Systems Laboratory, Spring 2025

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notes

- welcome to the course!
- on this side of this document you will find notes that accompany the text typically visualized in class
- these notes are meant to convey the messages that are not displayed in the text on the side, and basically constitute what the teacher intends to say in class

Table of Contents I

notes

- this is the table of contents of this document; each section corresponds to a specific part of the course

what is an impulse response

notes

Contents map

developed content units	taxonomy levels
Dirac delta	u1, e1
impulse response	u1, e1

prerequisite content units	taxonomy levels
superposition principle	u1, e1
LTI ODE	u1, e1

notes

Main ILO of sub-module "what is an impulse response"

Describe what the impulse response of an LTI system is in practice

notes

- by the end of this module you shall be able to do this

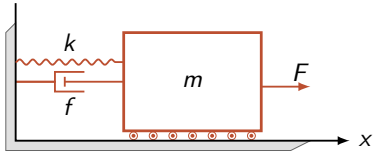
Impulse response ↔ superposition principle ↔ LTI system

talking about the impulse response of a nonlinear system is such a big mistake that may make you fail the exam on the spot

notes

- not joking here; we will see in this module that the concept is a direct consequence, and not having understood that an impulse response is meaningful only for LTI systems indicates a complete misunderstanding of the basis of the course

Practical example: spring-mass system

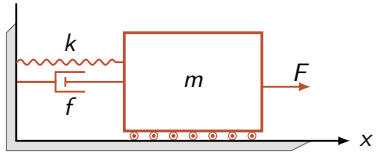


- output = position
- input = force (in Newtons)

notes

- In the mass-spring-damper system the impulse is a quick push (force times time) applied to the mass. The impulse response is the displacement of the mass over time after the push. The units of the impulse response are m/Ns.

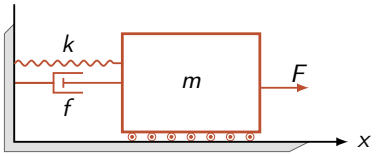
What if I push the cart with a force of 1 Newton?



notes

- note that here the position is x, but also $y = x$

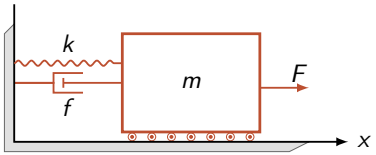
What if I push the cart with a force of 0.5 Newtons?

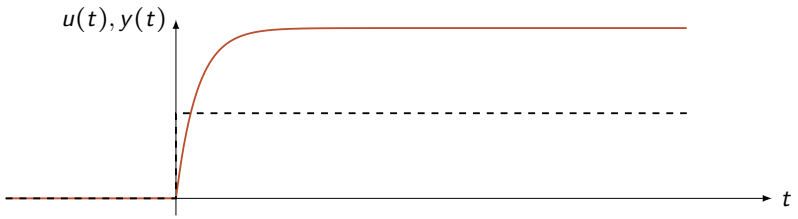


notes

- note that here the position is x, but also $y = x$

What if I push the cart with a force of 2 Newtons?

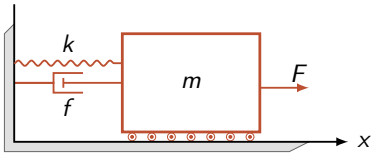


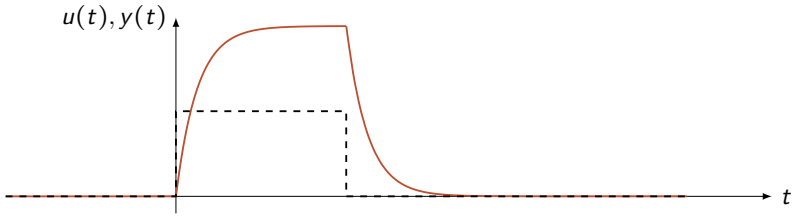


notes

- note that here the position is x, but also $y = x$

What if I push the cart with a force of 2 Newtons for 2 seconds?

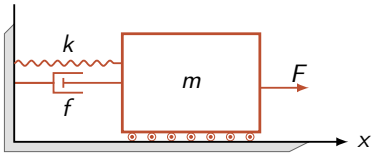


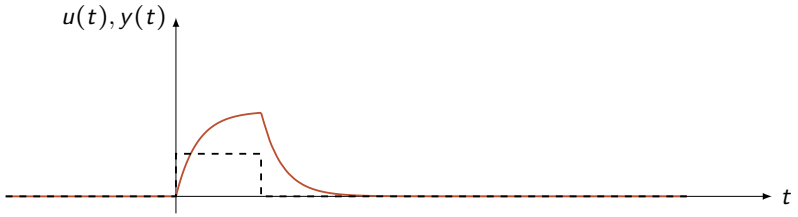


notes

- note that here the position is x, but also $y = x$

What if I push the cart with a force of 1 Newton for 1 second?

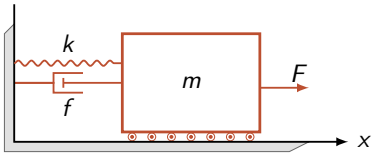


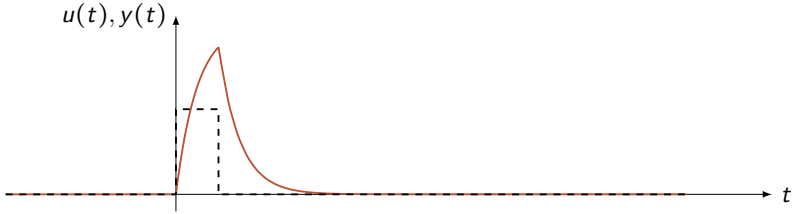


notes

- note that here the position is x, but also $y = x$

What if I push the cart with a force of 2 Newtons for 0.5 seconds?

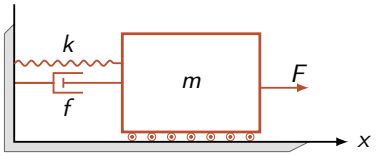


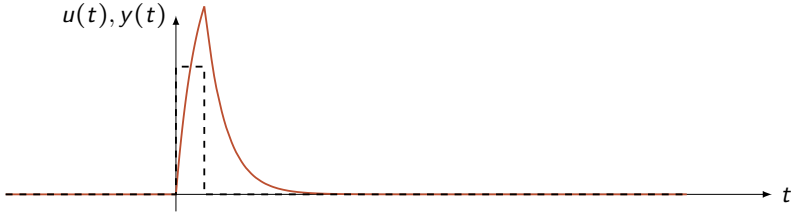


notes

- note that here the position is x, but also $y = x$

What if I push the cart with a force of 3 Newtons for 1/3 of a second?

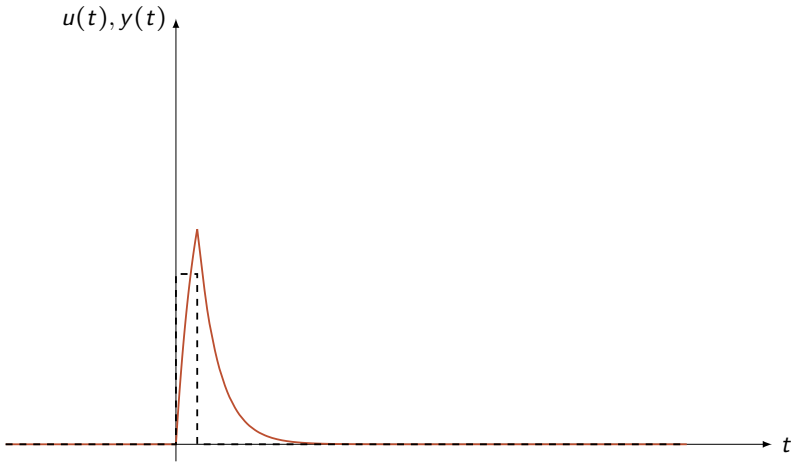




notes

- note that here the position is x, but also $y = x$

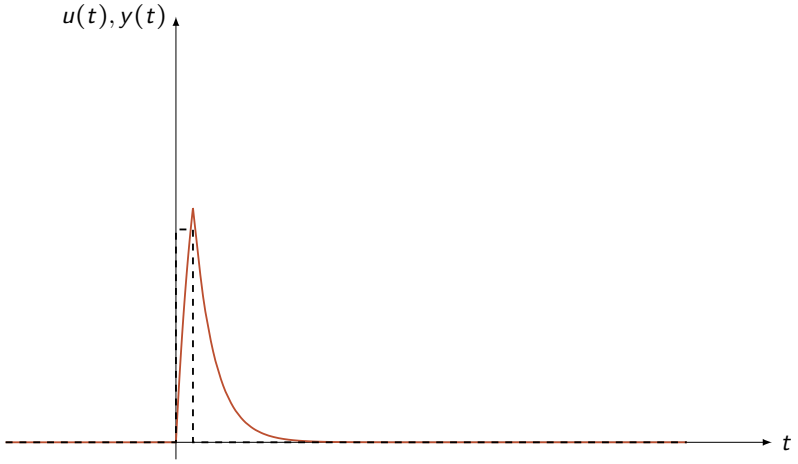
4 Newtons for 1/4 of a second?



notes

- note that here the position is x, but also $y = x$

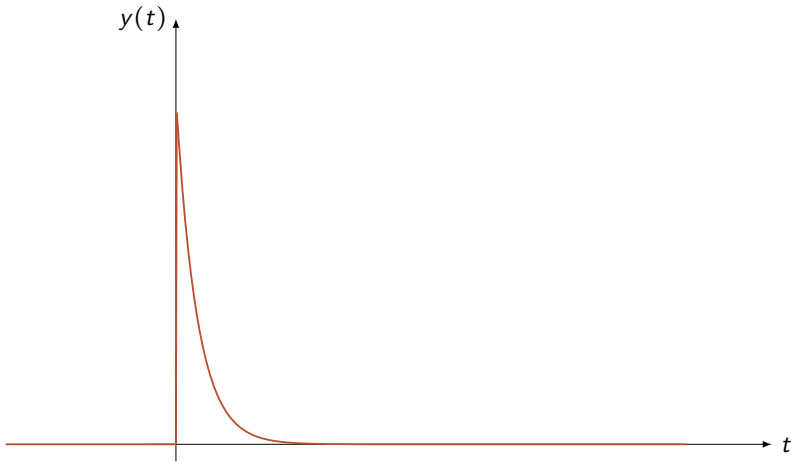
5 Newtons for 1/5 of a second?



notes

- note that here the position is x, but also $y = x$

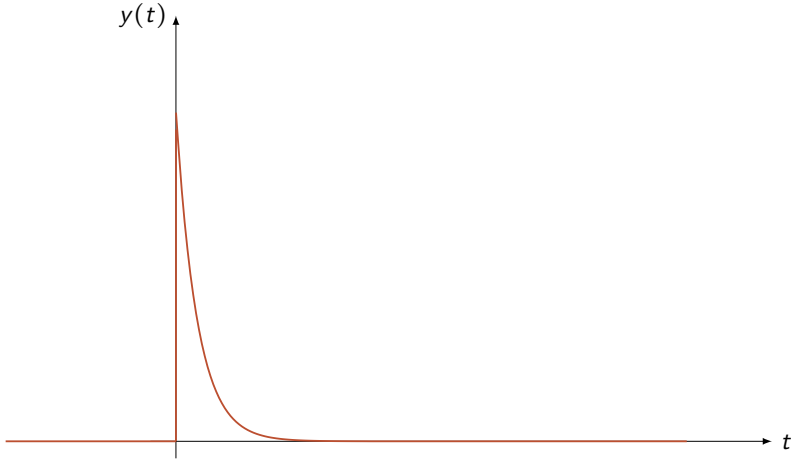
100 Newtons for 1/100 of a second?



notes

- here we do not plot u, since it is out of scale

1000 Newtons for 1/1000 of a second?

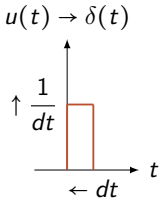


notes

- here we do not plot u, since it is out of scale

Where are we going with the input signal?

↦ Dirac's delta, i.e., pushing an unitary mass within an infinitesimal space



notes

- let's introduce this signal here
- note how for dt that goes to zero the signal becomes a sort of spike
- see also `https://en.wikipedia.org/wiki/Dirac_delta_function`

ESSENTIAL POINT

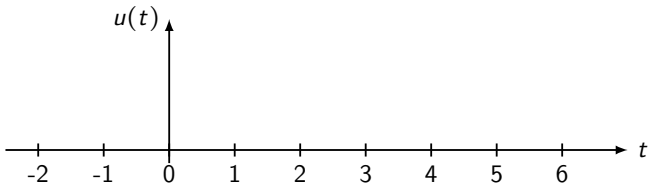
this game has sense only because the system is assumed to be LTI

notes

- otherwise we would not be guaranteed that we were having this sort of well defined limit behavior that we may explicitly compute by opportunely modifying the step response. If the system is not LTI, the step response and the impulse response are not connected, and this (as will be clear later on) implies we cannot use the response to the impulse as a template for computing all the potential responses that the system may give for any generic input

Discussion

Where would you draw $\delta(t-4)$?

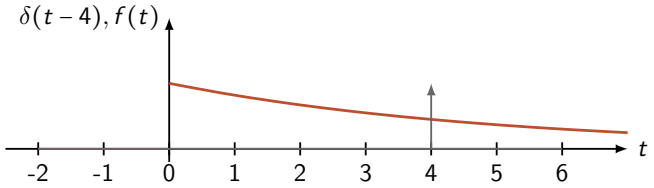


notes

- for this the spike is in $t = 10$ (because the spike is where the argument is zero, and in this case the argument is $t - 10$)

Discussion

$$\int_{-\infty}^{+\infty} f(\tau)\delta(\tau-4)d\tau = ?$$



notes

- it will be $f(10)$ because the δ has infinite mass in 10 and thus the product of the two functions is everywhere 0 but in $f(10)$
- when within an integral, δ thus somehow reveals that function within the integral in a specific point

Summarizing

Describe what the impulse response of an LTI system is in practice

- an opportune limit behavior of a transformation of the step response of a LTI system
- it is though a transformation that makes sense only if the system is LTI

notes

- you should now be able to do this, following the pseudo-algorithm in the itemized list

Most important python code for this sub-module

notes

Important libraries / methods

- `https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.impulse.html`
- `https://python-control.readthedocs.io/en/latest/generated/control.impulse_response.html`

notes

- with these methods you can compute the impulse response of a LTI system

Self-assessment material

notes

Question 1

What is the impulse response of an LTI system?

Potential answers:

I: **(wrong)** The output of the system when the input is a sinusoidal function.
II: **(wrong)** The output of the system when the input is a ramp function.
III: **(wrong)** The output of the system when the input is a step function.
IV: **(correct)** The output of the system when the input is a Dirac delta function.
V: **(wrong)** I do not know.

Solution 1:

The impulse response of an LTI system is the output of the system when the input is a Dirac delta function. This response characterizes the system's behavior and can be used to predict its output for any arbitrary input.

notes

- see the associated solution(s), if compiled with that ones :)

Question 2

Why is the impulse response meaningful only for LTI systems?

Potential answers:

I: **(correct)** Because the impulse response is a direct consequence of the superposition principle, which applies only to LTI systems.
II: **(wrong)** Because nonlinear systems do not respond to impulses.
III: **(wrong)** Because the impulse response is too complex to compute for nonlinear systems.
IV: **(wrong)** Because nonlinear systems have infinite impulse responses.
V: **(wrong)** I do not know.

Solution 1:

The impulse response is meaningful only for LTI systems because it relies on the superposition principle, which is a fundamental property of LTI systems. Nonlinear systems do not satisfy this principle, making the concept of an impulse response invalid for them.

notes

- see the associated solution(s), if compiled with that ones :)

Question 3

What happens to the mass-spring-damper system when the input force is a Dirac delta function?

Potential answers:

I: **(wrong)** The mass oscillates indefinitely without damping.
II: **(correct)** The mass exhibits a transient response that decays over time due to damping.
III: **(wrong)** The mass remains stationary because the impulse is too short to affect it.
IV: **(wrong)** The mass moves with constant velocity.
V: **(wrong)** I do not know.

Solution 1:

- what is an impulse response 4

When the input force is a Dirac delta function, the mass-spring-damper system exhibits a transient response that decays over time due to the damping effect. This response is the impulse response of the system.

notes

- see the associated solution(s), if compiled with that ones :)

Question 4

What is the integral of $f(\tau)\delta(\tau - 4)$ from $-\infty$ to $+\infty$?

Potential answers:

I: **(wrong)** $\int_{-\infty}^{+\infty} f(\tau)d\tau$
II: **(wrong)** 0
III: **(correct)** $f(4)$
IV: **(wrong)** $\delta(4)$
V: **(wrong)** I do not know.

Solution 1:

The integral of $f(\tau)\delta(\tau - 4)$ from $-\infty$ to $+\infty$ is $f(4)$. This is because the Dirac delta function "samples" the function $f(\tau)$ at $\tau = 4$.

- what is an impulse response 5

notes

- see the associated solution(s), if compiled with that ones :)

Recap of sub-module "what is an impulse response"

- impulse responses are directly connected to step responses
- actually this connection is valid only if the system is LTI

notes

- the most important remarks from this sub-module are these ones