Coupling of quarks to W in the S.M. (1)

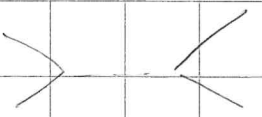
$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

↳ Vector of spinors of "u" type quarks ↳ mass (or production) eigenstates

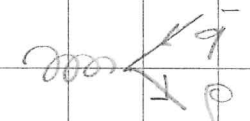
Production in collider

- Mostly QED or neutral W.I. in e^+e^- colliders

$$e^+e^- \to Z^0/\gamma \to q\bar{q}$$



- QCD in hadron colliders



$\alpha_W = \alpha_{em}/\sin^2\vartheta$

C.C. W.I. Lagrangian:

$$\ldots \frac{\alpha}{\sin^2\vartheta_W} \bar{U} \gamma^\rho \frac{1-\gamma^5}{2} D' W_\rho + \ldots$$

$D' = VD \Rightarrow V$: unitary matrix

$V^+ V = \mathbb{1}$

(applying V to D is a pure convention could likewise apply V^+ to $\bar{U}$)

Parameters for mixing matrix

$V = [V_{ij}]$ $\quad i = 1, N \quad j = 1, N$ = # of generations

complex parameters

$V_{ij} = \rho_{ij} e^{i\phi_{ij}} \longrightarrow \begin{cases} N^2 \text{ moduli} \\ N' \text{ phases} \end{cases} \longrightarrow 2N^2 \text{ real parameters}$

but

$V^\dagger V = 1$ (unitarity)

$V^*_{ki} V_{ik} = 1$

$V^*_{ik} V_{ik} = 1 \Rightarrow |V_{ik}| = 1$

$\sum_k |V_{ik}|^2 = 1 \quad i = N \Rightarrow N$ constraints

double counting

$V^*_{ki} V_{ij} = 0 \quad k \neq j \quad \left(\frac{1}{2}\right) N(N-1) \times 2$ → complex equations

Quark phases

$J^\rho_{q_1 q_2} = \bar{\psi}_{q_1} \gamma^\rho \left(\frac{1 - \gamma^5}{2}\right) \psi_{q_2}$

$\psi'_{q_1} = e^{i\alpha_1} \psi_{q_1} \qquad \psi'_{q_2} = e^{i\alpha_2} \psi_{q_2}$

invariant

$J'^\rho_{q_1 q_2} = e^{i(\alpha_2 - \alpha_1)} J^\rho_{q_1 q_2}$

but $|\eta| \propto |J^\rho J_\rho|^2 \longrightarrow$ overall phases irrelevant

N quarks $\Rightarrow 2N - 1$ relative phases

$N_{dof} = 2N^2 - N - \frac{1}{2} N(N-1) \times 2 - N + 1$

$= 2N^2 - N^2 + N - 2N + 1 =$

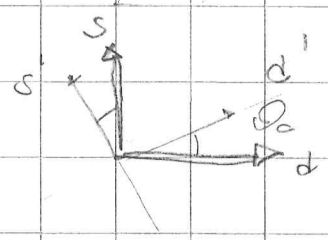
$= (N-1)^2$

$N = 2 \rightarrow 1$ parameter $\Rightarrow$ rotation in a (2,

$\begin{pmatrix} d \\ s \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$

$d' = d \cos\theta_c + s \sin\theta_c$

$s' = -d \sin\theta_c + s \cos\theta_c$



$N = 3$



$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$



$(N-1)^2 = 4$ parameters

$\rightarrow$ 3 rotations (Euler angles)

$\rightarrow$ 1 phase $\rightarrow$ accounting for CP violation in the SM

$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

[see PDG review for full product]

Hierarchy (experimental)

$\theta_{12} \gg \theta_{23} \gg \theta_{13}$

$\left[V_{ud} \gg V_{cb} \gg V_{ub} \right]$

$\lambda = \sin\theta_{12} \simeq \sin\theta_c = 0.22431$ (85)

= expand in power series of λ

Reparameterize through a power series expansion

$$(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \longrightarrow (\lambda, A, \rho, \eta) \qquad (3)$$

which respects unitarity at each and any step

$$\lambda = s_{12} = \frac{V_{us}}{\sqrt{|V_{us}|^2 + |V_{ud}|^2}} = \sin\theta_c = 0.224$$

$$A\lambda^2 = s_{23} = \lambda \left| \frac{V_{cb}}{V_{us}} \right| \qquad A \simeq 0.8$$

$\to$ parameter of power series expansion

$$A\lambda^3(\rho + i\eta) = s_{13} e^{i\delta} = V_{ub}^* \qquad \rho, s \quad O(1)$$

also $$(\rho + i\eta)\left(1 - \frac{\lambda^2}{2} + O(\lambda^4)\right) = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

Five terms contain the phase, but only 2 it's significant:

Note that

$$c_{12} = \sqrt{1 - s_{12}^2} \simeq 1 - \tfrac{1}{2} s_{12}^2 = 1 - \lambda^2/2$$

$$c_{23} = \sqrt{1 - s_{23}^2} \simeq 1 - \tfrac{1}{2} s_{23}^2 = 1 - \tfrac{1}{2} A^2 \lambda^4 \sim 1$$

$$c_{13} = \sqrt{1 - s_{13}^2} \simeq 1 - \tfrac{1}{2} s_{13}^2 = 1 - \tfrac{1}{2} A^2 \lambda^6 (\rho + i\eta)^2 \sim 1$$

So, for instance

$$V_{cs} = -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta}$$

$$\simeq -\left(1 - \frac{\lambda^2}{2}\right) A\lambda^2 - \lambda \cdot 1 \cdot A\lambda^3 (\rho + i\eta)$$

$$= -A\lambda^2 + \frac{1}{2} A\lambda^4 - \lambda^4 A(\rho + i\eta)$$

$$= -A\lambda^2 \left(1 - \lambda^2(\rho + i\eta)\right)$$

$$\simeq -A\lambda^2$$

$\to$ violating term $O(\lambda^2)$

$\lambda^* =$

whence

$$V_{td} = s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}$$
$$\simeq A\lambda^3 - \left(1-\frac{\lambda^2}{2}\right)\cdot 1\cdot A\lambda^3(\rho+i\eta)$$
$$= A\lambda^3(1-\rho-i\eta)$$

We finally get the so called "Wolfenstein Parametrization"

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

To note

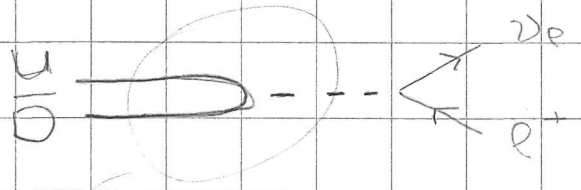
- matrix unitarity at any order $\sum_{jk} V_{ji}^* V_{ki} = \delta$
- "Cabibbo" doublets (u, d, c, s) almost decoupled from the other, ruled from just one parameter, the Cabibbo angle
- $|V_{ub}/V_{cb}|^2 \simeq (A\lambda^2)^2 = 2\%$
  $|V_{td}/V_{tb}|^2 \simeq (A\lambda^3)^2 = 10^{-6}$
  } t decays only to b quark
- CPV must contain V_{ub} or V_{td} $\propto A\lambda^3$
  ⊗ → few are known.

The Cabibbo sector

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

Measurements $\begin{cases} \text{leptonic decays of charged mesons} \\ \text{semileptonic decays of charged \& neutral hadrons} \end{cases}$

$P \to \ell \nu_\ell$ $\quad \left(P = \begin{cases} \pi^+ = |u\bar{d}\rangle \to V_{ud} \\ K^+ = |u\bar{s}\rangle \to V_{us} \\ D^+ = |c\bar{d}\rangle \to V_{cd} \\ D_s^+ = |c\bar{s}\rangle \to V_{cs} \end{cases} \right.$



$\ell = e$ for π decay
e, μ for K decay
e, μ, τ for D decay

$$\Gamma_P = \Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P m_P m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 |V_{UD}|^2 \left(1 + \frac{\alpha}{\pi} C_P\right)$$

QED corrections $O(\%)$

$\to$ annihilation

P Pseudoscalar meson, $J^P = 0^-$ $\to$ negative parity

$\Rightarrow$ only axial-vector current contributes

$$J_\mu^P = i p_\mu f_P = \langle 0 | \bar{D} \gamma_\mu \gamma^5 U | P(p) \rangle$$

$f_P \longleftrightarrow$ "wave function overlap" $\propto |\psi(0)|^2$
computed with Lattice QCD techniques

$$\Gamma_P = \Gamma \cdot Br(P \to \ell\nu) = \frac{1}{\tau} \cdot Br(P \to \ell\nu)$$

$(\pi^+ \to \mu^+ \nu_\mu \; B_\tau \sim 100\% \qquad \tau = 26.033(5)\ ns)$

$K^+ \to \mu^+ \nu_\mu$

$f_\pi = 130.2 \pm 0.8$ MeV (6 ‰)

$f_K = 155.7 \pm 0.7$ MeV (4.5 ‰)

$f_K / f_\pi = 1.1916(34)$ (2.9 %)

EXP TH

$|V_{ud}|\, f_{\pi^+} = 127.13 \pm 0.01 \pm 0.13$ MeV (1 ‰)

$|V_{us}|\, f_{K^+} = 35.08 \pm 0.04 \pm 0.03$ MeV (1.4 ‰)

$\left|\frac{V_{us}}{V_{ud}}\right| \cdot \frac{f_{K^+}}{f_{\pi^+}} = 0.2760 \pm 0.0003 \pm 0.0002$ (1.5 %)

$|V_{ud}| = 0.9764 \pm 0.006$

$|V_{us}| = 0.2253 \pm 0.001$

$\Big\}\ |V_{ud}|^2 + |V_{us}|^2 =$

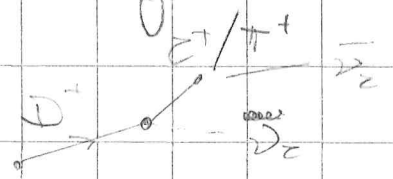
Charm

$$\frac{D^+ \to \tau^+ \nu}{D^+ \to \mu^+ \nu} \propto \frac{m_\tau \left(1 - (m_\tau/m_D)^2\right)^2}{m_\mu \left(1 - (m_\mu/m_D)^2\right)^2} = 15\,!$$

$m_\tau / m_{D^+} = 1.776 / 1.869 = 0.95$

$\rho(D \to \tau \nu_\tau) = 0.1$

Decay to τ is favored! But harder to see (2 ✓)



still better exp result from μ decay

How to manage the missing neutrino? (5)

Charm factory BES 3

$\psi(3S) \to D^+ D^-$ (~50%) $D^0 \bar{D}^0$ (~50%)

Tag & Probe:

Fully reconstruct a D^+ meson

$D^+ \to K^+ \pi^+ \pi^-, K_S \pi^+, K_S K^-, \ldots$



$E_{Tag} = E_{Probe} = E_{beam}$

$\vec{p}_{Tag} = \sum_i \vec{p}_i$

$m^2_{Tag} = \sqrt{E^2_{Tag} - \vec{p}^2_{Tag}}$

Probe $\begin{cases} \vec{p}_p = -\vec{p}_{Tag} \\ \vec{p}_\nu = \vec{p}_p - \vec{p}_\mu \end{cases}$

$E_\nu = p_\nu$

$E_\nu + E_\mu = E_p = \frac{\sqrt{s}}{2}$

→ overconstrained event

$m^2_\nu = E^2_\nu - p^2_\nu = \ldots$

