Is this time series a solution of this recurrence relation?

Contents map

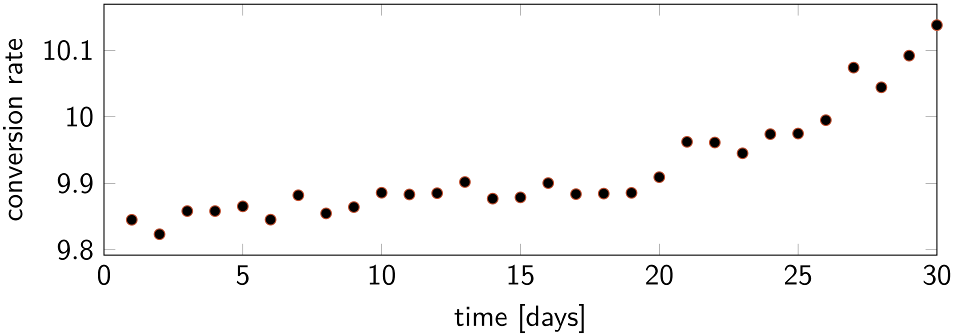
developed content units	taxonomy levels
recurrence relation	u1, e1

prerequisite content units	taxonomy levels
delay operator	u1, e1

Main ILO of sub-module
“Is this time series a solution of this recurrence relation?”

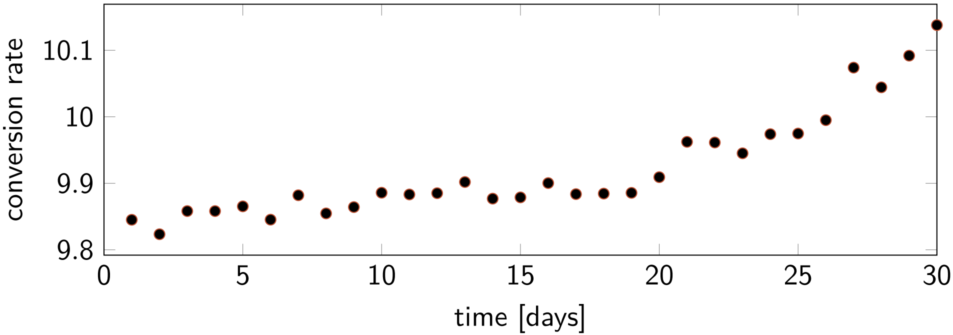
Decide whether a given time series is a solution to
a specified recurrence relation by direct verification

What is a time series?



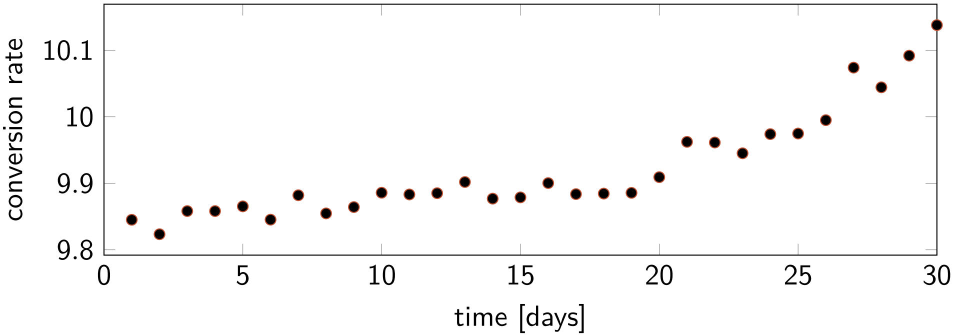
$y(kT)$, or $y[k]$ for simplicity of notation (assuming time discrete in this module)

What is the equivalent of a derivative for discrete-time signals?



$$y[k] \quad \mapsto \quad \frac{y[k]-y[k-1]}{T} = \frac{y[k]-y[k]q^{-1}}{T} = y[k]\left(\frac{1-q^{-1}}{T}\right)$$

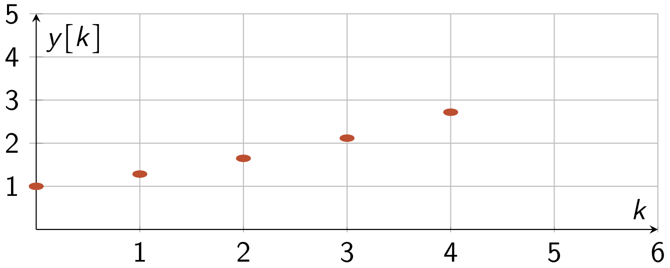
Would you say that $y[k] = y[k]q^{-1}$, in this case?



"uhm, where are we going with all this stuff?"

$\mapsto$ be able to do forecasts

would you be able to compute $y[6]$ from this graph, if you knew that $y[k] = 1.2y[k+1]$?



a RR is a tool to produce forecasts

Example: $y[k+1] = 0.5y^2[k]u[k]$

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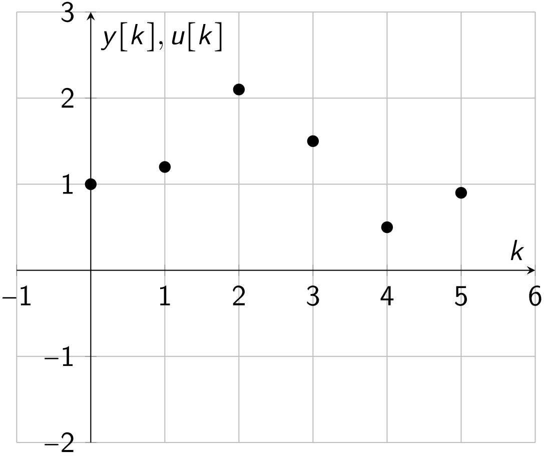
k	$y[k]$	$u[k]$
0		1
1		1.2
2		2.1
3		1.5
4		0.5
5		0.9



a RR is a tool to produce forecasts

Example: $$y[k+1] = 0.5y^2[k]u[k]$$

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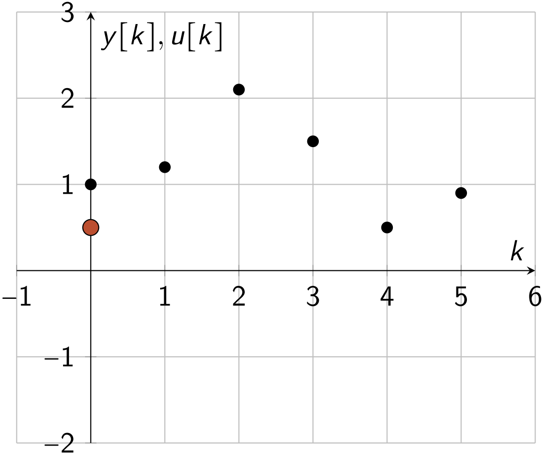

- Is this time series a solution of this recurrence relation?

a RR is a tool to produce forecasts

Example: $y[k+1] = 0.5y^2[k]u[k]$

k	$y[k]$	$u[k]$
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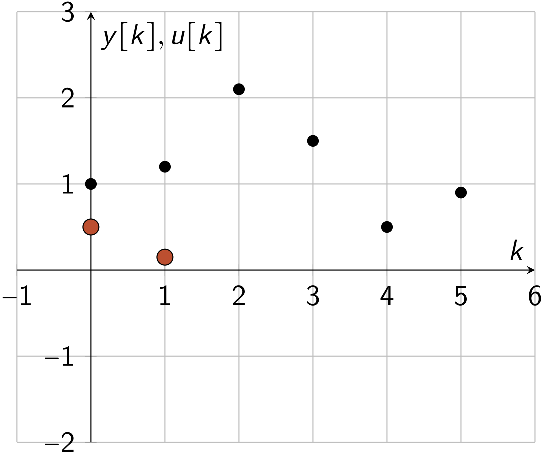

a RR is a tool to produce forecasts

Example: $y[k+1] = 0.5y^2[k]u[k]$

k	$y[k]$	$u[k]$
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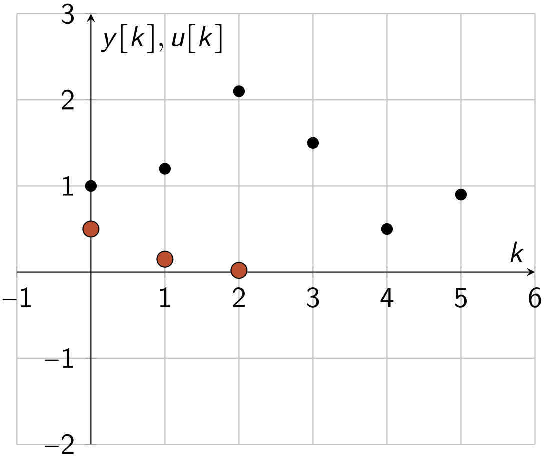

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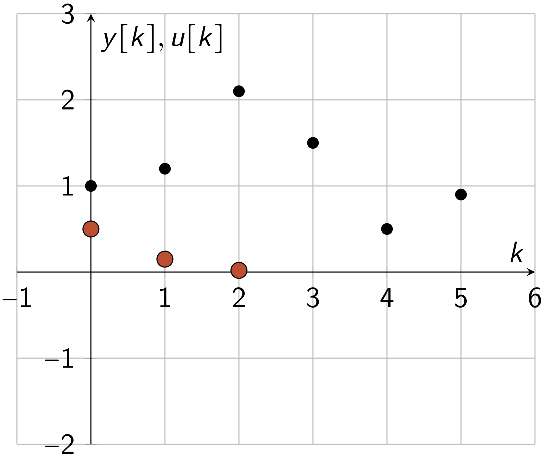
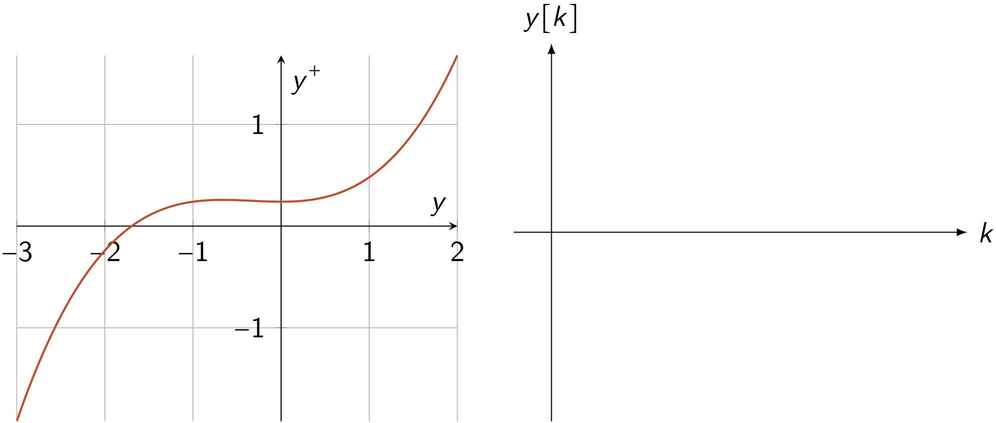


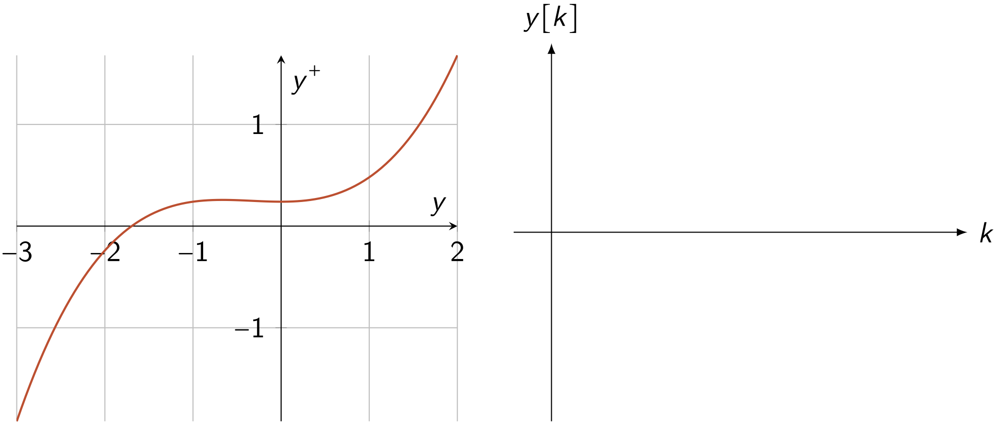
Notation

symbol	$\mapsto$	meaning	alternative 1	alternative 2
$\vdots$		$\vdots$	$\vdots$	$\vdots$
y^{+++}		$y[k+3]$	yq^{3}	$y^{[3]}$
y^{++}		$y[k+2]$	yq^{2}	$y^{[2]}$
y^{+}		$y[k+1]$	yq^{1}	$y^{[1]}$
y		$y[k]$	yq^{0}	$y^{[0]}$
y^{-}		$y[k-1]$	yq^{-1}	$y^{[-1]}$
y^{--}		$y[k-2]$	yq^{-2}	$y^{[-2]}$
y^{---}		$y[k-3]$	yq^{-3}	$y^{[-3]}$
$\vdots$		$\vdots$	$\vdots$	$\vdots$

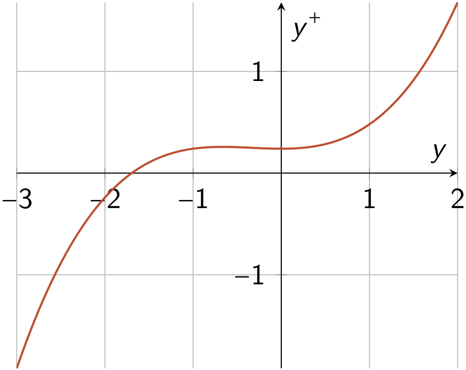
But what does it mean to solve a RR, graphically?

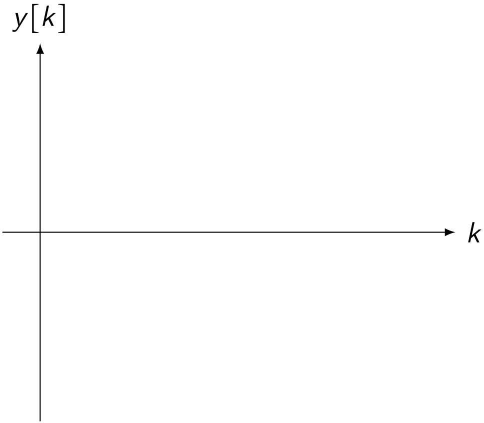


Is knowing the RR enough to be able to generate a trajectory?



Does $\{y[k] = \cos[k], y[0] = 1\}$ solve this RR?





Are we done with this?

Decide whether a given function is a solution to a specified RR by direct verification

→ no, there are still a lot of cases we shall cover

Notation time!

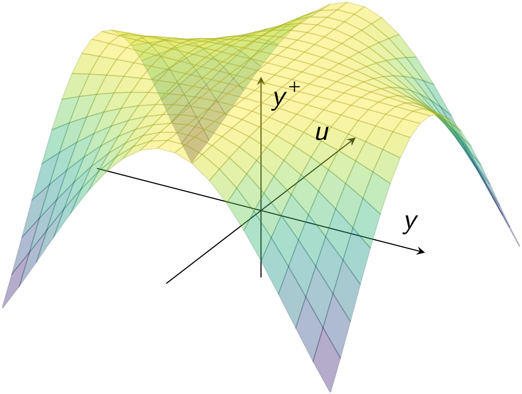
In control, modelling a dynamical system = defining

$$\boldsymbol{y}^{+} = \boldsymbol{f}\left(\boldsymbol{y}, \boldsymbol{u}, \boldsymbol{d}, \boldsymbol{\theta}\right),$$

thus defining:

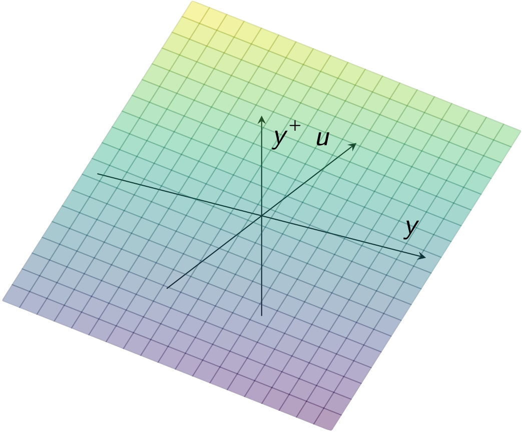
- the variables
  - $\boldsymbol{u}$ = inputs *(i.e., what we can steer)*
  - $\boldsymbol{d}$ = disturbances *(i.e., what we cannot steer but that still influences the system)*
  - $\boldsymbol{y}$ = outputs *(i.e., what we are interested into)*
- the shape of $\boldsymbol{f}$
- the value of its parameters $\boldsymbol{\theta}$
- bold font = vector

A graphical example of $y^+ = f(y, u)$



https://www.geogebra.org/classic/mmppe6hs

The special RR $y^+ = ay + bu$



Watch out: converting an ODE to a RR is not just substituting 'dots' with 'pluses'

temperature of the center of a cake in an oven whose temperature is 200 degrees:

$$\dot{T} = -0.5(T - 200) \quad ? \mapsto ? \quad T^{+} = -0.5(T - 200)$$


Ceci n'est pas une pipe.

Important point: model $\neq$ real world

Ceci n'est pas un gâteau.

$$T^{+} = f(T)$$

Summarizing

Decide whether a given function is a solution to a specified RR by direct verification

- check y, compute $f(y)$, compute y^+
- does $f(y) = y^+$?
- same apply for higher orders / more complex RRS from notational perspectives

Most important python code for this sub-module

Solving RRs

`https://www.geeksforgeeks.org/recurrence-relation-in-python/`

Self-assessment material

Question 1

Which of the following best describes what it means for a function $y[k]$ to be a solution of a RR?

Potential answers:

I: It satisfies the RR for at least one value of k.
II: It satisfies the RR for all values of k in its domain.
III: It approximately satisfies the RR within a certain error margin.
IV: It satisfies the RR only at integer values of k.
V: I do not know

Question 2

What additional information is needed to uniquely determine a solution of a RR?

Potential answers:

I: The function $y[k]$ itself.
II: An initial condition specifying the value of y at a given time.
III: A boundary condition at two different points.
IV: The highest-order difference of y.
V: I do not know

Question 3

Given the RR $y^+ = y$, which of the following functions is a solution?

Potential answers:

I: $y[k] = 0$
II: $y[k] = C$, where C is a constant.
III: $y[k] = \sin k$
IV: $y[k] = \dfrac{1}{k+1}$
V: I do not know

Recap of sub-module
“Is this time series a solution of this recurrence relation?”

- a function is a solution of a RR if it satisfies the equation for all values in its domain
- initial conditions are necessary to uniquely determine a solution

?