Tema 2

Es 1 $f(x) = x(\lg x)^4 + 1$

$D: \{x > 0\} = (0, +\infty)$. Dato che $(\lg x)^4 \geq 0$ ho che $f(x) \geq 1 > 0$ $\forall x \in D$

non ho simmetrie né periodicità

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \underbrace{x(\lg x)^4}_{\downarrow 0} + 1 = 1$

$x = 0$ è singolarità eliminabile e la aggiungo al dominio $f(0) = 1$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} (\lg x)^4 + \frac{1}{x} = +\infty$

NON HO ASINTOTO ORIZZONTALE NÉ OBLIQUO

f è continua nel dominio esteso

$$f'(x) = (\lg x)^4 + \cancel{x} \cdot 4(\lg x)^3 \frac{1}{\cancel{x}} = (\lg x)^3 [\lg x + 4]$$

per $x > 0$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (\lg x)^3 (\lg x + 4) = +\infty$$

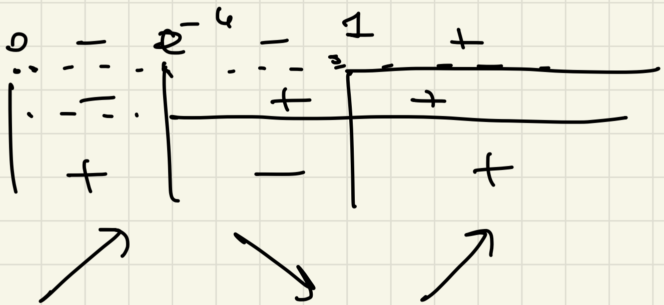
in $x = 0$ ho attacco verticale

$$f'(x) \ge 0 \iff (\lg x)^3 (\lg x + 4) \ge 0$$

$(\lg x)^3 \ge 0 \iff x \ge 1$

$\lg x + 4 \ge 0 \iff x \ge e^{-4}$

$\lg x \ge -4 = \lg e^{-4}$



f è crescente in $(0, e^{-4})$ e $(1, +\infty)$

$x = e^{-4}$ è pto di massimo locale (NON ASSOLUTO)

$x = 0$, $x = 1$ sono pti di minimo locale e anche assoluto

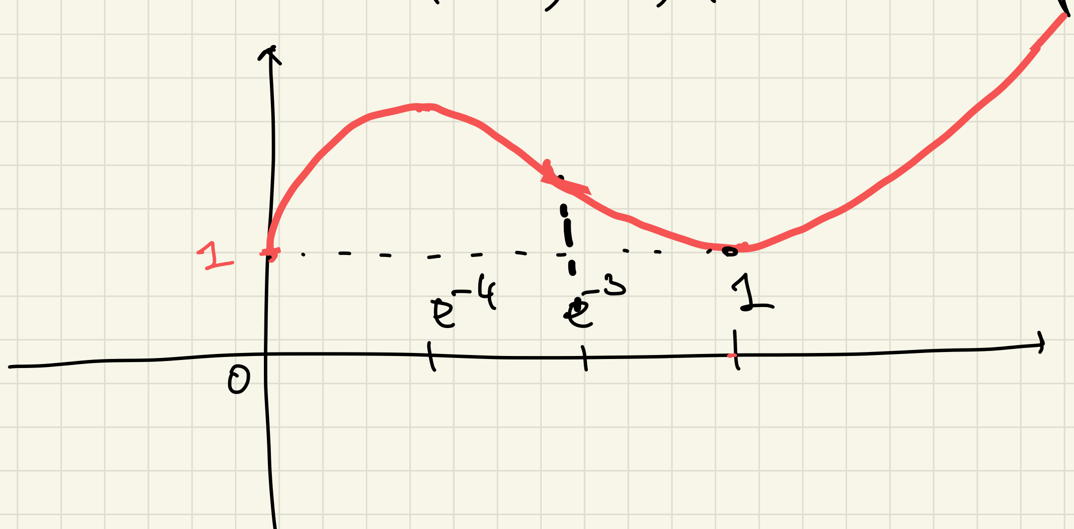
$$f(0) = f(1) = 1 \le f(x) \ \forall x \in D.$$

$$f''(x) = 3(\lg x)^2 \frac{1}{x}\left[\lg x + 4\right] + (\lg x)^3 \cdot \frac{1}{x} =$$

$$= (\lg x)^2 \frac{1}{x}\left[3\lg x + 12 + \lg x\right] = \frac{4}{x}(\lg x)^2 \cdot \left[\lg x + 3\right]$$

$$f''(x) \geq 0 \iff \lg x + 3 \geq 0 \iff x \geq e^{-3}$$

f è convessa in $(e^{-3}, +\infty)$. $x = e^{-3}$ è pto di flesso.



Es 2 : $e^{x^2} = 1 + x^2 + \frac{1}{2} x^4 + o(x^4)$

$$x \operatorname{arctg} x = x\left(x - \frac{1}{3} x^3 + o(x^3)\right) = x^2 - \frac{1}{3} x^4 + o(x^4)$$

$$\frac{e^{x^2} - 1 - x \operatorname{arctg} x}{x^\alpha} = \frac{\cancel{1} + \cancel{x^2} + \frac{1}{2} x^4 + o(x^4) \cancel{-1} - \cancel{x^2} + \frac{1}{3} x^4 + o(x^4)}{x^\alpha}$$

$$= \frac{\frac{5}{6} x^4 + o(x^4)}{x^\alpha} = \frac{x^4}{x^\alpha}\left(\frac{5}{6} + o(1)\right) \longrightarrow \begin{cases} \frac{5}{6} & \alpha = 4 \\ +\infty & \alpha > 4 \\ 0 & \alpha < 4 \end{cases}$$

Es 3 $\displaystyle\int_{\lg 4}^{\lg 5} \frac{e^x}{e^{2x} - 2e^x - 3}\, dx = \left[\begin{array}{l} y = e^x \\ x = \lg y \\ dx = \frac{1}{y} dy \end{array}\right] = \int_4^5 \frac{\cancel{y}}{y^2 - 2y - 3} \frac{1}{\cancel{y}} dy$

fratti semplici $y^2 - 2y - 3 = (y+1)(y-3)$

$$\frac{1}{y^2 - 2y - 3} = \frac{A}{(y+1)} + \frac{B}{(y-3)} \longrightarrow \begin{cases} A + B = 0 \\ -3A + B = 1 \end{cases} \quad \begin{cases} A = -\frac{1}{4} \\ B = 1/4 \end{cases}$$

$$\int \frac{1}{y^2-2y-3}\,dy = -\frac{1}{4}\int \frac{1}{y+1}\,dy + \frac{1}{4}\int \frac{1}{y-3}\,dy = \frac{1}{4}\lg\left|\frac{y-3}{y+1}\right| + c$$

$$\int_4^5 \frac{1}{y^2-2y-3}\,dy = \frac{1}{4}\lg\frac{5-3}{5+1} - \frac{1}{4}\lg\frac{4-3}{4+1} = \frac{1}{4}\lg\frac{2}{6} - \frac{1}{4}\lg\frac{1}{5}$$

$$= \frac{1}{4}\lg\left(\frac{5}{3}\right).$$

$$\lim_{M\to+\infty}\int_{\lg 4}^{M} \frac{e^x}{e^{2x}-2e^x-3}\,dx = \lim_{M\to+\infty}\int_4^{e^M} \frac{1}{y^2-2y-3}\,dy =$$

$$= \lim_{M\to+\infty} \frac{1}{4}\lg\left(\frac{e^M-3}{e^M+1}\right) - \frac{1}{4}\lg\left(\frac{1}{5}\right) = \lim_{M\to+\infty} \frac{1}{4}\overbrace{\lg\left(\frac{\cancel{e^M}(1-\frac{3}{e^M})}{\cancel{e^M}(1+\frac{1}{e^M})}\right)}^{\lg 1 = 0} +$$

$$-\frac{1}{4}\lg\left(\frac{1}{5}\right) = -\frac{1}{4}\lg\frac{1}{5} = +\frac{1}{4}\lg 5$$

Es 4 $\quad \sum_{n=0}^{+\infty} \frac{3^n}{n!}$

criterio del rapporto $\quad a_n = \frac{3^n}{n!} \quad a_{n+1} = \frac{3^{n+1}}{(n+1)!} = \frac{3^n 3}{n!(n+1)}$

$$\lim_n \frac{a_{n+1}}{a_n} = \lim_n a_{n+1} \cdot \frac{1}{a_n} = \lim_n \frac{\cancel{3^n} \cdot 3}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{3^n}} = 0 < 1$$

la serie converge