compute the equilibria of the system

# Contents map

| developed content units | taxonomy levels |
|---|---|
| equilibrium | u1, e1 |

| prerequisite content units | taxonomy levels |
|---|---|
| RR | u1, e1 |

# Main ILO of sub-module "compute the equilibria of the system"

**Compute** the equilibria of a RR by solving for stationary points

# Is this in equilibrium?

# Are these in equilibrium, while falling?

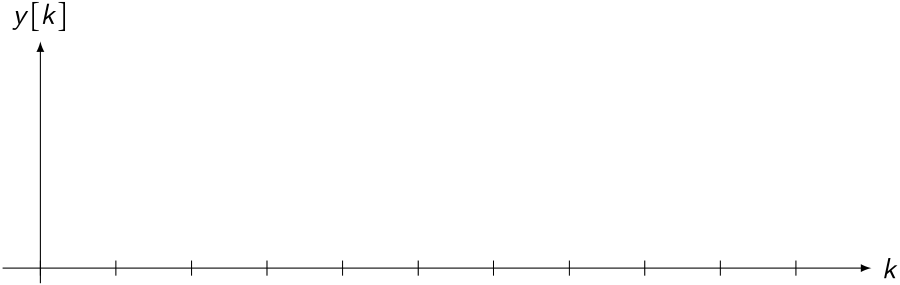
# Equilibrium = a trajectory that is constant in time

$$y[k] = \text{constant}$$

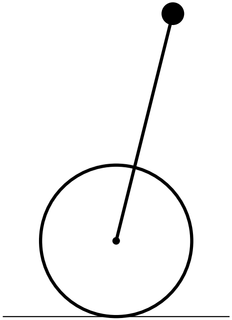
# Example

average temperature of a yogurt taken out from the fridge into a very large room whose temperature is 20 degrees, forward-Euler discretized with a sampling time $T = 1$ seconds:

$$\dot{y} = -0.5\left(y - 20\right) \quad \mapsto \quad y^{+} = 0.5y + 10$$



## What does it mean that this system is in equilibrium from an intuitive point of view?



# What does it mean that this system is in equilibrium from a mathematical point of view?

equilibrium means $\boldsymbol{y}^+ = \boldsymbol{y}$

# What does it mean that this system is in equilibrium from a mathematical point of view?

$$\boxed{\text{equilibrium means } \boldsymbol{y}^{+} \quad = \quad \boldsymbol{y}}$$

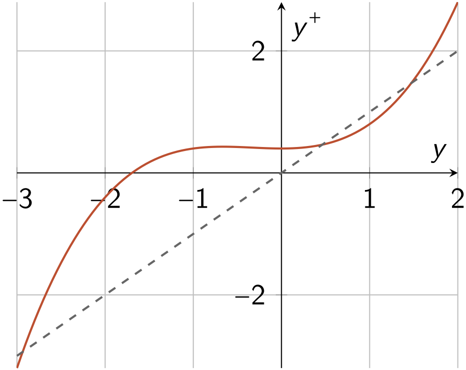
this implies

$$\boldsymbol{y}_{\text{eq}}, \boldsymbol{u}_{\text{eq}} \text{ is an equilibrium point iff } \boldsymbol{y}^{+} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$$

i.e., the equilibria of a system are the zeros of $\boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$

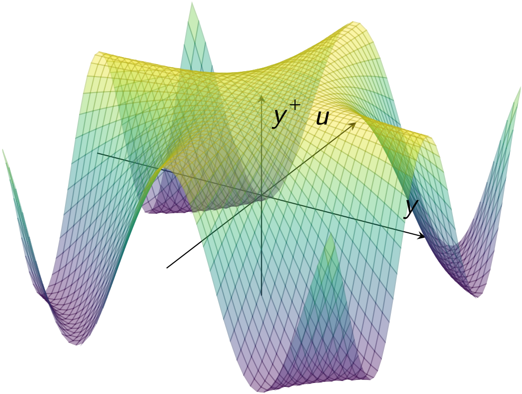
# Equilibra as the zeros of $y^+ = f(y)$, graphically

Exemplified situation of *autonomous* single output systems:



# Equilibra as the zeros of $\boldsymbol{f}$, graphically

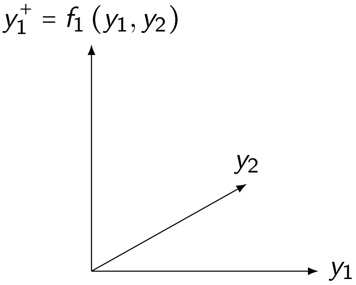
Exemplified situation of SISO (single input single output) systems:

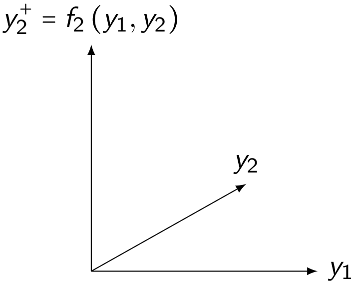



*https://www.geogebra.org/classic/mmppe6hs*

# Equilibra as the zeros of $\boldsymbol{f}$, graphically

Exemplified situation of automonous multiple output systems:



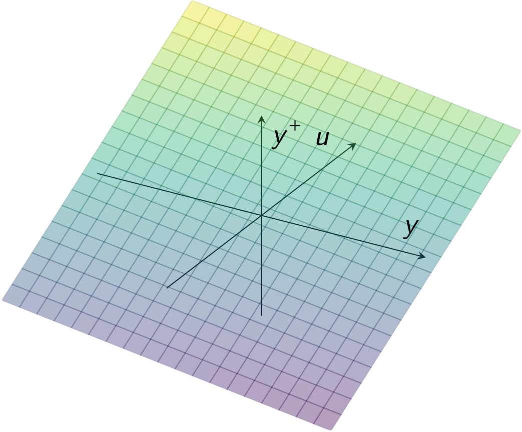

(remember: $\boldsymbol{y}_{\text{eq}}, \boldsymbol{u}_{\text{eq}}$ equilibrium iff $\boldsymbol{f}(\boldsymbol{y}_{\text{eq}}, \boldsymbol{u}_{\text{eq}}) = \boldsymbol{y}_{\text{eq}}$, i.e., all the components simultaneously!)

$$\boxed{\text{equilibria} = \text{extremely important topic in automatic control!}}$$

will be analysed in more details later on in this course
and much more extensively in others
*(feat. Lyapunov, Krasovskii, La-Salle among others)*

# Discussion: what are the equilibria in this case?

$$y^+ = ay + bu$$



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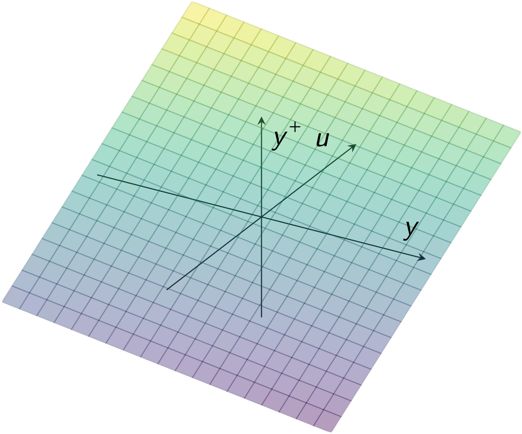
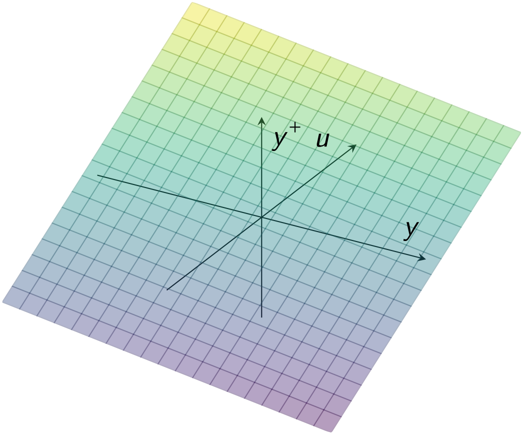



$(y_0 = 0, u = 0)$ is always an equilibrium for these systems!

## Discussion: can we have for this specific RR an equilibrium if $u \neq \text{constant}$?

$$y^+ = ay + bu$$



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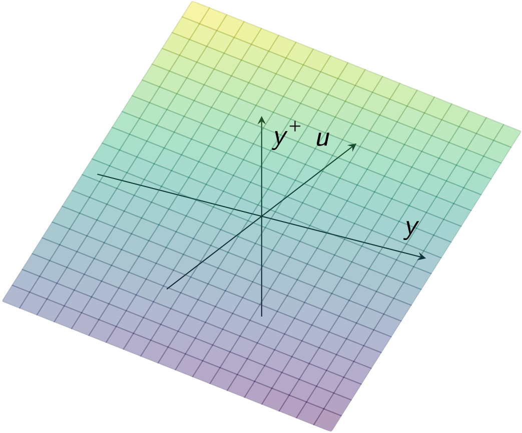
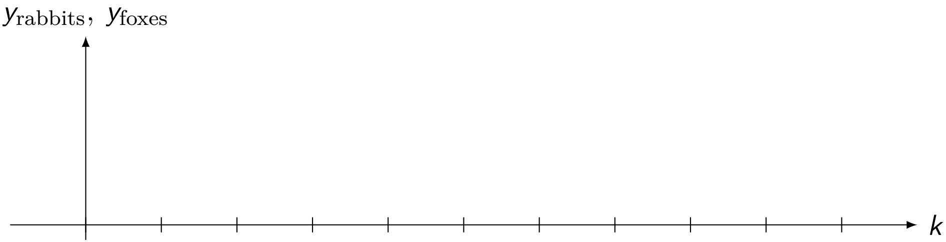


# Another example: a Lotka-Volterra model ($\neq$ real world):

$$\begin{cases} y^+_{\text{rabbits}} &= \;\; 1.4 \cdot y_{\text{rabbits}} - 0.5 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \\ y^+_{\text{foxes}} &= 0.7 \cdot y_{\text{foxes}} + 0.7 \cdot y_{\text{rabbits}} \cdot y_{\text{foxes}} \end{cases}$$




- compute the equilibria of the system 

# Summarizing

**Compute** the equilibria of a RR by solving for stationary points

- put $y = f(y, u) \implies y - f(y, u) = 0$, and compute the corresponding points. It may be that there is the need to put all the various inputs $u = \mathrm{constant}$ (or disturbances $d = \mathrm{constant}$)

# Most important python code for this sub-module

# Root finding in python

`https://pythonnumericalmethods.studentorg.berkeley.edu/notebooks/chapter19.05-Root-Finding-in-Python.html`

# Self-assessment material

# Question 1

Which of the following best defines an equilibrium point in a dynamical system?

**Potential answers:**

I: A point where the system's state constantly increases.
II: A point where the system's state remains unchanged over time.
III: A point where the system's state oscillates periodically.
IV: A point where the system's state diverges exponentially.
V: I do not know

# Question 2

How can equilibrium points be identified in a graphical representation of $y^+ = f(y)$?

**Potential answers:**

I: At points where $f(y)$ reaches its maximum value.
II: At points where $f(y)$ crosses the $y$-axis.
III: At points where $f(y)$ is strictly increasing.
IV: At points where $f(y) = y$, indicating that the system remains unchanged.
V: I do not know

# Question 3

Which of the following systems is most likely to be in equilibrium?

**Potential answers:**

I: A ball rolling down a hill.
II: A balancing robot standing perfectly upright and not moving.
III: A pendulum swinging back and forth.
IV: A falling hailstone.
V: I do not know

# Question 4

For the discrete-time system $y^+ = 0.5y + 10$, what is the equilibrium value of $y$?

**Potential answers:**

I: 0
II: 10
III: 20
IV: 40
V: I do not know

# Question 5

In a linear time-invariant system, an equilibrium point can be computed by:

**Potential answers:**

I: Setting the system dynamics to zero and solving for state and input values.
II: Taking the time derivative of the system matrix.
III: Finding the eigenvalues of the system matrix.
IV: Taking the integral of the system dynamics.
V: I do not know

# Recap of sub-module "compute the equilibria of the system"

- Equilibria in dynamical discrete time systems correspond to points where the system's state does not change over time.
- Autonomous time-varying RRs can have equilibria, but their location may vary with time.
- Some dynamical systems may not have equilibria, particularly if they involve unbounded growth.
- Non-autonomous LTI RRs can have equilibria only if the input $u(t)$ remains constant over time.

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