explain what BIBO stability means

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|---|---|
| unbounded signal | u1, e1 |
| bounded signal | u1, e1 |
| BIBO stability | u1, e1 |

| **prerequisite content units** | **taxonomy levels** |
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| RR | u1, e1 |

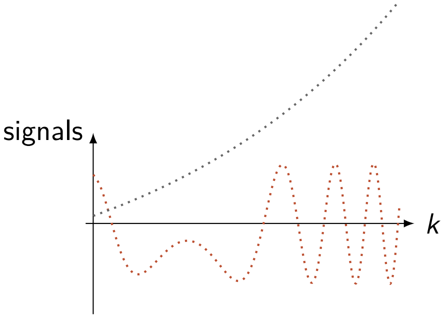
- explain what BIBO stability means 

# Main ILO of sub-module "explain what BIBO stability means"

**Graphically explain** the definition of BIBO (Bounded-Input Bounded-Output) stability and its connection to system behavior

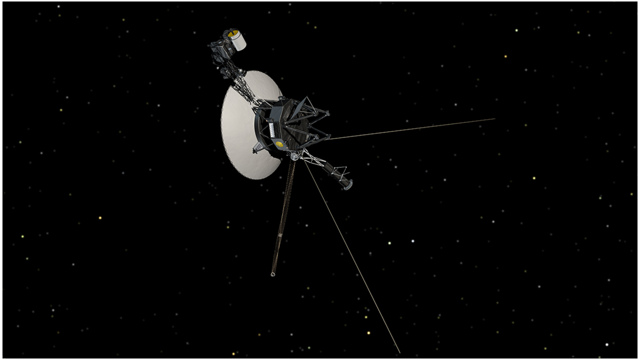
**Give examples** of systems that are BIBO stable or not, and motivate their properties with physical intuitions

# Definitions: bounded and unbounded signals



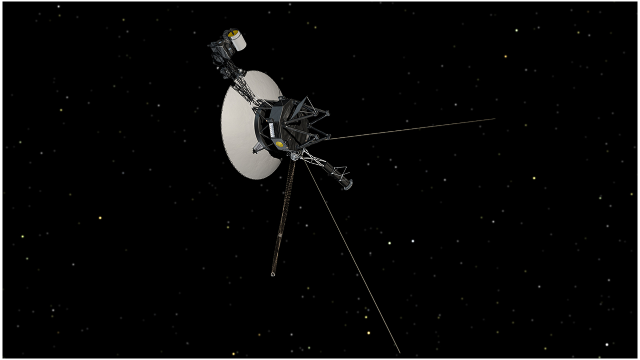

**bounded:** I can find an $M$ for which $|y[k]| < M$ for every $k$. **Unbounded** otherwise

# Intuition



simplified model = single integrator: $v^+ = v + bu$

# Intuition



simplified model = single integrator: $v^+ = v + bu$

$$u[k] = \text{const.} \neq 0 \implies \lim_{t \to +\infty} v[k] = ?$$

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# Interesting difference

$$v^+ = v + bu \quad \implies \quad \text{constant non null } u[k] \text{ causes } v[k] \text{ unbounded}$$

$$v^+ = av + bu, \ |a| < 1 \quad \implies \quad \text{constant non null } u[k] \text{ causes } v[k] \text{ bounded}$$

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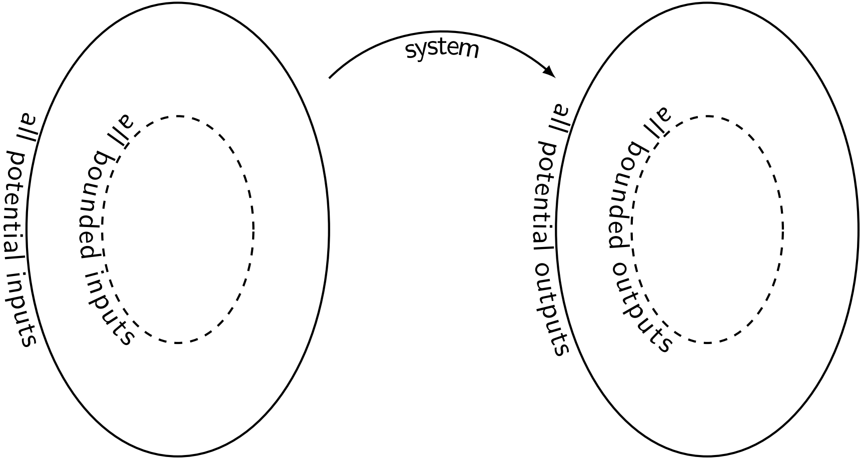
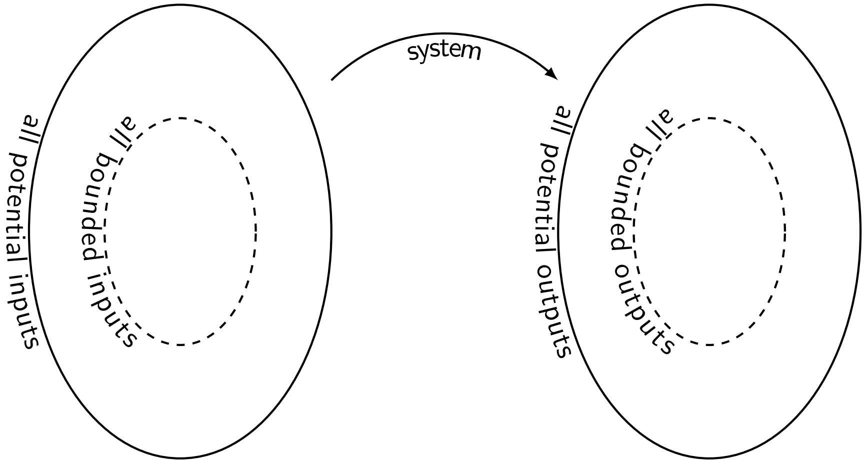
# Visualizing this with avocados

for $v^+ = v + bu$ some bounded $u[k]$'s lead to unbounded $v[k]$'s



# Visualizing this with avocados

for $v^+ = v + bu$ not all the bounded $u[k]$'s lead to unbounded $v[k]$'s, though

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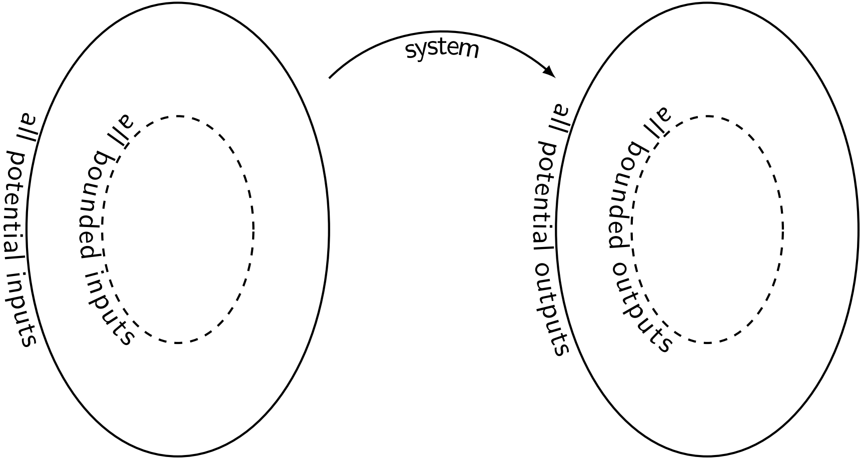
# Visualizing this with avocados

for $v^+ = av + bu$, $|a| < 1$ all the bounded $u[k]$'s lead to bounded $v[k]$'s



Important difference: the two systems have two different "stability" properties relative to how inputs get mapped into outputs!

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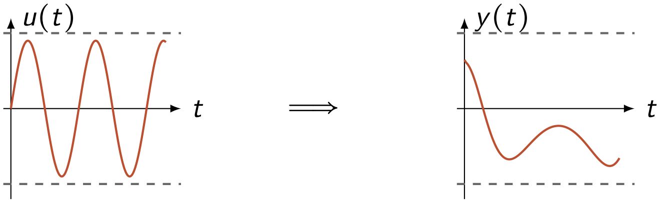
# Remark

- "Stability" referring to specific equilibria:
  - simply stable equilibrium
  - convergent equilibrium
  - asymptotically stable equilibrium

- "Stability" referring to specific systems:
  - Bounded Input Bounded Output (BIBO) stable systems (we are seeing this now)
  - Input to State Stable (ISS) systems (we will not see this in this module)

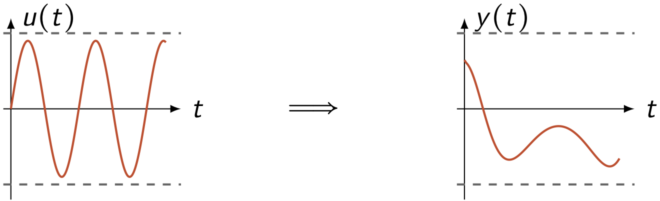
# Definition: BIBO stability

If $\boldsymbol{y}^{+} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$ is so that if $\boldsymbol{u}$ is bounded then $\boldsymbol{y}$ will be for sure bounded, then that system is said BIBO stable



# Definition: BIBO stability

If $\boldsymbol{y}^{+} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$ is so that there exists at least one $\boldsymbol{u}$ bounded for which the corresponding $\boldsymbol{y}$ will be unbounded, then that system is said **not** BIBO stable



# Example

Is $y^+ = 2y + u$ BIBO stable?

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# Example

Is $y^+ = yu$ BIBO stable?

# Example

Is $y^+ = u$ BIBO stable?

# Example

Is $y^+ = -y$ BIBO stable?

# Summarizing

**Graphically explain** the definition of BIBO (Bounded-Input Bounded-Output) stability and its connection to system behavior

**Give examples** of systems that are BIBO stable or not, and motivate their properties with physical intuitions

- if starting from the kernel of the input-avocado we are guaranteed to end up in the kernel of the output-avocado, then we have BIBO stability
- if not, not
- BIBO stability is a concept that relates to non-autonomous system - and it is not necessary that it it LTI, may also be nonlinear. But it needs to have at least one input

# Most important python code for this sub-module

# No dedicated python libraries for this ...

... but one can use the `control` library for checking the properties of the transfer function or impulse response of the system, if LTI of course

# Self-assessment material

# Question 1

Which of the following statements best describes BIBO (Bounded-Input Bounded-Output) stability?

**Potential answers:**

I: A system is BIBO stable if all outputs remain constant regardless of the input.
II: A system is BIBO stable if every bounded input leads to a bounded output.
III: A system is BIBO stable if it has at least one bounded output for an unbounded input.
IV: A system is BIBO stable if it is asymptotically stable.
V: I do not know

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# Question 2

Which of the following systems is NOT BIBO stable?

**Potential answers:**

I: $y^+ = -2y + u.$

II: $y^+ = y + u.$

III: $y^+ = -y + 3u.$

IV: $y^+ = -0.5y + u.$

V: I do not know

# Question 3

Which graphical interpretation correctly illustrates a system that is NOT BIBO stable?

**Potential answers:**

I: A system where all bounded inputs correspond to bounded outputs.
II: A system where at least one bounded input results in an unbounded output.
III: A system where all unbounded inputs lead to unbounded outputs.
IV: A system where the impulse response is always decreasing over time.
V: I do not know

# Question 4

A spacecraft is modeled as a single integrator $v^+ = v + bu$. What can be said about its BIBO stability?

**Potential answers:**

I: The system is NOT BIBO stable because a constant nonzero input leads to an unbounded velocity.

II: The system is BIBO stable because velocity is a smooth function of time.

III: The system is BIBO stable because acceleration remains bounded.

IV: The system is BIBO stable because it eventually reaches a steady-state velocity.

V: I do not know

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# Question 5

Can one define the BIBO properties of an autonomous system?

**Potential answers:**

I: No, BIBO stability is defined in terms of input-output behavior.
II: Yes, every system has a well-defined BIBO stability property.
III: Only if the system is linear and time-invariant.
IV: Yes, but only for discrete-time systems.
V: I do not know

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# Question 6

If a system has two inputs, can it be BIBO stable with respect to one input but not the other?

**Potential answers:**

I: Yes, different inputs can excite different dynamics in the system.
II: No, BIBO stability is a system-wide property.
III: Only if the system is nonlinear.
IV: Only for discrete-time systems.
V: I do not know

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# Question 7

BIBO stability properties are connected to equilibrium points, so if a system has three equilibria, do we need to analyze BIBO stability separately for each equilibrium?

**Potential answers:**

I: No, BIBO stability is an input-output property and does not depend on equilibria.
II: Yes, because each equilibrium defines a different stability region.
III: Only if the system is nonlinear.
IV: Only for continuous-time systems.
V: I do not know

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# Question 8

Is an integrator a BIBO stable system?

**Potential answers:**

I: No, an integrator accumulates input over time, leading to unbounded output for bounded input.
II: Yes, because the output remains predictable.
III: Only in discrete-time systems.
IV: Only if the initial condition is zero.
V: I do not know

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# Recap of sub-module "explain what BIBO stability means"

- BIBO stability means "a bounded input must imply a bounded output"
- it is a concept that in general it is disconnected to that of marginal stability / convergence of an equilibrium

?