Systems Laboratory, Spring 2025

Damiano Varagnolo –

what is an impulse response

Contents map

developed content units	taxonomy levels
Dirac delta	u1, e1
impulse response	u1, e1

prerequisite content units	taxonomy levels
superposition principle	u1, e1
LTI ODE	u1, e1

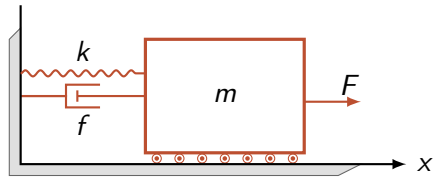
Main ILO of sub-module "what is an impulse response"

Describe what the impulse response of an LTI system is in practice

Impulse response ↔ superposition principle ↔ LTI system

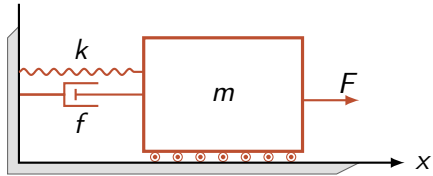
talking about the impulse response of a nonlinear system is such a big mistake that may make you fail the exam on the spot

Practical example: spring-mass system

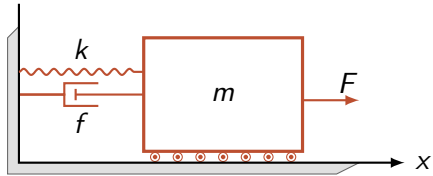


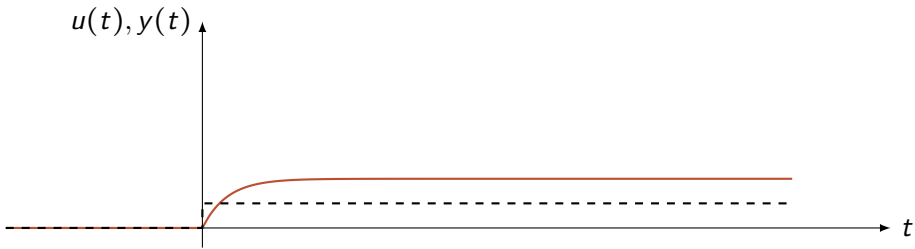
- output = position
- input = force (in Newtons)

What if I push the cart with a force of 1 Newton?

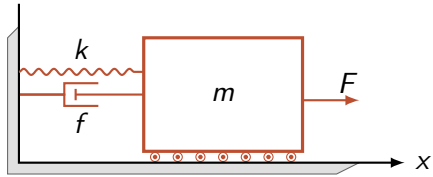


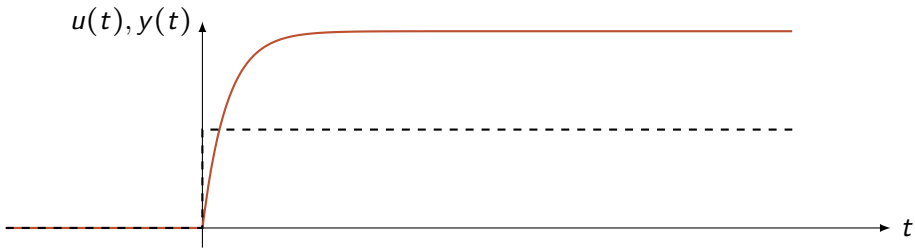
What if I push the cart with a force of 0.5 Newtons?



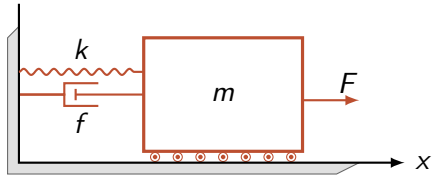


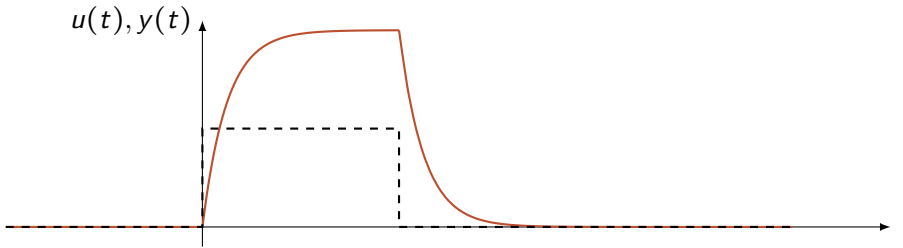
What if I push the cart with a force of 2 Newtons?



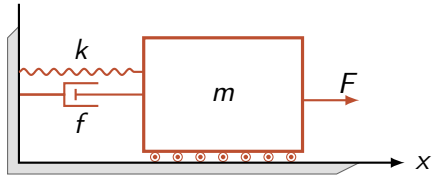


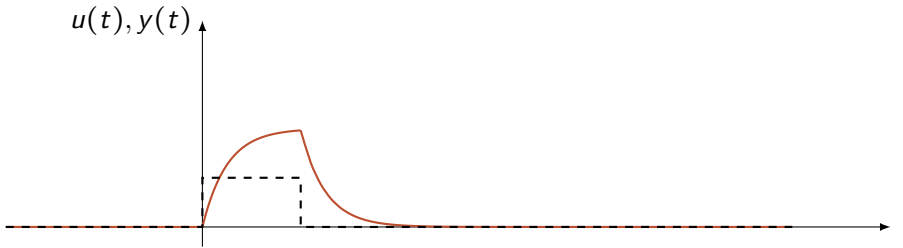
What if I push the cart with a force of 2 Newtons for 2 seconds?



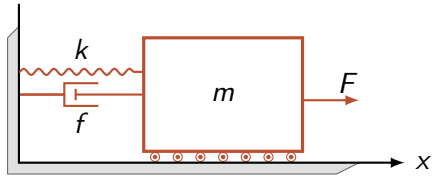


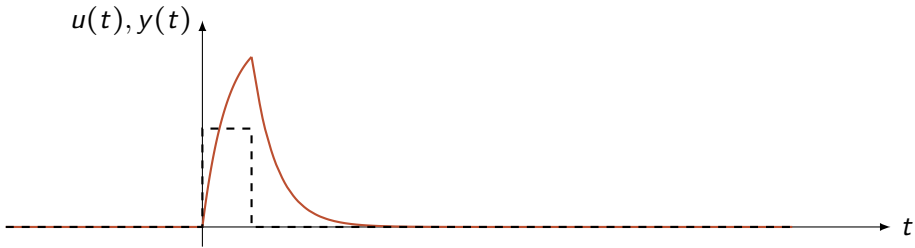
What if I push the cart with a force of 1 Newton for 1 second?



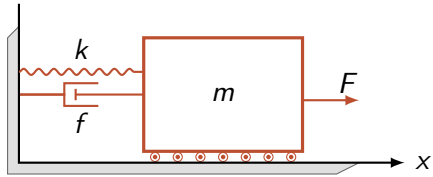


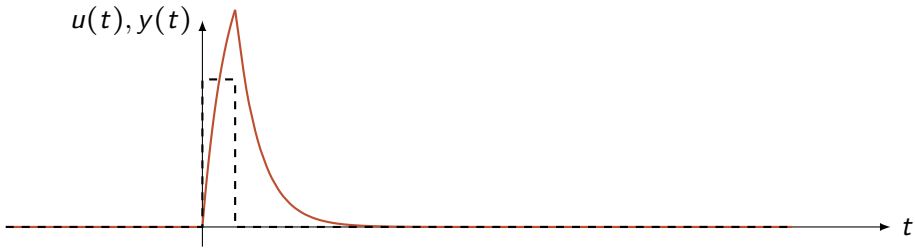
What if I push the cart with a force of 2 Newtons for 0.5 seconds?



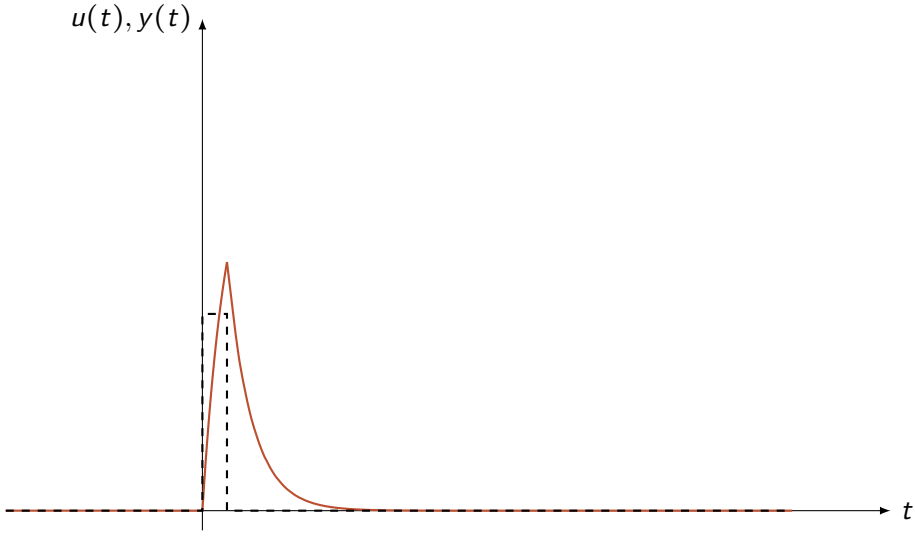


What if I push the cart with a force of 3 Newtons for 1/3 of a second?

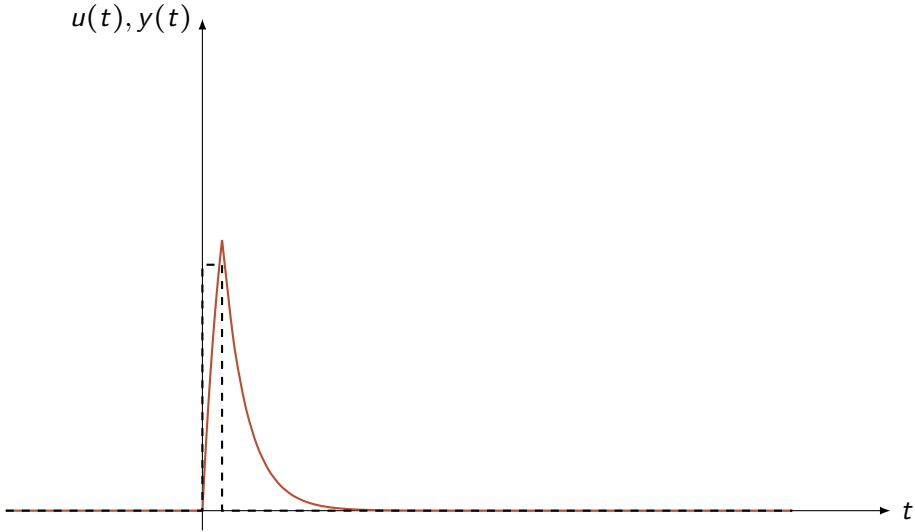




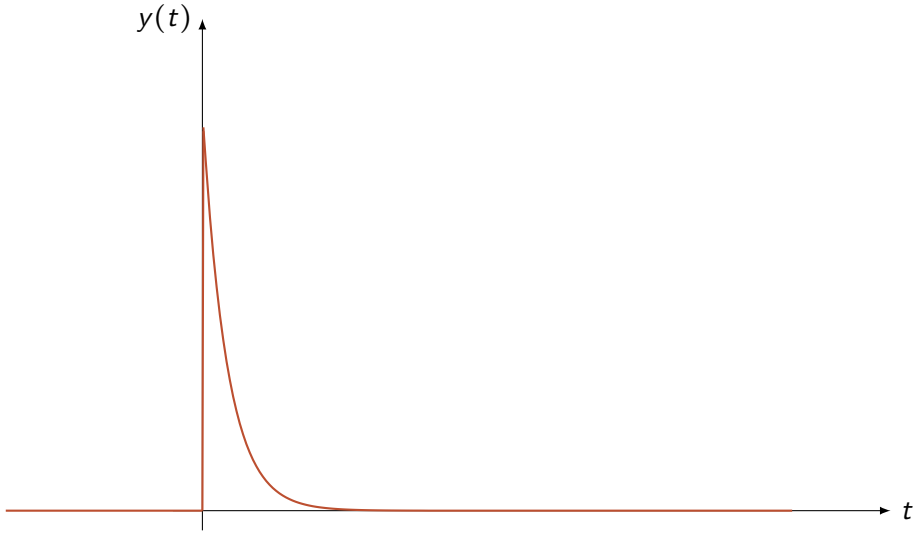
4 Newtons for 1/4 of a second?



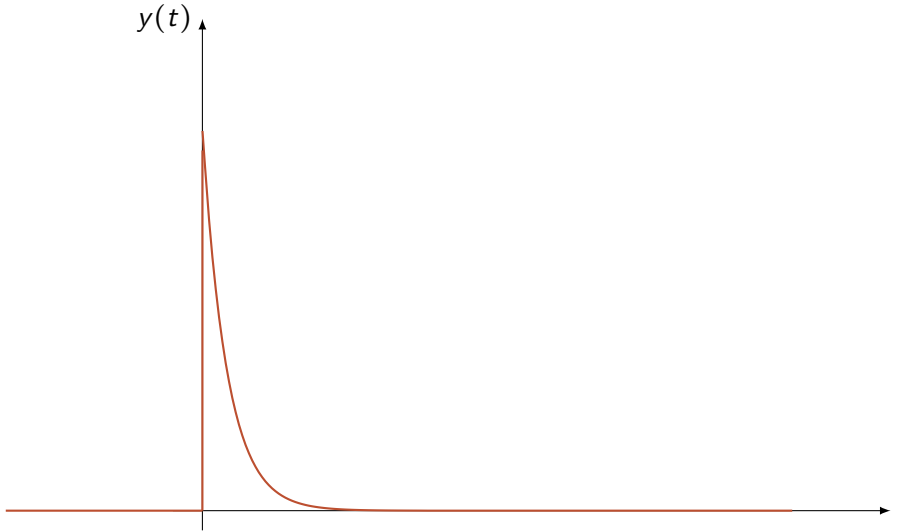
5 Newtons for 1/5 of a second?



100 Newtons for 1/100 of a second?

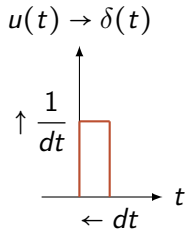


1000 Newtons for 1/1000 of a second?



Where are we going with the input signal?

↦ Dirac's delta, i.e., pushing an unitary mass within an infinitesimal space

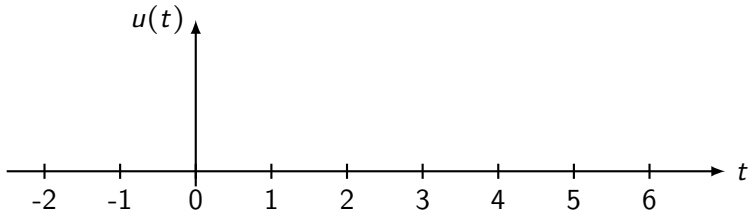


ESSENTIAL POINT

this game has sense only because the system is assumed to be LTI

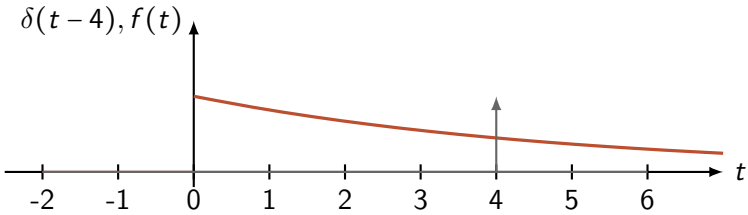
Discussion

Where would you draw $\delta(t-4)$?



Discussion

$$\int_{-\infty}^{+\infty} f(\tau)\delta(\tau-4)d\tau = ?$$



Summarizing

Describe what the impulse response of an LTI system is in practice

- an opportune limit behavior of a transformation of the step response of a LTI system
- it is though a transformation that makes sense only if the system is LTI

Most important python code for this sub-module

Important libraries / methods

- https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.impulse.html
- https://python-control.readthedocs.io/en/latest/generated/control.impulse_response.html

Self-assessment material

Question 1

What is the impulse response of an LTI system?

Potential answers:

I: The output of the system when the input is a sinusoidal function.
II: The output of the system when the input is a ramp function.
III: The output of the system when the input is a step function.
IV: The output of the system when the input is a Dirac delta function.
V: I do not know.

Question 2

Why is the impulse response meaningful only for LTI systems?

Potential answers:

I: Because the impulse response is a direct consequence of the superposition principle, which applies only to LTI systems.

II: Because nonlinear systems do not respond to impulses.

III: Because the impulse response is too complex to compute for nonlinear systems.

IV: Because nonlinear systems have infinite impulse responses.

V: I do not know.

Question 3

What happens to the mass-spring-damper system when the input force is a Dirac delta function?

Potential answers:

I: The mass oscillates indefinitely without damping.
II: The mass exhibits a transient response that decays over time due to damping.
III: The mass remains stationary because the impulse is too short to affect it.
IV: The mass moves with constant velocity.
V: I do not know.

Question 4

What is the integral of $f(\tau)\delta(\tau - 4)$ from $-\infty$ to $+\infty$?

Potential answers:

I: $\int_{-\infty}^{+\infty} f(\tau)d\tau$

II: 0

III: $f(4)$

IV: $\delta(4)$

V: I do not know.

Recap of sub-module "what is an impulse response"

- impulse responses are directly connected to step responses
- actually this connection is valid only if the system is LTI

?