Visualizing systems with block schemes

Contents map

developed content units	**taxonomy levels**
block scheme	u1, e1

prerequisite content units	**taxonomy levels**
ODEs	u1, e1

Main ILO of sub-module "Visualizing systems with block schemes"

Explain the purpose of block diagrams in control systems by comparing their industrial and analytical applications

Construct block diagram representations of first-order differential equations by identifying and connecting appropriate functional blocks

Distinguish between static and dynamic blocks by analyzing their mathematical representations and memory requirements

Simplify complex block diagrams to single equivalent blocks by applying series, parallel, and feedback reduction rules

Interpret feedback loops in block diagrams by relating their presence to system equations and dynamic behavior

Roadmap

- the most common block schemes
- first order systems as block schemes

Block diagrams - why?

- used very often in companies
- aid visualization *(until a certain complexity is reached. . .)*
- enable “drag & drop” way of programming

- in this course, primarily used for interpretations

Block diagrams - why? Part 2

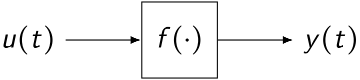
Convenient operation in control systems analysis

- step 1: identify single-input single-output subsets (blocks)
- step 2: represent the overall system as an interconnection of such subsets

Static block (a.k.a. memoryless block)

= representation of a static (i.e., instantaneous) relationship between input and output

$$y(t) = f\big(u(t)\big)$$



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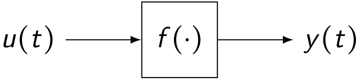
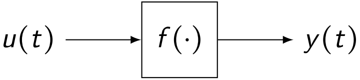


they can be linear or nonlinear, depending on $f(\cdot)$

Dynamic block (a.k.a. block with memory)

= representation of a dynamic relationship between input and output (i.e., such that the output $y(t)$ at time t does not depend only on the input $u(t)$ at the same time t, but also on its behavior at different times – potentially not only past)

$$y(t) = f\left(u, y\right)$$



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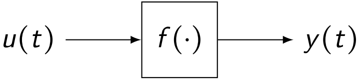
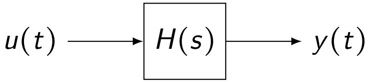


they can be linear or nonlinear, depending on $f(\cdot)$

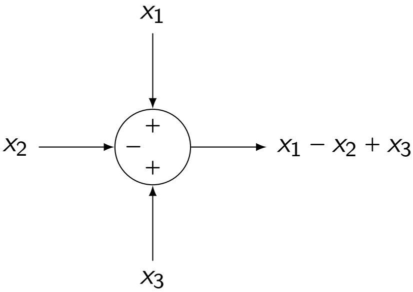
Example of dynamic block: transfer function

$$y(t) = y_\ell(t) + u * h(t)$$

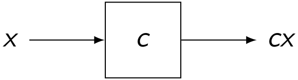


Operations with the block schemes

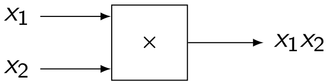
Most common block diagrams - sum of n signals



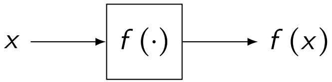
Most common block diagrams - multiplication for a constant



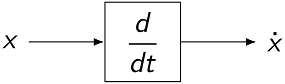
Most common block diagrams - multiplication of two signals



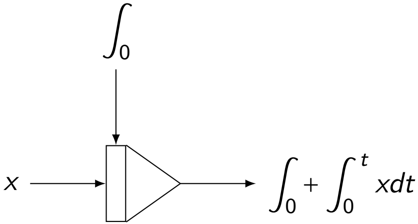
Most common block diagrams - generic functions



Most common block diagrams - derivatives



Most common block diagrams - integrals

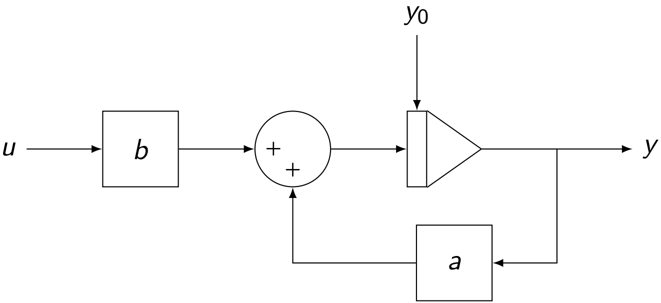


Discussion: how do we represent a first order differential equation with a block scheme?

$$\dot{y} = ay + bu$$

The solution

$$\dot{y} = ay + bu$$



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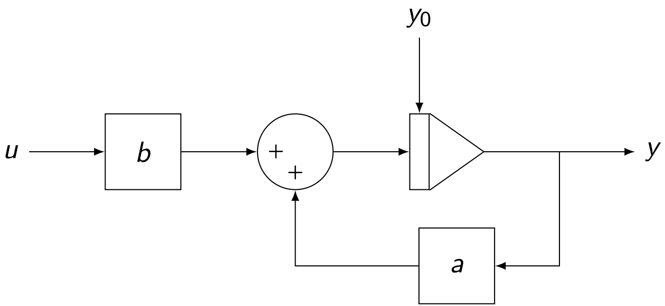
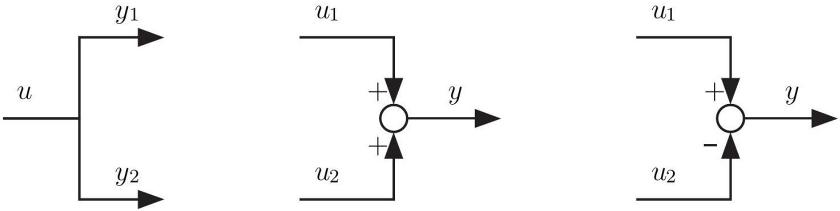


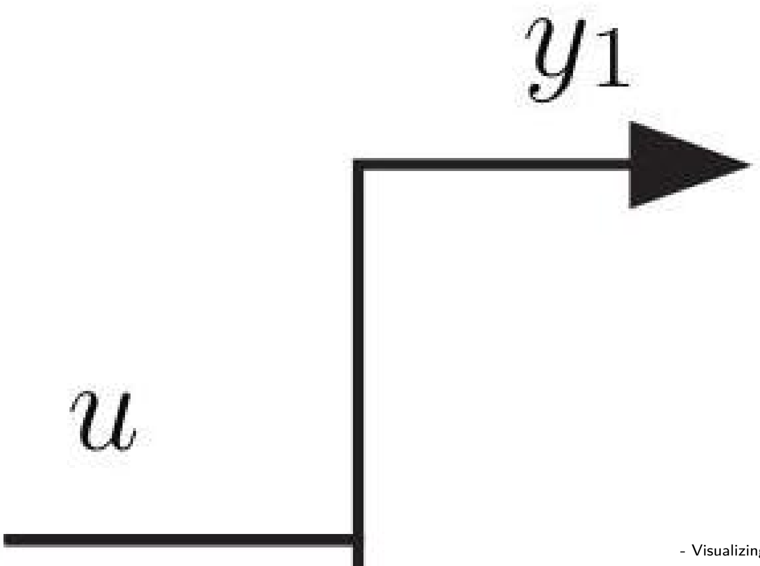
Discussion: do you note the presence of a feedback loop?

Interconnections

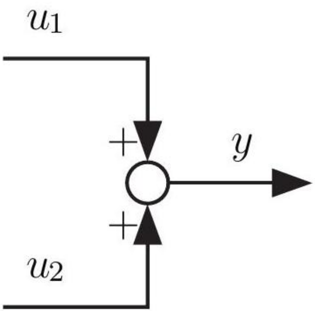
Branching points and summing junctions

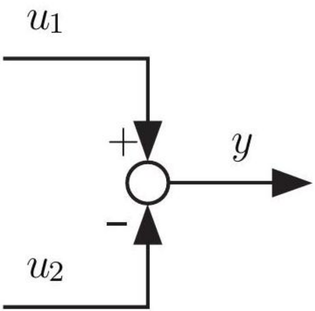


Branching points



Junctions





= blocks with two inputs $u_1(t)$, $u_2(t)$ and one output $y(t)$, in which

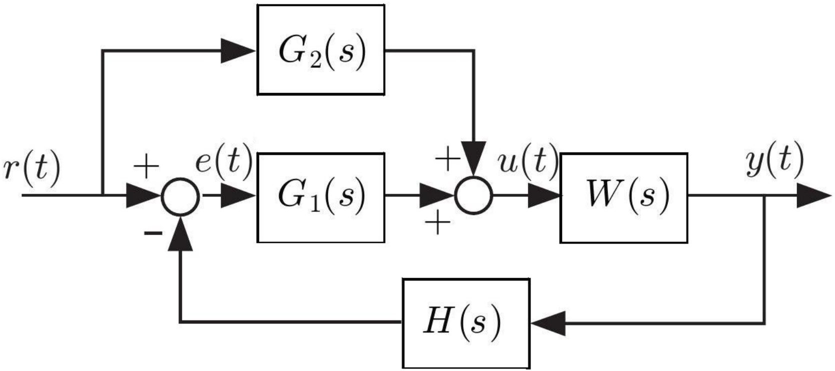
$$y(t) = u_1(t) + u_2(t)$$

or

$$y(t) = u_1(t) - u_2(t)$$

Reduction of block schemes to a single block

Example



complicated $\implies$ can we rewrite it in a simpler way?

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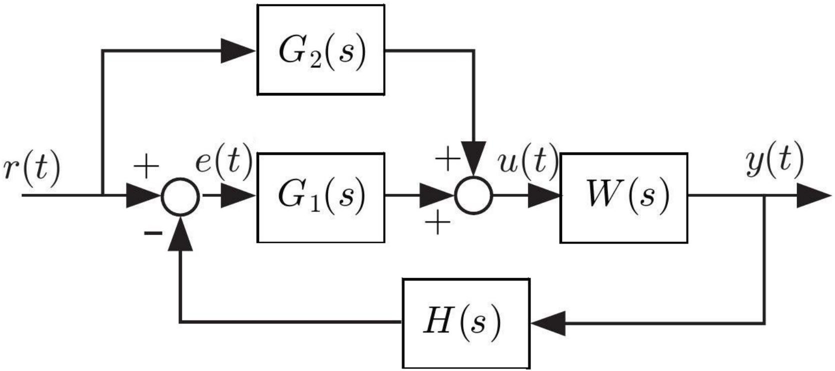

complicated $\implies$ can we rewrite it in a simpler way? Algebraic relations:

$$Y(s) = W(s)U(s)$$

$$U(s) = G_1(s)E(s) + G_2(s)R(s)$$

$$E(s) = R(s) - H(s)Y(s)$$

Example, part 2

$$
\begin{aligned}
Y(s) &= W(s)U(s) \\
U(s) &= G_1(s)E(s) + G_2(s)R(s) \\
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\end{aligned}
$$

implies

$$
\begin{aligned}
Y(s) &= W(s)\left[G_1(s)E(s) + G_2(s)R(s)\right] \\
&= W(s)\left[G_1(s)(R(s) - H(s)Y(s)) + G_2(s)R(s)\right]
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$$Y(s) = W(s)\left[G_1(s)E(s) + G_2(s)R(s)\right]$$
$$= W(s)\left[G_1(s)(R(s) - H(s)Y(s)) + G_2(s)R(s)\right]$$

multiplying and grouping terms in $Y(s)$ and in $R(s)$ separately we obtain

$$(1 + W(s)G_1(s)H(s))\,Y(s) = W(s)\,(G_1(s) + G_2(s))\,R(s)$$

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$$
\left(1 + W(s)G_1(s)H(s)\right)Y(s) = W(s)\left(G_1(s) + G_2(s)\right)R(s)
$$

therefore

$$
\frac{Y(s)}{R(s)} = \frac{W(s)\left(G_1(s) + G_2(s)\right)}{1 + W(s)G_1(s)H(s)}
$$

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and so the relationship between the input \$r(t)\$ and the output \$y(t)\$ can be described by a single block with transfer function

Example, part 2

$$Y(s) = W(s)U(s)$$
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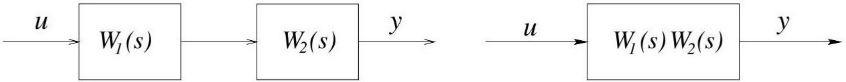
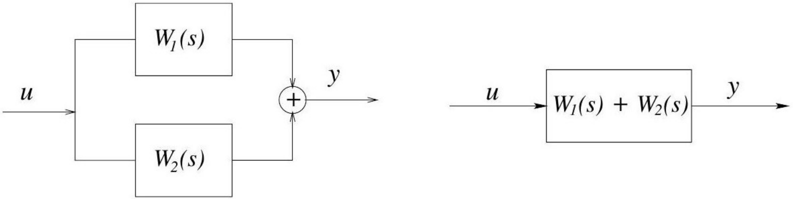
Reduction of block diagrams to a single block

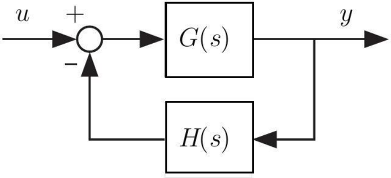
Specific case: series or parallel blocks

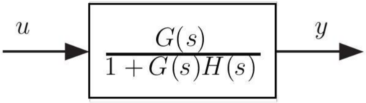




Reduction of block diagrams to a single block

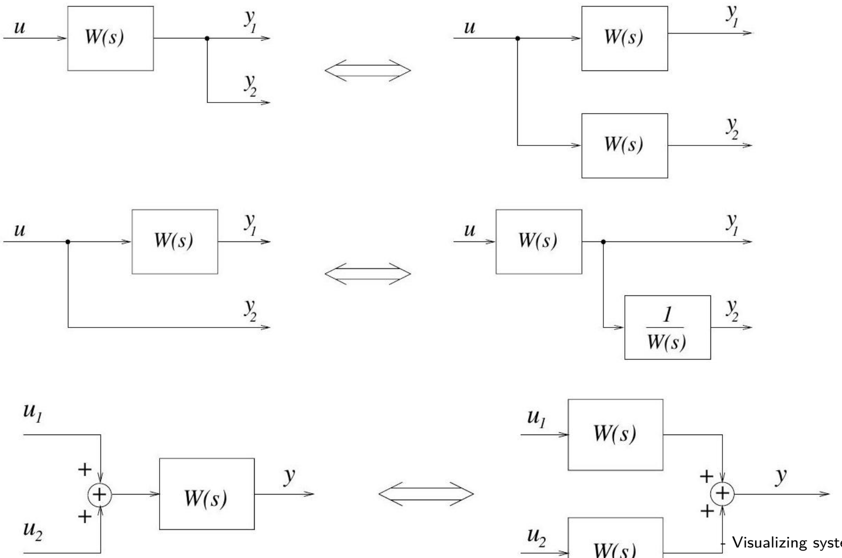
Specific case: feedback





Reduction of block diagrams to a single block

Specific case: moving blocks around



Suggested videos introducing Simulink

- https://www.youtube.com/watch?v=pFICO_syIIs (10 minutes)
- https://www.youtube.com/watch?v=QIAxyLchf4k (50 minutes)

Summarizing

Explain the purpose of block diagrams in control systems by comparing their industrial and analytical applications

Construct block diagram representations of first-order differential equations by identifying and connecting appropriate functional blocks

Distinguish between static and dynamic blocks by analyzing their mathematical representations and memory requirements

Simplify complex block diagrams to single equivalent blocks by applying series, parallel, and feedback reduction rules

Interpret feedback loops in block diagrams by relating their presence to system equations and dynamic behavior

Most important python code for this sub-module

The control package

Example:

- # Series connection
  series = control.series(G1, G2)
- # Parallel connection
  parallel = control.parallel(G1, G2)
- # Feedback connection feedback = control.feedback(G1, G2)

Self-assessment material

Question 1

In a block diagram representation of a first-order differential equation $\dot{y} = ay + bu$, why does the feedback path emerge?

Potential answers:

I: Because we need to implement a controller
II: Because the output y affects its own rate of change $\dot{y}$
III: Because all dynamic systems require feedback
IV: Because it represents the input signal $u(t)$
V: I do not know

Question 2

What is the fundamental difference between a branching point and a summing junction in block diagrams?

Potential answers:

I: Branching points perform calculations while summing junctions don't
II: Branching points duplicate signals while summing junctions combine them
III: Summing junctions can only handle two inputs while branching points can have many outputs
IV: Branching points require memory while summing junctions are memoryless
V: I do not know

Question 3

When reducing a complex block diagram to a single equivalent block, what does the denominator of the resulting transfer function typically represent?

Potential answers:

I: The gain of the input signal
II: The time delay of the system
III: The feedback characteristics of the system
IV: The nonlinearities in the system
V: I do not know

Question 4

Why does a dynamic block require memory while a static block doesn't?

Potential answers:

I: Because dynamic blocks are always digital implementations
II: Because dynamic blocks depend on past values of input/output
III: Because static blocks can only represent linear relationships
IV: Because dynamic blocks operate at higher frequencies
V: I do not know

Question 5

What is the conceptual reason why series-connected blocks can be reduced by multiplying their transfer functions?

Potential answers:

I: Because multiplication is commutative
II: Because it's an arbitrary convention
III: Because each block's output becomes the next block's input
IV: Because the Laplace transform requires it
V: I do not know

Recap of module "Visualizing systems with block schemes"

- block representations are alternative representations
- they enable graphical coding, that is used quite a lot in big companies

?