Tema 4

Es 1 $f(x) = x(\lg x)^2 + 3$

$D: \{x > 0\} = (0, +\infty)$. Dato che $(\lg x)^2 \geq 0$ ho che $f(x) \geq 3 > 0$ $\forall x \in D$

non ho simmetrie né periodicità

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \underbrace{x(\lg x)^2}_{\downarrow 0} + 3 = 3$$

$x = 0$ è singolarità eliminabile e la aggiungo al dominio $f(0) = 3$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} (\lg x)^2 + \frac{3}{x} = +\infty$$

NON HO ASINTOTO ORIZZONTALE NÉ OBLIQUO

f è continua nel dominio esteso

$f'(x) = (\lg x)^2 + \cancel{x} \cdot 2(\lg x)\frac{1}{\cancel{x}} = (\lg x)[\lg x + 2]$

per $x > 0$

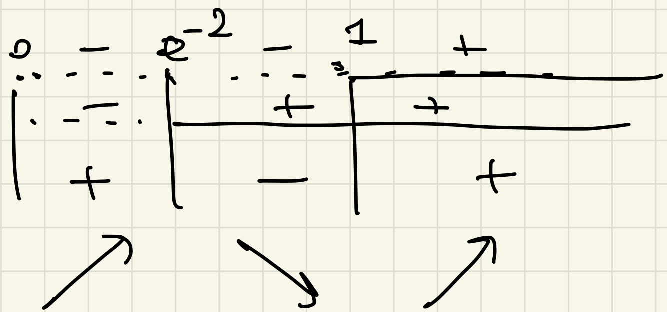
$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (\lg x)(\lg x + 2) = +\infty$ in $x = 0$ ho attacco verticale

$f'(x) \geq 0 \iff (\lg x)(\lg x + 2) \geq 0$

$\lg x \geq 0 \iff x \geq 1$

$\lg x + 2 \geq 0 \iff x \geq e^{-2}$

$\lg x \geq -2 = \lg e^{-2}$



f è crescente in $(0, e^{-2})$ e $(1, +\infty)$

$x = e^{-2}$ è pto di massimo locale (NON ASSOLUTO)

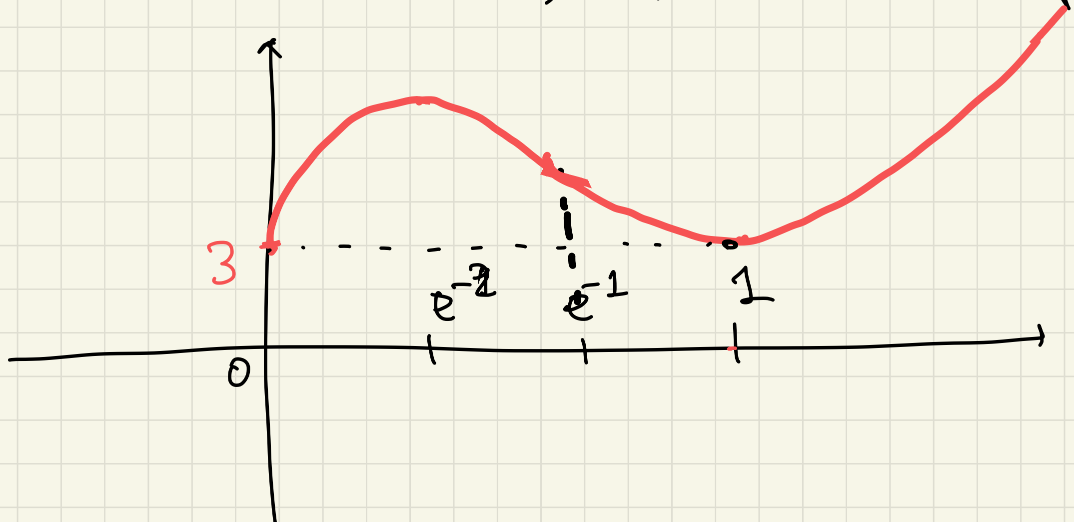
$x = 0$, $x = 1$ sono pti di minimo locale e anche assoluto $f(0) = f(1) = 3 \leq f(x) \ \forall x \in D$.

$$f''(x) = \frac{1}{x}(\lg x + 2) + \lg x \cdot \frac{1}{x} = \frac{1}{x}[\lg x + \lg x + 2] =$$

$$= \frac{1}{x}[2\lg x + 2] = \frac{2}{x}[\lg x + 1]$$

$f''(x) \geq 0 \iff \lg x + 1 \geq 0 \iff x \geq e^{-1}$

f è convessa in $(e^{-1}, +\infty)$. $x = e^{-1}$ è pto di flesso.



Es 2 : $\lg(1+x^2) = x^2 - \frac{1}{2}x^4 + o(x^4)$

$\sin x = x - \frac{1}{6}x^3 + o(x^3)$

$$\frac{\lg(1+x^2) - x\sin x}{x^\alpha} = \frac{\cancel{x^2} - \frac{1}{2}x^4 + o(x^4) \cancel{-x^2} + \frac{1}{6}x^4 + o(x^4)}{x^\alpha}$$

$$= \frac{-\frac{1}{3}x^4 + o(x^4)}{x^\alpha} = \frac{x^4\left(-\frac{1}{3} + o(1)\right)}{x^\alpha} \Rightarrow \begin{cases} -\frac{1}{3} & \text{se } \alpha = 4 \\ -\infty & \text{se } \alpha > 4 \\ 0 & \text{se } \alpha < 4 \end{cases}$$

Es 3 $\displaystyle\int_{\lg 2}^{\lg 3} \frac{e^x}{e^{2x} + e^x - 2}\,dx = \begin{bmatrix} y = e^x \end{bmatrix} = \int_2^3 \frac{\cancel{y}}{y^2 + y - 2}\,\frac{1}{\cancel{y}}\,dy$

$x = \lg y$

$dx = \frac{1}{y}dy$

Fratti semplici

$y^2 + y - 2 = (y-1)(y+2)$

$$\frac{1}{y^2+y-2} = \frac{A}{y-1} + \frac{B}{y+2} = \frac{Ay + 2A + By - B}{(y-1)(y+2)}$$

$$\begin{cases} A + B = 0 \\ 2A - B = 1 \end{cases} \qquad B = -\frac{1}{3} \quad A = \frac{1}{3}$$

$$\int \frac{1}{y^2+y-2}\,dy = \frac{1}{3}\int \frac{1}{y-1}\,dy - \frac{1}{3}\int \frac{1}{y+2}\,dy = \frac{1}{3}\lg|y-1| - \frac{1}{3}\lg|y+2| + c$$

$$= \frac{1}{3}\lg\frac{|y-1|}{|y+2|} + c$$

$$\int_2^3 \frac{1}{y^2+y-2}\,dy = \frac{1}{3}\lg\left(\frac{3-1}{3+2}\right) - \frac{1}{3}\lg\left(\frac{2-1}{2+2}\right) = \frac{1}{3}\lg\left(\frac{2}{5}\right) - \frac{1}{3}\lg\left(\frac{1}{4}\right)$$

$$= \frac{1}{3}\lg\left(\frac{8}{5}\right).$$

$$\lim_{M\to+\infty}\int_{\lg 2}^{M} \frac{e^x}{e^{2x}+e^x-2}\,dx = \lim_{M\to+\infty}\int_2^{e^M}\frac{1}{y^2+y-2}\,dy =$$

$$= \lim_{M\to+\infty}\frac{1}{3}\lg\left(\frac{e^M-1}{e^M+2}\right) - \frac{1}{3}\lg\left(\frac{1}{4}\right) = \lim_{M\to+\infty}\lg\left(\overbrace{\frac{\cancel{e^M}\left(1-\frac{1}{e^m}\right)}{\cancel{e^M}\left(1+\frac{2}{e^m}\right)}}^{1}\right) - \frac{1}{3}\lg\frac{1}{4} =$$

$\lg 1 = 0$

$$= -\frac{1}{3}\lg\frac{1}{4} = \frac{1}{3}\lg 4.$$

Es 4 $\sum_{n=0}^{+\infty} \frac{2^n}{n!}$

criterio del rapporto $a_n = \frac{2^n}{n!}$ $a_{n+1} = \frac{2^{n+1}}{(n+1)!} = \frac{2^n \cdot 2}{n!(n+1)}$

$\lim_n \frac{a_{n+1}}{a_n} = \lim_n a_{n+1} \cdot \frac{1}{a_n} = \lim_n \frac{\cancel{2^n}\, 2}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{2^n}} = 0 < 1$

la serie converge