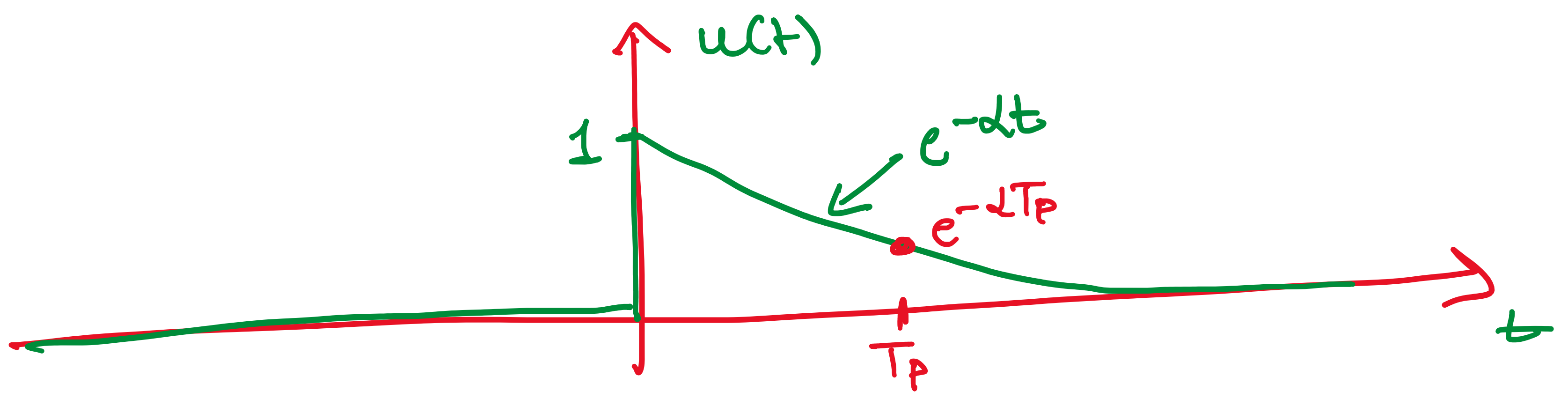
Le06

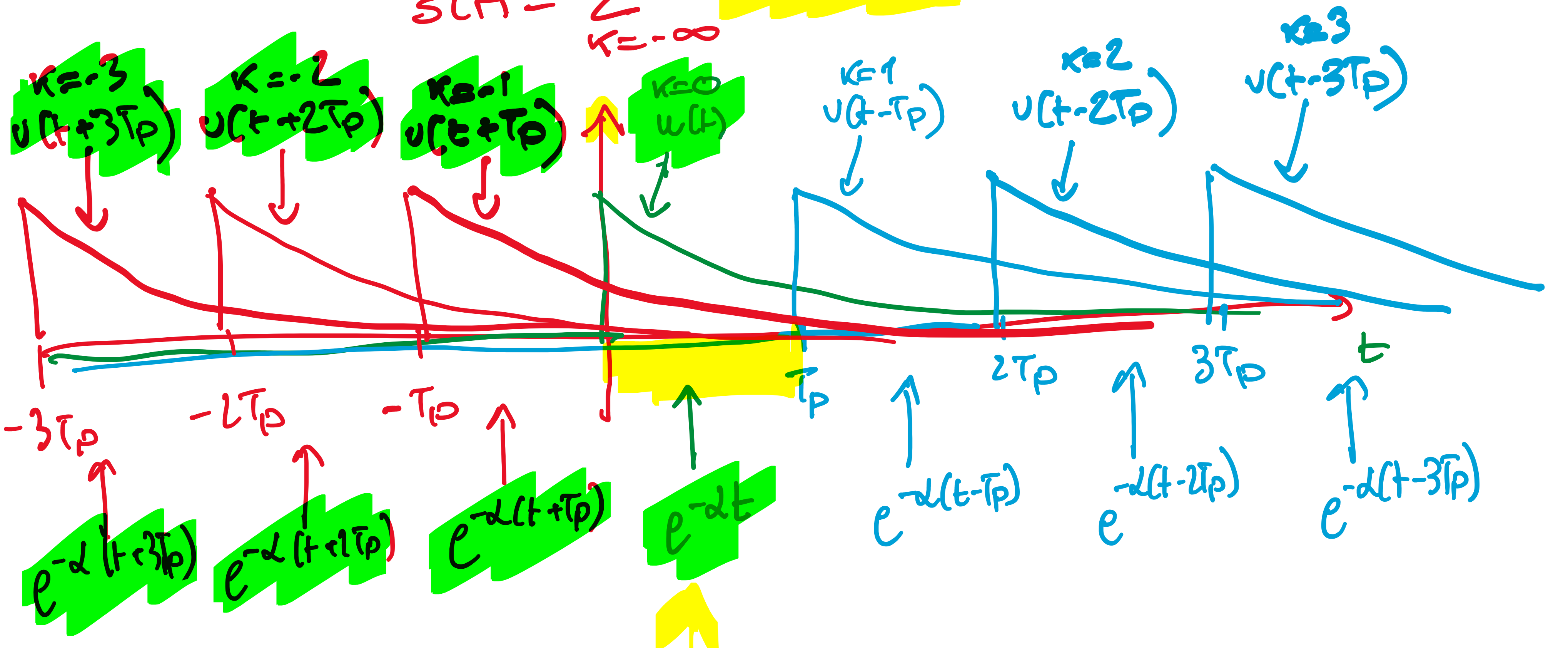
Friday, 7 March 2025 08:51

ES 1 TROVARE E DISEGNARE $s(t) = \text{rep}_{T_P}\, u(t)$

CON $u(t) = 1(t)\, e^{-\alpha t}$ $\quad \alpha > 0$



$$s(t) = \sum_{k=-\infty}^{+\infty} u(t - kT_P)$$



CI CONCENTRIAMO SUL PERIODO $(0, T_P)$

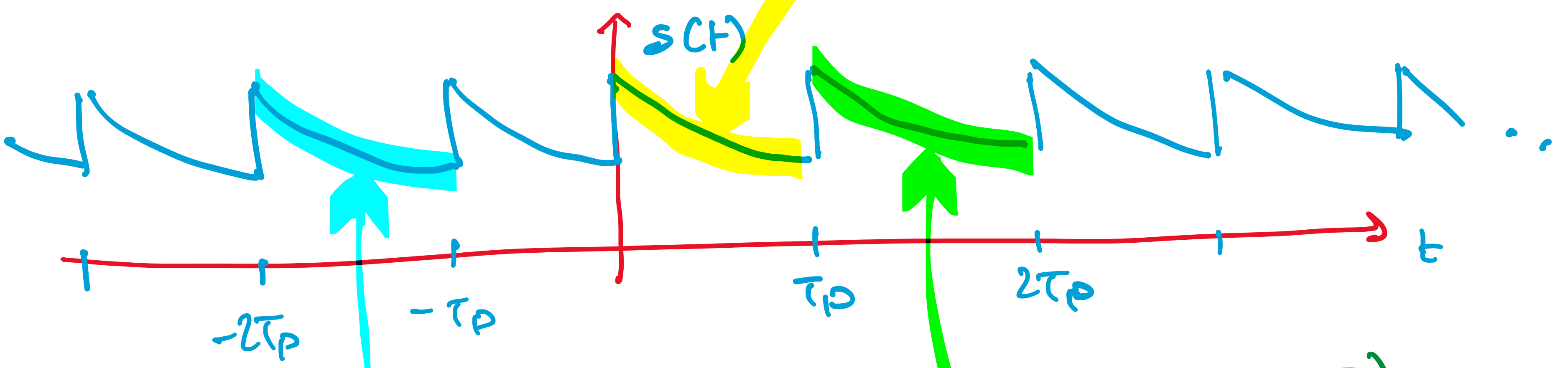
PER $t \in (0, T_P) \rightarrow s(t) = \sum_{k=-\infty}^{+\infty \to 0} u(t-kT_P)\, e^{-\alpha(t-kT_P)}$

$$= \sum_{k=-\infty}^{0} e^{-\alpha t}\, e^{\alpha T_P k}$$

$m = -k$

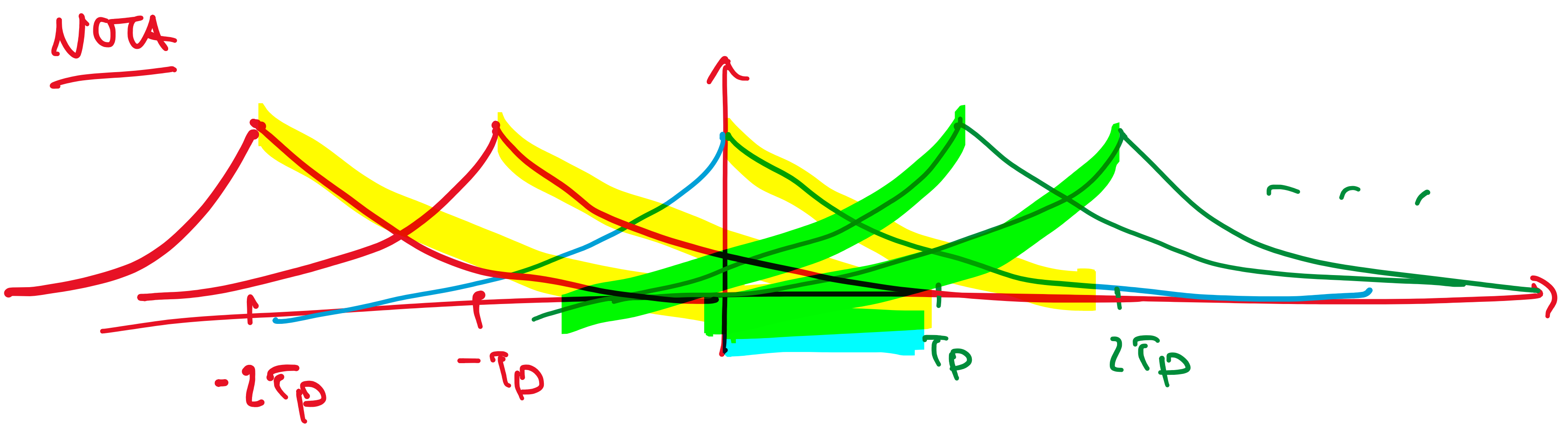
$$= e^{-\alpha t} \sum_{m=0}^{+\infty} \left(e^{-\alpha T_P}\right)^m$$

$$= e^{-\alpha t} \cdot \frac{1}{1 - e^{-\alpha T_P}} = v(t)$$

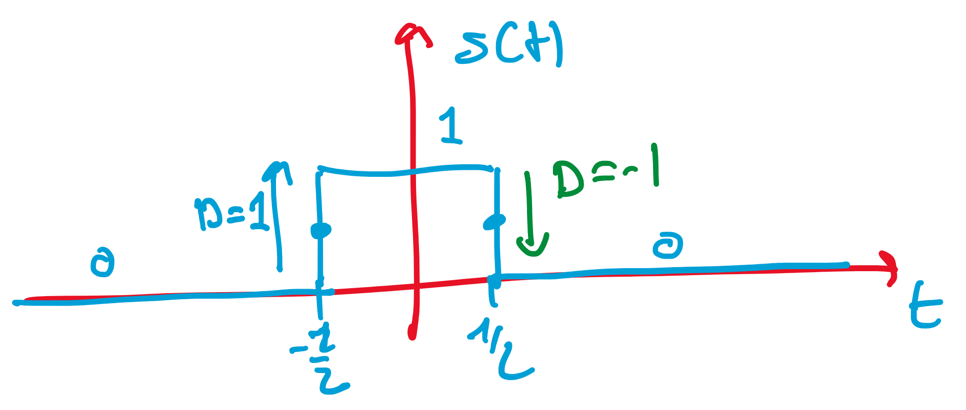


$$v(t+2T_P) = \frac{e^{-\alpha(t+2T_P)}}{1 - e^{-\alpha T_P}} \qquad v(t-T_P) = \frac{e^{-\alpha(t-T_P)}}{1 - e^{-\alpha T_P}}$$

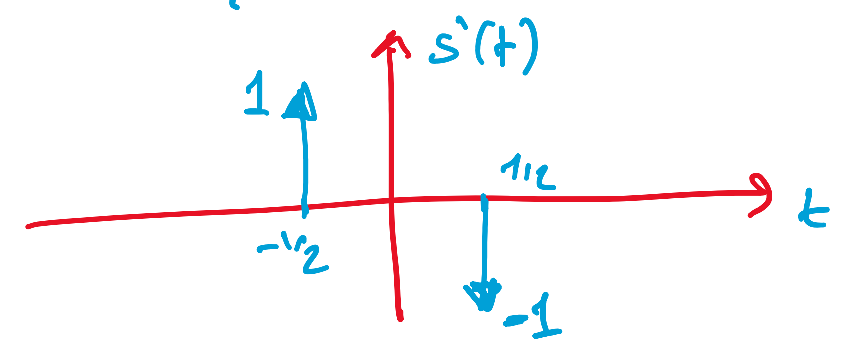
NOTA



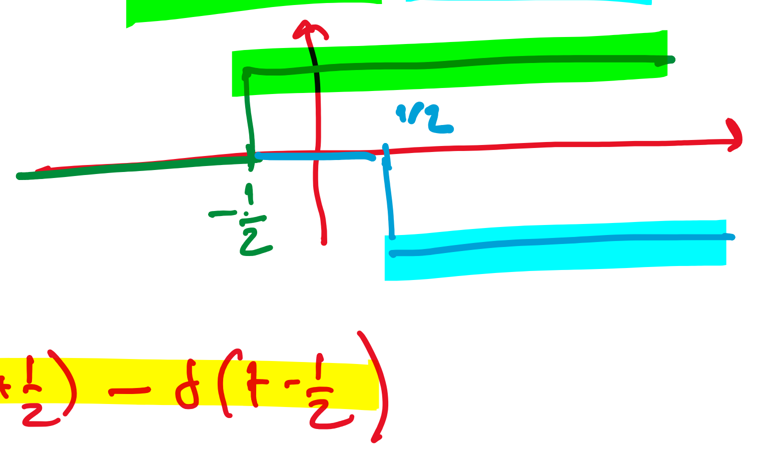
ES 2 TROVARE LA DERIVATA (GENERALIZZATA) DI $\text{rect}(t)$



$$s'(t) = 1 \cdot \delta\left(t + \tfrac{1}{2}\right) - 1\, \delta\left(t - \tfrac{1}{2}\right)$$

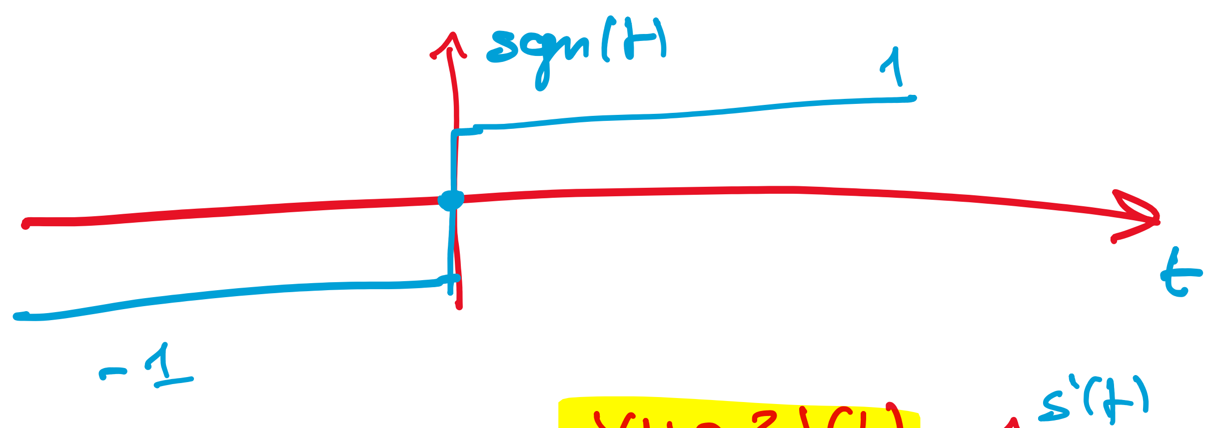


$$s(t) = \text{rect}(t) = 1\left(t + \tfrac{1}{2}\right) - 1\left(t - \tfrac{1}{2}\right)$$

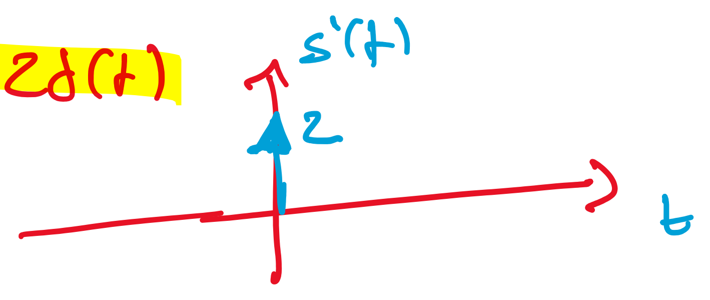


$$s'(t) = \delta\left(t + \tfrac{1}{2}\right) - \delta\left(t - \tfrac{1}{2}\right)$$

ES 3 TROVARE $s'(t)$ PER $s(t) = \text{sgn}(t)$

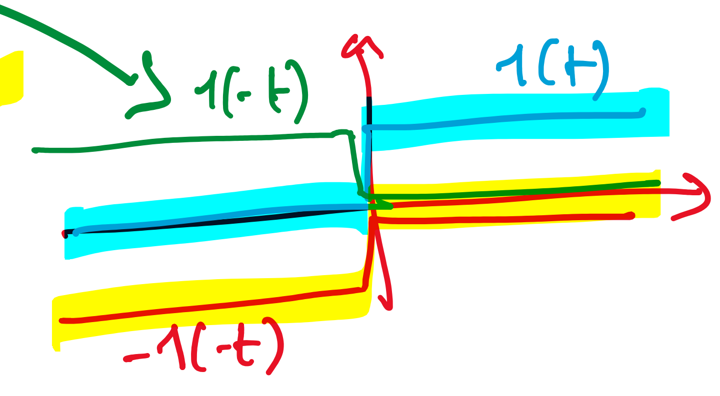


$$s'(t) = 2\delta(t)$$



$$s(t) = 1(t) - 1(-t)$$

$$s'(t) = \delta(t) + \delta(-t) = 2\delta(t)$$



$$s(t) = -1 + 2 \cdot 1(t)$$

$$s'(t) = 0 + 2 \cdot \delta(t)$$