building and interpreting phase portraits for RRs

Contents map

developed content units	**taxonomy levels**
phase portrait	u1, e1

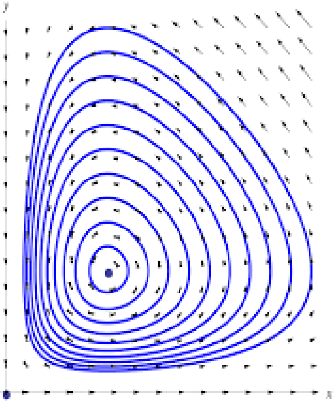
prerequisite content units	**taxonomy levels**
ODE	u1, e1
RR	u1, e1

Main ILO of sub-module “building and interpreting phase portraits for RRs”

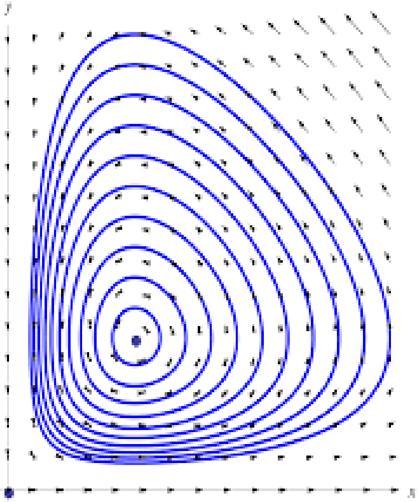
Construct and interpret phase portraits of first- and second-order autonomous RRs using qualitative analysis techniques

Starting with a CT example: a Lotka-Volterra model:

$$\begin{cases} \dot{y}_{\text{rabbits}} &= 0.4 \cdot y_{\text{rabbits}} - 0.5 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \\ \dot{y}_{\text{foxes}} &= -3 \cdot y_{\text{foxes}} + 0.7 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \end{cases}$$



Phase Portrait in CT = a graphical representation of the trajectories of a dynamical system in the state space



But what happens if we discretize the system?

$$\begin{cases} \dot{y}_{\text{rabbits}} &= \quad 0.4 \cdot y_{\text{rabbits}} - 0.5 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \\ \dot{y}_{\text{foxes}} &= -3 \cdot y_{\text{foxes}} + 0.7 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} y^{+}_{\text{rabbits}} &= \quad 1.3 \cdot y_{\text{rabbits}} - 0.7 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \\ y^{+}_{\text{foxes}} &= 0.8 \cdot y_{\text{foxes}} + 0.9 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \end{cases}$$

But what happens if we discretize the system?

$$\begin{cases} \dot{y}_{\mathrm{rabbits}} &= \ \ 0.4 \cdot y_{\mathrm{rabbits}} - 0.5 \cdot y_{\mathrm{rabbits}} \cdot y_{\mathrm{foxes}} \\ \dot{y}_{\mathrm{foxes}} &= -3 \cdot y_{\mathrm{foxes}} + 0.7 \cdot y_{\mathrm{rabbits}} \cdot y_{\mathrm{foxes}} \end{cases}$$

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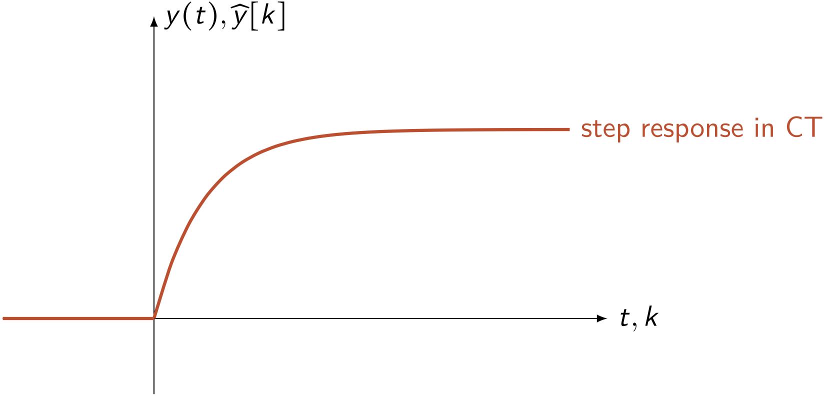
changing discretization time T will make the parameters change, and thus get different approximations

Important concept: as T decreases, DT discretizations become more and more accurate representations of the CT dynamics



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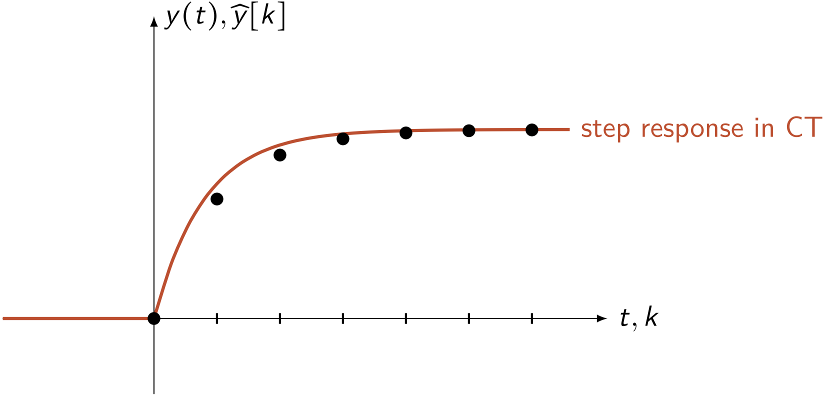

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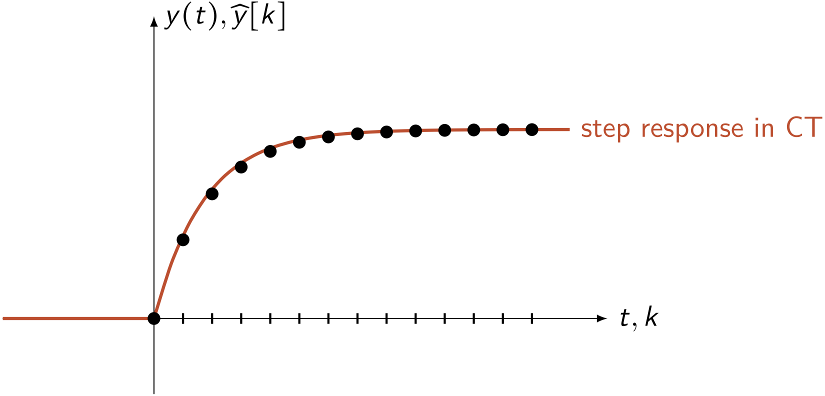
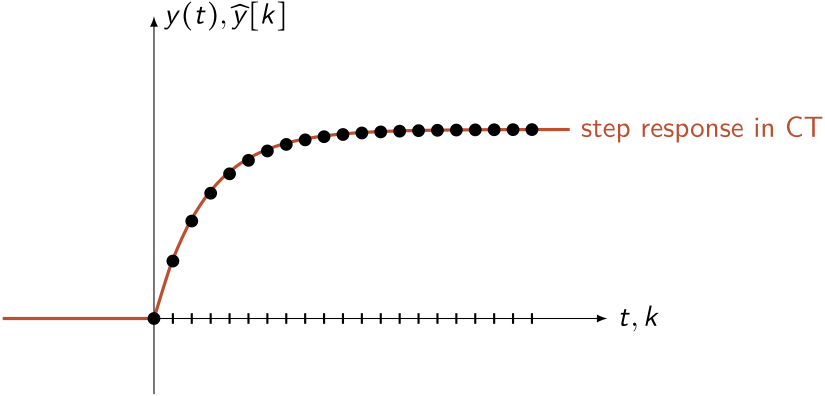


Important concept: as T decreases, DT discretizations become more and more accurate representations of the CT dynamics



Summarizing

Construct and interpret phase portraits of first- and second-order autonomous RRs using qualitative analysis techniques

- the plots are basically the same that one may get from the phase portraits relative to ODEs, but they are discrete versions of them
- the accuracy of the discretization depends on the sampling period, and thus one shall always be a bit wary of the results one get

Most important python code for this sub-module

Tutorial on how to plot phase portraits

`https://aleksandarhaber.com/`

`phase-portraits-of-state-space-models-and-differential-equations-in-python`

Self-assessment material

Question 1

What is the primary purpose of a phase portrait for a discrete-time system?

Potential answers:

I: To compute the exact solution of the system
II: To visualize the qualitative behavior of the system's trajectories in state space
III: To determine the numerical stability of the system
IV: To solve the system's differential equations analytically
V: I do not know

Question 2

What happens to the accuracy of a discrete-time system's phase portrait as the discretization time T decreases?

Potential answers:

I: The phase portrait becomes less accurate

II: The phase portrait becomes more accurate, converging to the continuous-time solution

III: The phase portrait remains unchanged

IV: The phase portrait becomes unstable

V: I do not know

Question 3

When discretizing the Lotka-Volterra model, what is the effect of changing the discretization time T?

Potential answers:

I: The system's equilibria change

II: The parameters of the discretized system change, leading to different approximations

III: The system becomes unstable

IV: The system's trajectories become chaotic

V: I do not know

Question 4

What does a stable equilibrium point in a discrete-time phase portrait indicate?

Potential answers:

I: Trajectories diverge away from the equilibrium point
II: Trajectories converge to the equilibrium point over time
III: The system exhibits periodic behavior
IV: The system becomes chaotic
V: I do not know

Question 5

What does a closed loop in a discrete-time phase portrait typically represent?

Potential answers:

I: A stable equilibrium point
II: An unstable equilibrium point
III: Periodic or quasi-periodic behavior
IV: Chaotic behavior
V: I do not know

Recap of sub-module "building and interpreting phase portraits for RRs"

- A phase portrait is a graphical representation of a dynamical systems trajectories in state space.
- Phase portraits provide qualitative insight into system behavior without requiring explicit solutions.
- First-order systems have a one-dimensional state space, while second-order systems require two dimensions, etc.
- The smaller the sampling period T, the closer the discrete-time phase portraits is to the one one would get from the continuous time version of the system

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