altrove} \end{cases} = \frac{1}{2}e^{j\varphi_0}\delta(K-1) + \frac{1}{2}e^{-j\varphi_0}\delta(K+1)$$



ES 5 CALCOLARE S_K CON $s(t) = 3 - \sin(2t) + 4\cos(2t) + 2\cos(6t - \pi/4)$

Tp = T

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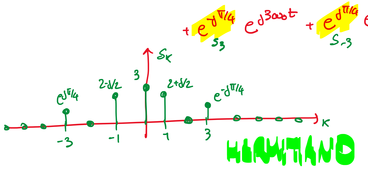
$$\omega_0 = \frac{2\pi}{T_p} = 2$$

$$s(t) = 3 - \sin(\omega_0 t) + 4\cos(\omega_0 t) + 2\cos(3\omega_0 t - \pi/4)$$

$$= 3 + j\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \frac{4}{2}\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + 2\,\frac{e^{j\pi/4}e^{j3\omega_0 t} + e^{-j\pi/4}e^{-j3\omega_0 t}}{2}$$

$$= 3 + \underbrace{\left(\frac{1}{2}+2\right)}_{S_1} e^{j\omega_0 t} + \underbrace{\left(-\frac{1}{2}+2\right)}_{S_{-1}} e^{-j\omega_0 t} + \underbrace{e^{j\pi/4}}_{S_3} e^{j3\omega_0 t} + \underbrace{e^{-j\pi/4}}_{S_{-3}} e^{-j3\omega_0 t}$$



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