

what is an impulse response

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| <u>prerequisite content units</u> | <u>taxonomy levels</u> |
|-----------------------------------|------------------------|
| superposition principle | u1, e1 |
| Euler backwards discretization | u1, e1 |
| LTI RR | u1, e1 |

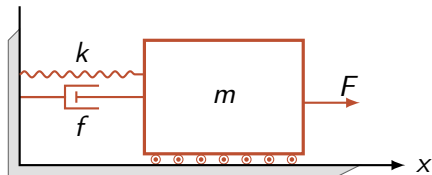
Main ILO of sub-module “what is an impulse response”

Describe what the impulse response of an LTI system is in practice

Impulse response \leftrightarrow superposition principle \leftrightarrow LTI system

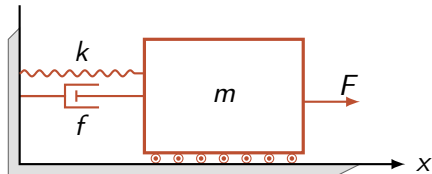
talking about the impulse response of a nonlinear system is such a big mistake that may make you fail the exam on the spot

Practical example: spring-mass system



- output = position
- input = force (in Newtons)

Discretizing the spring-mass system with a Euler backwards scheme

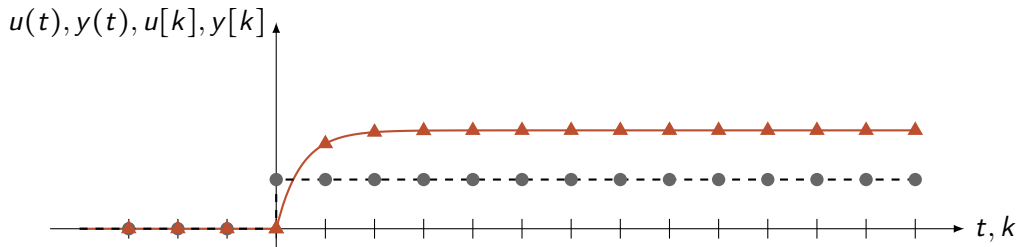
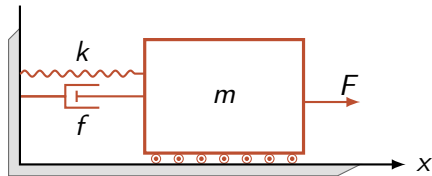


$$\text{Newton} \mapsto \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t)$$

becomes, with Euler,

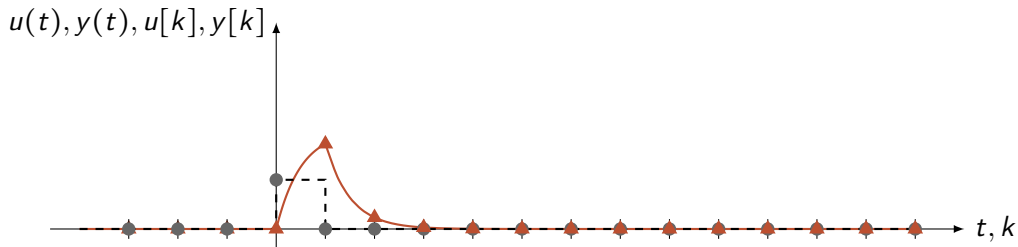
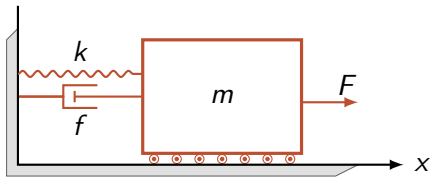
$$y^{++} + \alpha_1 y^+ + \alpha_0 y = \beta u$$

What does it mean to apply a control signal $u[k] = \text{step}$?

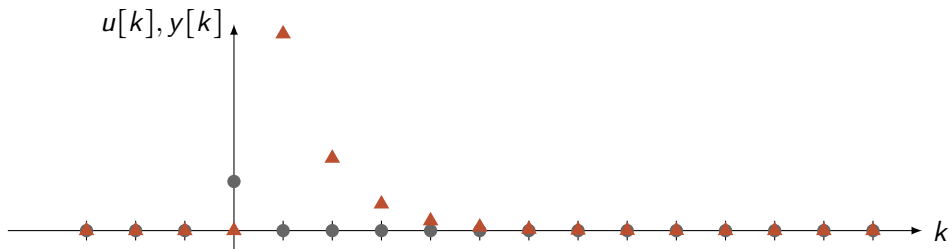


And $u[k] = 1$ only for $k = 0$, and 0 otherwise?

(i.e., $u[k] = \delta[k]$)

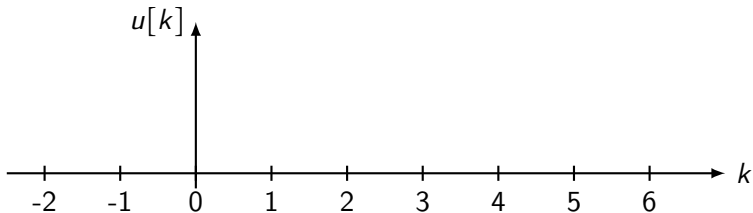


The impulse response for DT LTI systems



Discussion

Where would you draw $\delta[k - 4]$?



Summarizing

Describe what the impulse response of an LTI system is in practice

- an opportune behavior of the response of a LTI system to a discrete Dirac's delta

Most important python code for this sub-module

Important libraries / methods

- <https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.impulse.html>
- https://python-control.readthedocs.io/en/latest/generated/control.impulse_response.html

Self-assessment material

Question 1

What is the impulse response of an LTI system?

Potential answers:

- I: The output of the system when the input is a discrete Dirac's delta.
- II: The output of the system when the input is a step function.
- III: The output of the system when the input is a sinusoidal signal.
- IV: The output of the system when the input is a random signal.
- V: I do not know

Question 2

Why is the impulse response meaningful only for LTI systems?

Potential answers:

- I: Because nonlinear systems do not have outputs.
- II: Because the impulse response relies on the superposition principle, which is valid only for LTI systems.
- III: Because the impulse response is only defined for continuous-time systems.
- IV: Because the impulse response is too complex for nonlinear systems.
- V: I do not know

Question 3

What happens to the impulse response of a discrete-time LTI system if the input is $\delta[k - 4]$?

Potential answers:

- I: The impulse response becomes zero.
- II: The impulse response is shifted by 4 time units.
- III: The impulse response is scaled by a factor of 4.
- IV: The impulse response becomes nonlinear.
- V: I do not know

Question 4

What is the practical significance of the impulse response in analyzing LTI systems?

Potential answers:

- I: It allows us to ignore the system's initial conditions.
- II: It characterizes the system's behavior and can be used to determine the output for any input.
- III: It ensures the system response is always sinusoidal.
- IV: It makes the system response independent of the input.
- V: I do not know

Question 5

In the context of a spring-mass-damper system, what does the impulse response represent?

Potential answers:

- I: The steady-state position of the mass.
- II: The displacement of the mass over time after an instantaneous force is applied.
- III: The force required to keep the mass at rest.
- IV: The frequency of oscillation of the mass.
- V: I do not know

Recap of sub-module “what is an impulse response”

- impulse responses are directly connected to step responses
- actually this connection is valid only if the system is LTI

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