Le04

Monday, 3 March 2025 10:29

$s(t) = A\cos(2\pi f_0 t + \varphi_0)$ $f_0 \neq 0$ $A \in \mathbb{R}$

$m_s = 0$

$P_s = \frac{A^2}{2}$

$s(t) = A e^{j2\pi f_0 t}$ $f_0 \neq 0$ $A \in \mathbb{C}$

$m_s = 0$

$P_s = |A|^2$

$s(t) = \sum_{k=1}^{K} A_k e^{j2\pi f_k t} + A_0$ con f_k TUTTE DIVERSE $f_k \neq 0$

$m_s = 0 + A_0$

$P_s = \sum_{k=1}^{K} |A_k|^2 + |A_0|^2$

$s(t) = \sum_{k=1}^{K} A_k \underbrace{\cos(2\pi f_k t + \varphi_k)}_{\frac{1}{2} e^{j(2\pi f_k t + \varphi_k)} + \frac{1}{2} e^{-j(2\pi f_k t + \varphi_k)}}$ con f_k TUTTE DIVERSE $f_k \neq 0$ $f_k > 0$

$m_s = 0$

$P_s = \sum_{k=1}^{K} \frac{A_k^2}{2}$

$$\left(\frac{A_k}{2} e^{j\varphi_k}\right) \cdot \underset{\substack{\uparrow \\ f_k}}{e^{j2\pi f_k t}} + \left(\frac{A_k}{2} e^{-j\varphi_k}\right) \cdot \underset{\substack{\uparrow \\ -f_k}}{e^{-j2\pi f_k t}}$$

$$P_s = \sum_{k=1}^{K} \frac{|A_k|^2}{4} + \frac{|A_k|^2}{4} = \sum_{k=1}^{K} \frac{|A_k|^2}{2}$$

NOTA

$\cos\left(\frac{\pi}{16} n\right)$

$2\pi \cdot f_0 T$ $f_0 T = \frac{\pi/16}{2\pi} = \frac{1}{32}$

PERIODO $N = 32$

$\cos(\theta_0 n)$ $f_0 T = \frac{\theta_0}{2\pi}$ SE RAZIONALE ALLORA E' PERIODICO

$\sin\left(\frac{7}{5}\pi n\right)$ $\theta_0 = \frac{7}{5}\pi$ $\frac{\theta_0}{2\pi} = \frac{7}{10}$

$N = 10$

$\cos(2n)$ $\theta_0 = 2$ $\frac{\theta_0}{2\pi} = \frac{1}{\pi}$

NON PERIODICO!

NOTA 2

$$\cos\Big(\underbrace{\tfrac{16}{3}\pi}_{\theta_0} n\Big) = \cos\left(\tfrac{16}{3}\pi n - 2\pi n\right)$$

$$= \cos\Big(\underbrace{\tfrac{10}{3}\pi}_{\theta_0 - 2\pi} n\Big)$$

$$= \cos\left(\tfrac{10}{3}\pi n - 2\pi n\right)$$

$$= \cos\Big(\underbrace{\tfrac{4}{3}\pi}_{\theta_0 - 4\pi} n\Big)$$

$$= \cos\left(\tfrac{4}{3}\pi n - 2\pi n\right)$$

$$= \cos\Big(\underbrace{-\tfrac{2}{3}\pi}_{\theta_0 - 6\pi} n\Big)$$

$$= \cos\left(\tfrac{2}{3}\pi n\right)$$

RIPORTARSI A

$\theta_0 \in [0, 2\pi)$

O

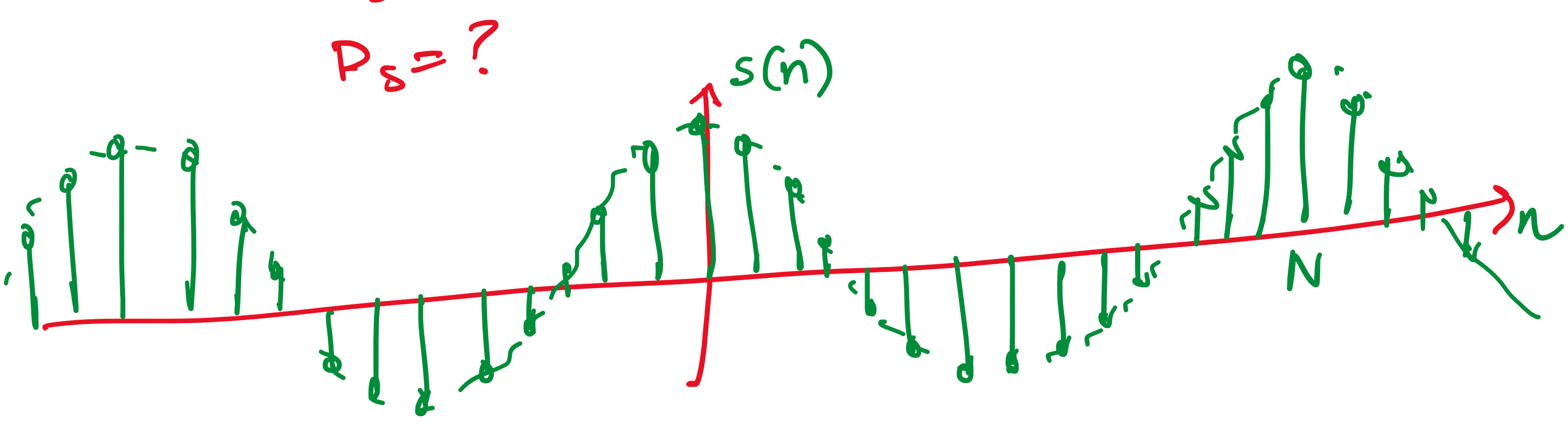
$\theta_0 \in [-\pi, \pi)$

ES 1 $s(n) = A\cos(2\pi f_0 n T)$ PERIODICO N

OVVERO CON $f_0 T \cdot N = K$ INTERO

$m_s = ?$

$P_s = ?$

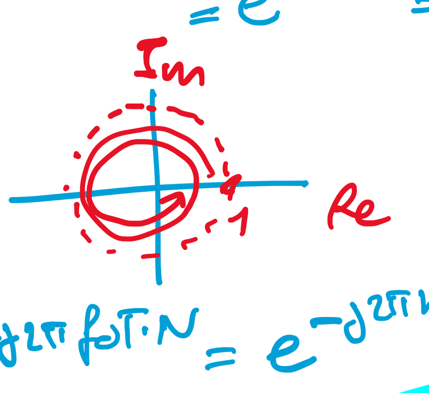


$$A_s(N) = \sum_{n=0}^{N-1} A\cos(2\pi f_0 n T)$$

$$= \sum_{n=0}^{N-1} \frac{A}{2} \underbrace{e^{j2\pi f_0 n T}}_{(e^{j2\pi f_0 T})^n = a^n} + \frac{A}{2} \underbrace{e^{-j2\pi f_0 n T}}_{(e^{-j2\pi f_0 T})^n = b^n}$$

$$= \sum_{n=0}^{N-1} \frac{A}{2} \frac{1 - a^N}{1 - a} + \frac{A}{2} \frac{1 - b^N}{1 - b} = 0$$

$a^N = e^{j2\pi f_0 T \cdot N} = e^{j2\pi \cdot K} = 1$



$b^N = e^{-j2\pi f_0 T \cdot N} = e^{-j2\pi K} = 1$

$m_s = 0$

$$|s(n)|^2 = s^2(n) = A^2 \cos^2(2\pi f_0 n T)$$

$$= \frac{A^2}{2} + \frac{A^2}{2} \cos(2\pi 2 f_0 n T)$$

$P_s = \frac{A^2}{2}$

ES 2 $s(n) = e^{j2\pi f_0 n T}$ con $f_0 T$ generico, NON NECESSARIAMENTE PERIODICO

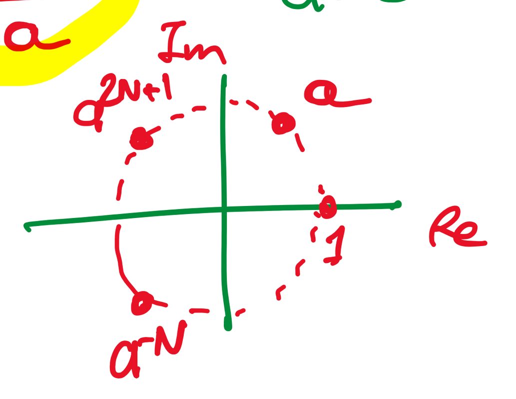
$= e^{j\theta_0 n}$

$$m_s = \lim_{N \to \infty} \frac{1}{1+2N} \sum_{n=-N}^{N} \cancel{s(n)}\ \cancel{e^{j\theta_0 n}}\ \underbrace{(e^{j\theta_0})^n}_{a^n} \quad a = e^{j\theta_0}$$

$$= \lim_{N \to \infty} \frac{a^{-N}}{1+2N} \sum_{m=0}^{2N} \cancel{a^{m-N}}\ a^m$$

$n = m - N$

$$= \lim_{N \to \infty} \frac{a^{-N}}{1+2N} \frac{1 - a^{2N+1}}{1 - a} \quad a = e^{j\theta_0}$$



$m_s = 0$

$|s(n)|^2 = |e^{j\theta_0 n}|^2 = 1$

$P_s = 1$