# Le09

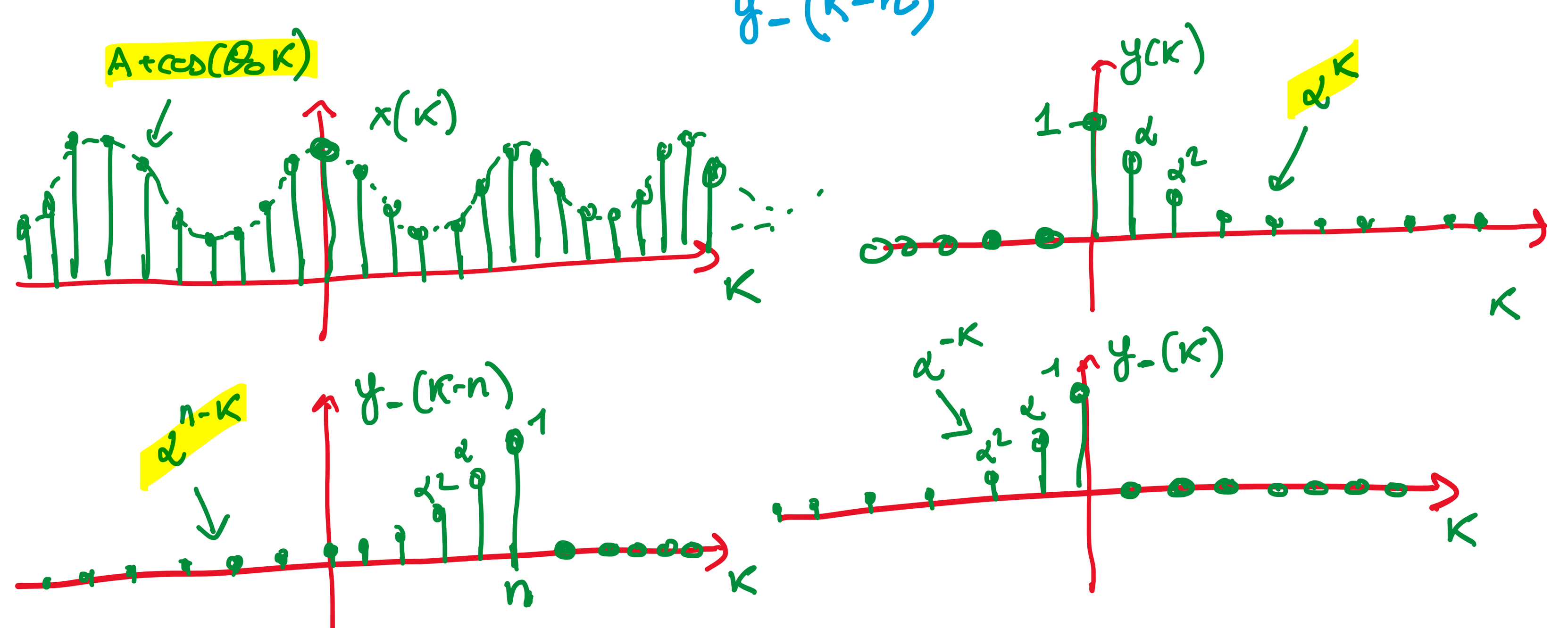
Friday, 14 March 2025 08:38

**ES1** CALCOLARE $z(n) = x * y(n)$

PER $x(n) = A + \cos(\theta_0 n)$

$y(n) = \alpha^n 1_0(n)$ , $\alpha$ REALE, $|\alpha| < 1$

$$z(n) = \sum_{k=-\infty}^{+\infty} x(k)\, \underbrace{y(n-k)}_{y_-(k-n)}$$




$$z(n) = \sum_{k=-\infty}^{\cancel{+\infty}\ n} \overset{(A+\cos(\theta_0 k))}{\cancel{x(k)}}\ \overset{\alpha^{n-k}}{\cancel{y(n-k)}}$$

$$= \sum_{k=-\infty}^{n} (A+\cos(\theta_0 k))\, \alpha^{n-k}$$

$$= \sum_{k=-\infty}^{n} \left(A + \frac{1}{2}e^{j\theta_0 k} + \frac{1}{2}e^{-j\theta_0 k}\right) \alpha^{n-k}$$

$$= \sum_{m=0}^{+\infty} \alpha^m \cdot \left(A + \frac{1}{2}(e^{j\theta_0})^{n-m} + \frac{1}{2}(e^{-j\theta_0})^{n-m}\right) \qquad \begin{aligned} m &= n-k \\ k &= n-m \end{aligned}$$

$$= \sum_{m=0}^{\infty} A\alpha^m + \frac{1}{2}\underbrace{\alpha^m (e^{j\theta_0})^{n-m}}_{e^{j\theta_0 n} \cdot \underbrace{\alpha^m (e^{j\theta_0})^{-m}}_{(\alpha e^{-j\theta_0})^m}} + \frac{1}{2}\underbrace{\alpha^m (e^{-j\theta_0})^{n-m}}_{e^{-j\theta_0 n} \cdot \underbrace{\alpha^m (e^{-j\theta_0})^{-m}}_{(\alpha e^{j\theta_0})^m}}$$

$$= A\sum_{m=0}^{\infty} \alpha^m + \frac{1}{2}e^{j\theta_0 n}\sum_{m=0}^{\infty} (\alpha e^{-j\theta_0})^m + \frac{1}{2}e^{-j\theta_0 n}\sum_{m=0}^{\infty} (\alpha e^{j\theta_0})^m$$

$$= \frac{A}{1-\alpha} + \frac{1}{2}\frac{e^{j\theta_0 n}}{1-\alpha e^{-j\theta_0}} + \frac{1}{2}\frac{e^{-j\theta_0 n}}{1-\alpha e^{j\theta_0}}$$

$$= \frac{A}{1-\alpha} + \mathrm{Re}\left[\frac{e^{j\theta_0 n}}{\underbrace{1-\alpha e^{-j\theta_0}}}\right]$$

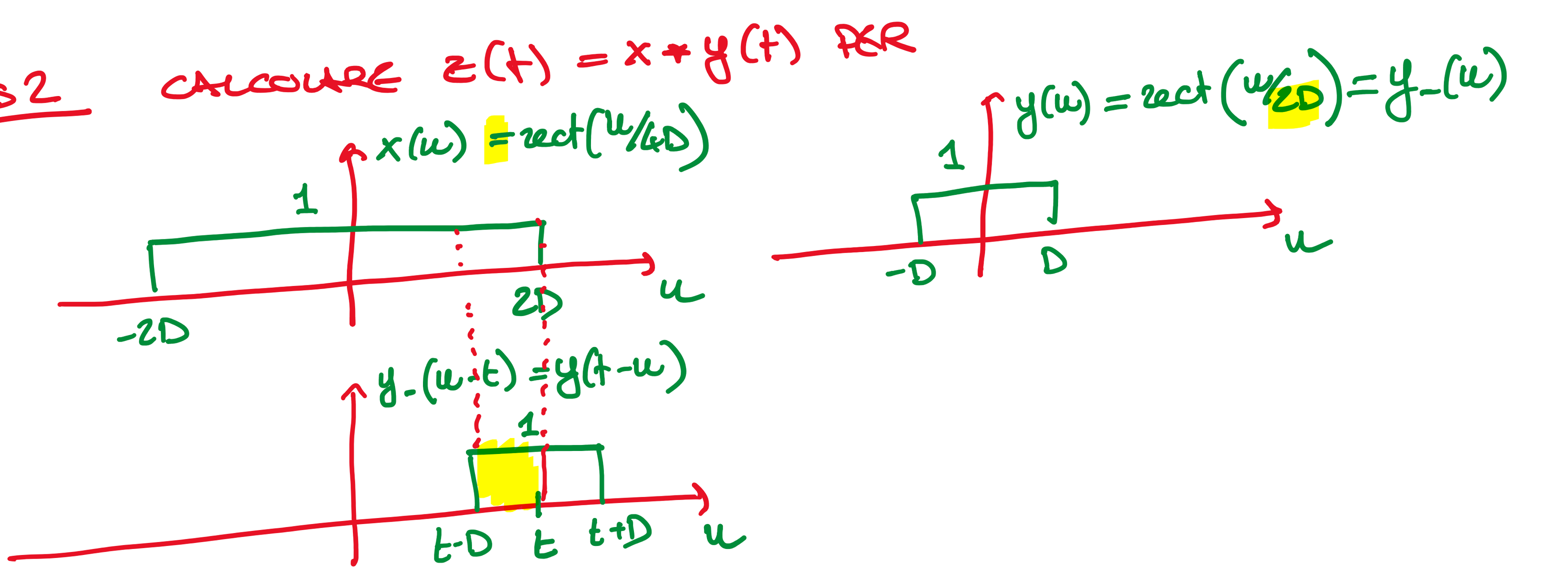
$$B = 1-\alpha e^{-j\theta_0} = |B|\, e^{j\varphi_B}$$

$$= \frac{A}{1-\alpha} + \mathrm{Re}\left[\frac{e^{j\theta_0 n}\, e^{-j\varphi_B}}{|B|\, \cancel{e^{j\varphi_B}}}\right] \longrightarrow e^{j(\theta_0 n - \varphi_B)}$$

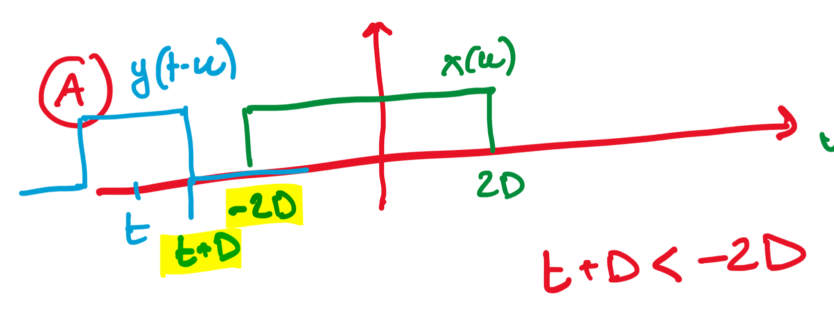
$$\boxed{z(n) = \frac{A}{1-\alpha} + \frac{\cos(\theta_0 n - \varphi_B)}{|B|} \qquad B = 1-\alpha e^{-j\theta_0}}$$

**ES2** CALCOLARE $z(t) = x * y(t)$ PER

$x(u) = \mathrm{rect}(u/4D)$ $\qquad$ $y(u) = \mathrm{rect}(u/2D) = y_-(u)$

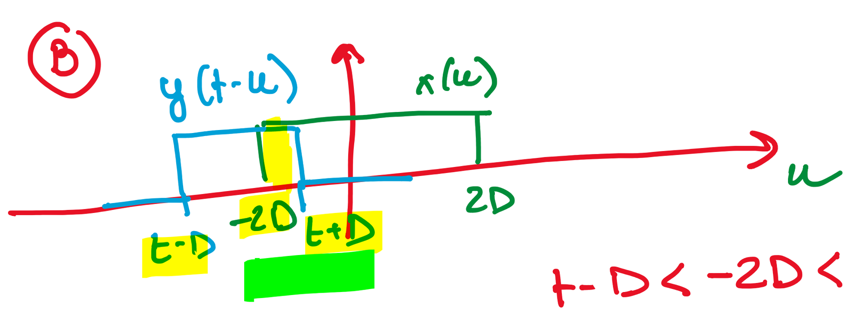



(A)




$t+D < -2D \longrightarrow t < -3D \quad z(t) = 0$

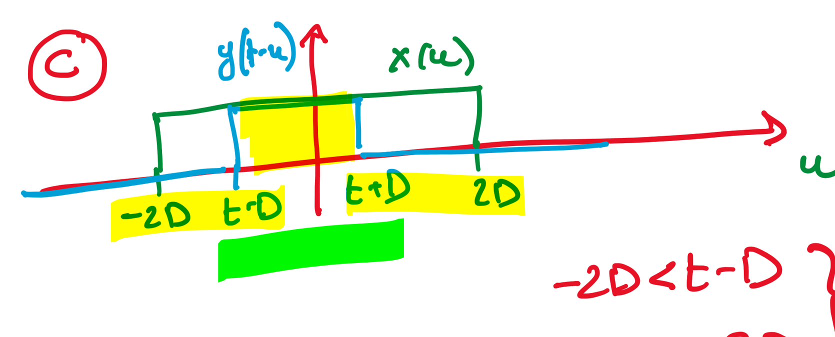
(B)




$t-D < -2D < t+D \longrightarrow -3D < t < -D$

$$z(t) = \int_{-2D}^{t+D} 1\, du = t+D-(-2D) = t+3D$$

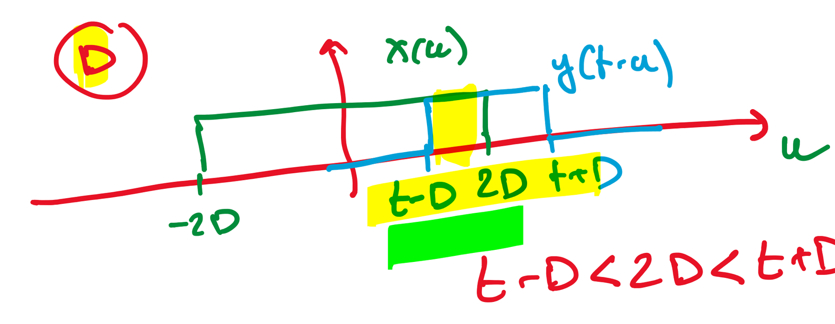
(C)




$\left.\begin{aligned} -2D &< t-D \\ t+D &< 2D \end{aligned}\right\} \longrightarrow -D < t < D$

$z(t) = 2D$

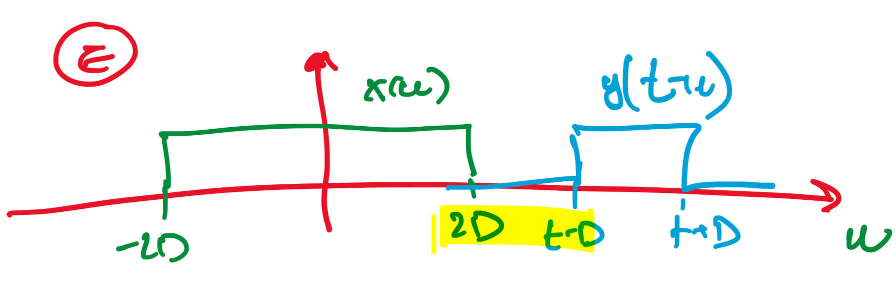
(D)




$t-D < 2D < t+D \longrightarrow D < t < 3D$

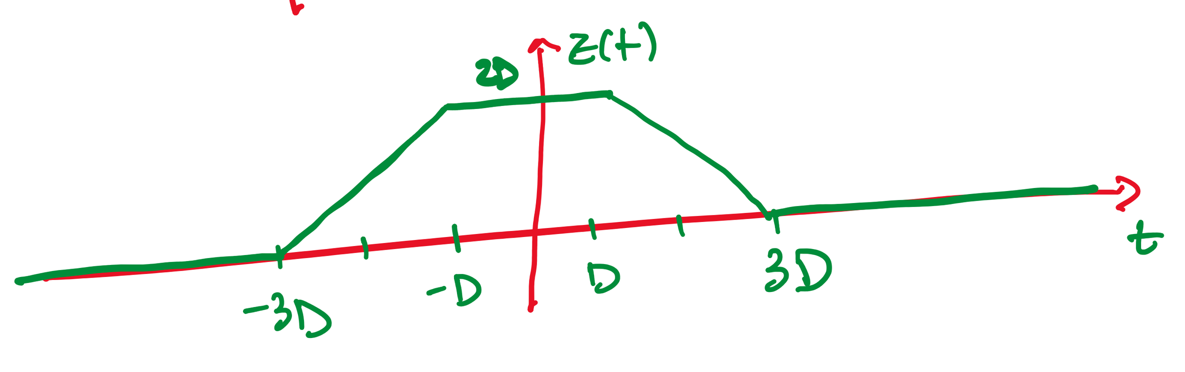
$$z(t) = \int_{t-D}^{2D} 1 \cdot du = 2D-(t-D) = 3D-t$$

(E)

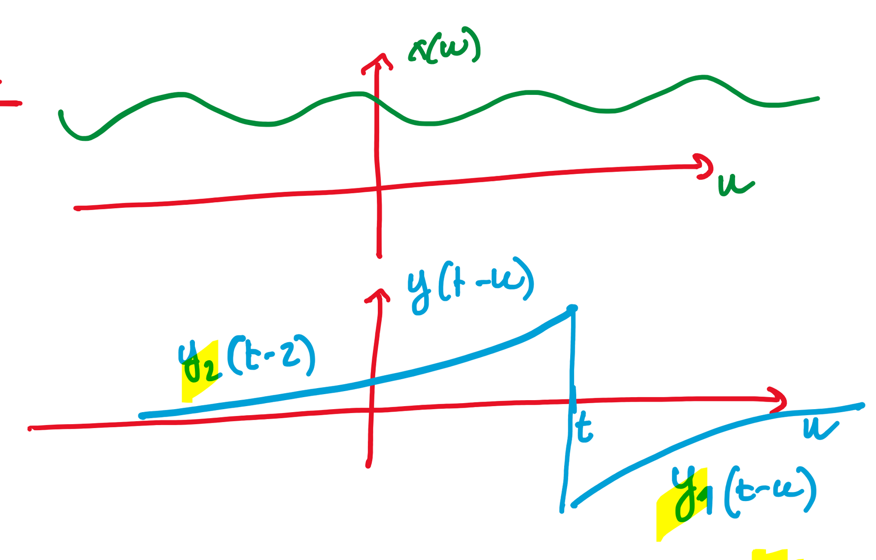



$2D < t-D \longrightarrow t > 3D \quad z(t) = 0$

$$z(t) = \begin{cases} 0 & t < -3D \\ 3D+t & -3D < t < -D \\ 2D & -D < t < D \\ 3D-t & D < t < 3D \\ 0 & t > 3D \end{cases}$$




**NOTA**




$$z(t) = \int_{-\infty}^{+\infty} x(u)\, y(t-u)\, du = \int_{-\infty}^{t} x(u)\, y_2(t-u)\, du + \int_{t}^{+\infty} x(u)\, y_1(t-u)\, du$$

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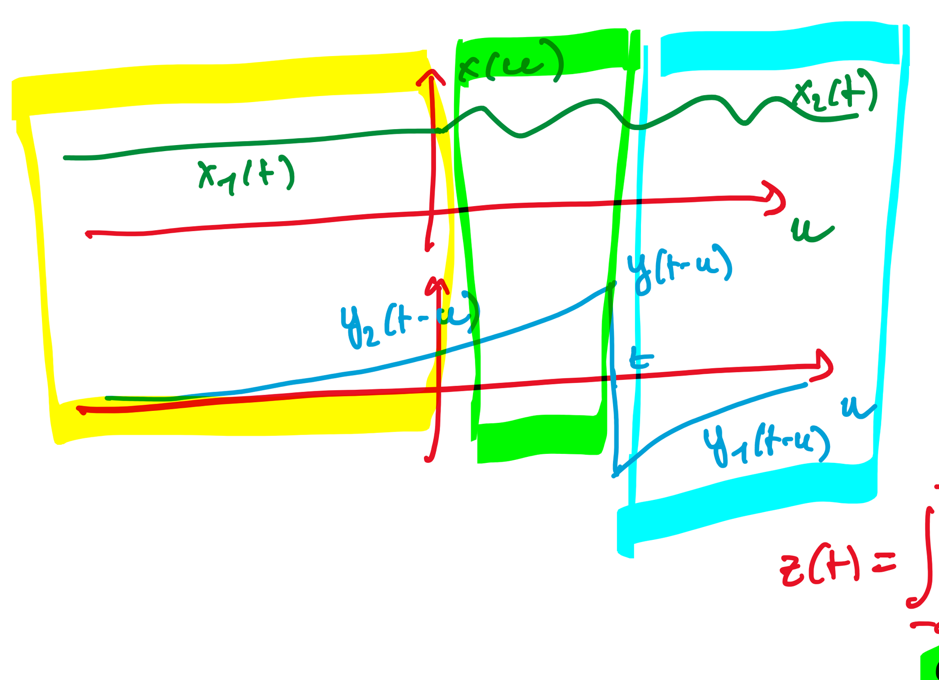
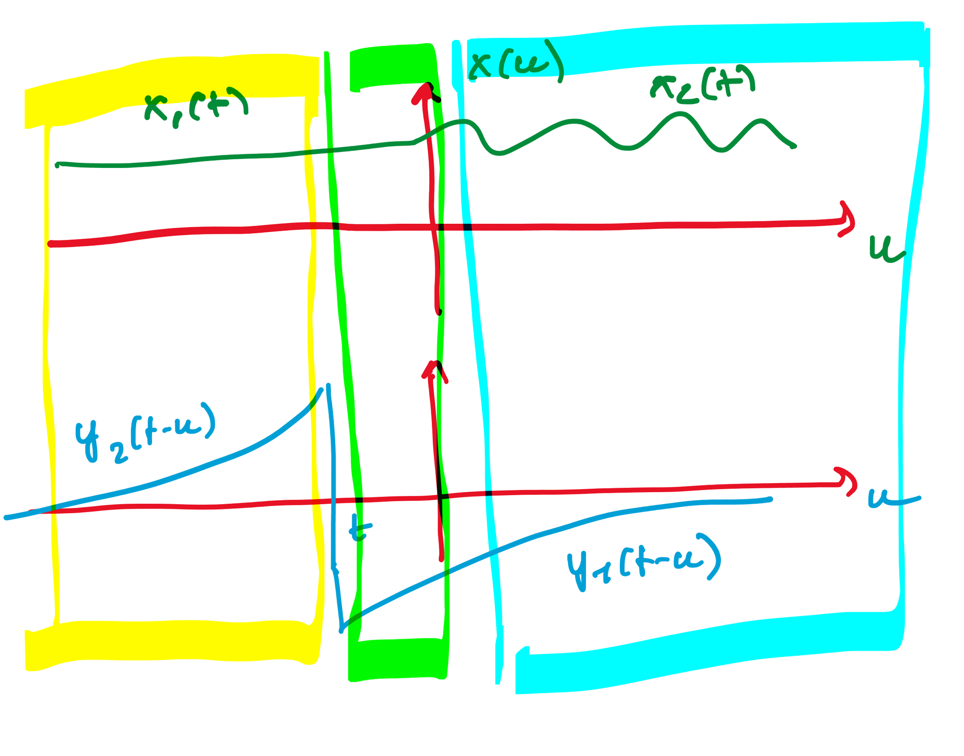



$t > 0$

$$z(t) = \int_{-\infty}^{+\infty} x(u)\, y(t-u)\, du$$

$$= \int_{-\infty}^{0} x_1(t)\, y_2(t-u)\, du + \int_{0}^{t} x_2(t)\, y_2(t-u)\, du + \int_{t}^{+\infty} x_2(t)\, y_1(t-u)\, du$$




$t < 0$

$$z(t) = \int_{-\infty}^{t} x_1(u)\, y_2(t-u)\, du + \int_{t}^{0} x_1(u)\, y_1(t-u)\, du + \int_{0}^{\infty} x_2(u)\, y_1(t-u)\, du$$