Weak Decays of «ground state» hadrons

Synopsys

- Proton stability and baryon number conservation
  - A sketch of visible and invisible proton decays
  - Baryon number conservation and the quark model
- Classification of weak decays
  - Hadronic
  - Semileptonic
  - Purely leptonic decays of
    - Pseudoscalar neutral mesons (*Flavor Changing Neutral Currents)*
    - Pseudoscalar charged mesons:
      - «helicity suppression»
      - $K \to \ell\nu_\ell \; vs \; \pi \to \ell\nu_\ell$

Proton stability

- Proton decays have been searched through:
  - Visible final states: $p \to \pi^0 \ell^+, p \to K^0 \ell^+, \ldots (\ell = e/\mu)$
    - Limits $\tau(p) > some\ 10^{33} years$, mode dependent (SuperKamiokande)
  - Invisible decays
    - SNO: tank filled with purified water.
    - p invisible decay would imply the transition $O^{16} \to N^{15\,*} \to N^{15}\gamma\ (6MeV)$
    - γ detected through Compton scattering with the water electrons: $\gamma e \to \gamma e$
    - The scattered e releases Cherenkov light in the tank, which is read out by photomultipliers
    - No signal:
    - Model independent

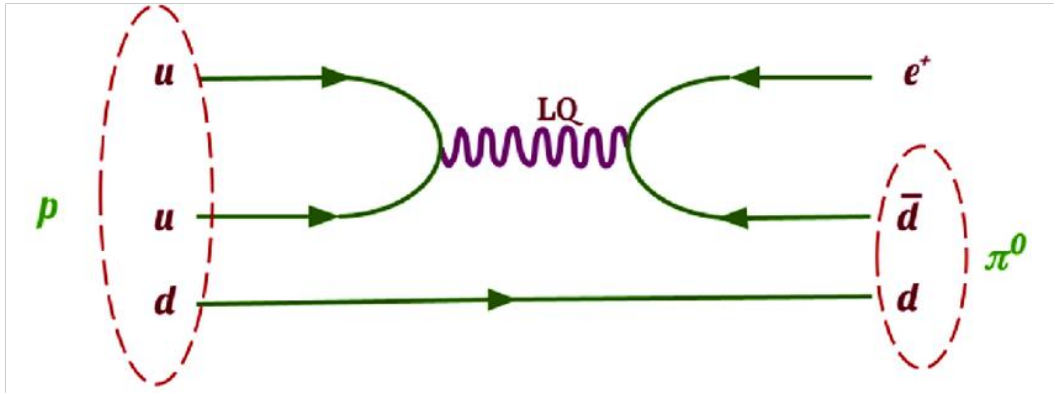
$$\boldsymbol{\tau(p) > 3 \times 10^{29} years}$$

The baryon quantum number

- Proton stability is not ensured by any fundamental symmetry
- The ***Baryon (B)*** quantum number conservation is imposed ad hoc
  - Unique case of a quantum number, ***respected by all interactions***, not dictated by fundamental symmetry.
  - We have $B(q) = -B(\bar{q}) = 1/3$
    - Half integer spin hadrons, composed by quark triplets $(q_1 q_2 q_3)$ states have B=*1*, are called baryons
    - Anti-baryons $(\bar{q}_1 \bar{q}_2 \bar{q}_3)$ have B=*-1*
    - B conservation forbids neutron-antineutron oscillations: $n \rightarrow \bar{n}$

B conservation in the quark model

- Proton decay would imply the transformation of *quarks* into *leptons*, mediated by some ad hoc new particles known as *leptoquark*
- These are predicted by some Grand Unified theories
- Proton stability imposes lower bounds upon *LQ* masses, lowe bound on their coupling to ordinary matter

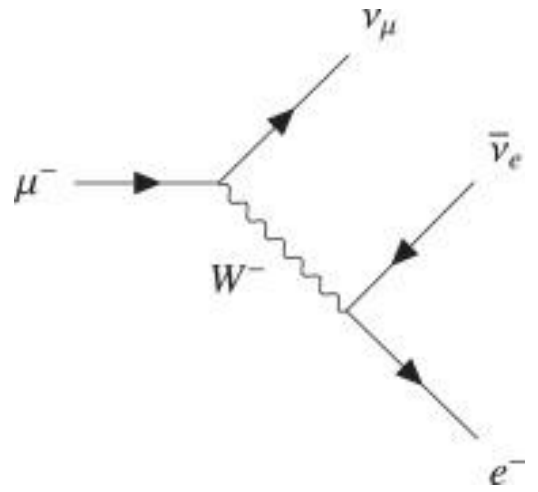


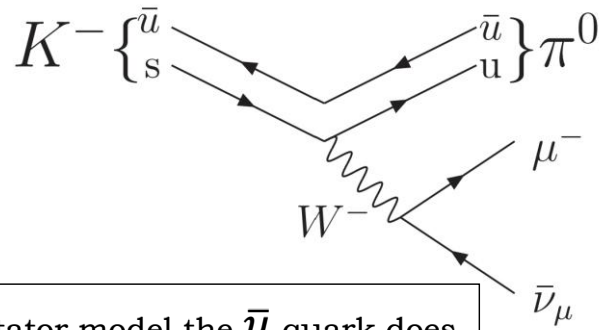
Weak hadronic decays

- Lowest mass pseudoscalar mesons and spin ½ baryons can only decay through flavor changing currents, therefore through *weak* or *e.mag*. Interactions
- We can have
  - Fully hadronic decays ($K^0 \to \pi^+\pi^-$, violating *S,I* and I_3, $\pi^0 \to \gamma\gamma$, violating *I*, …)
    - Quite difficult to compute- we will not discuss them further
  - Semileptonic decays ($n \to pe^- \bar{\nu}_e, \pi^+ \to \pi^0 e^+ \bar{\nu}_e, \ldots$)
    - (“*one hadronic current is easier than two*” G.Altarelli)
  - Leptonic decays ($h^+ \to \ell^+ \nu_\ell, h = \pi, K, D^+, D_s^+, B^+, \ell = e, \mu, \tau$)

Semileptonic Decays of Baryons and Mesons

- "Spectator model"
  - The most massive quark decays like the muon while the other(s) (*spectators*) stay unperturbed
  - True only in a naïve description (o(10%) precision)
  - In fact, we must account for strong interactions between the decaying quark, the decay product, and the spectators

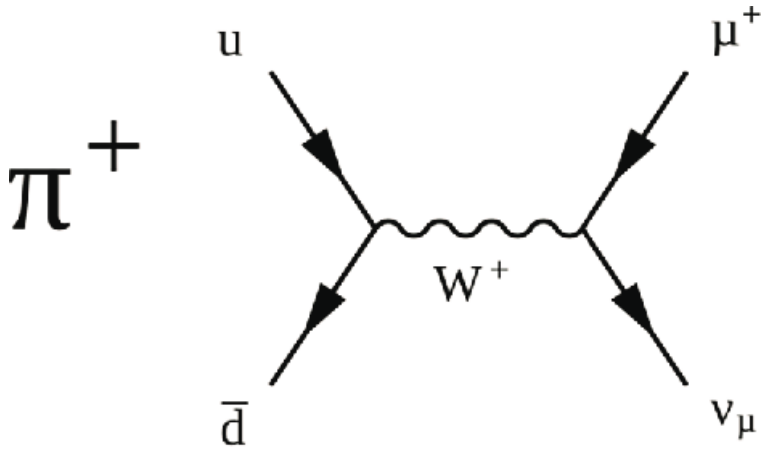




In the spectator model the $\bar{u}$ quark does not take part in the transition. For a precise description we must also consider gluon exchange with the initial *s* and final *u* quarks

Leptonic Decays of charged mesons

- Charged pion decay proceeds through the annihilation of the u *and* $\bar{d}$ quarks which produced a virtual W^+ which then materializes to $\ell^+\nu_\ell$



- According to PDG we have:
  - $\frac{Br(\pi^+\to e^+\nu_e)}{Br(\pi^+\to\mu^+\nu_\mu)} = 1.2\times10^{-4}$, but…
    - $Q(\pi\to\mu) = M(\pi) - M(\mu) \simeq 35\ MeV$
    - $Q(\pi\to e) = M(\pi) - M(e) \simeq M(\pi) \simeq\ 140\ MeV$
    - $\frac{\rho(\pi\to e)}{\rho(\pi\to\mu)} = \frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2} = 2.3$, "phase space" ratio
- Why is decay to a muon favored?

The phase space is a statistical term accounting for the number of different configurations allowed for the final state particles. The larger the p.s. the more likely the transition.
In a two-body decay we have :

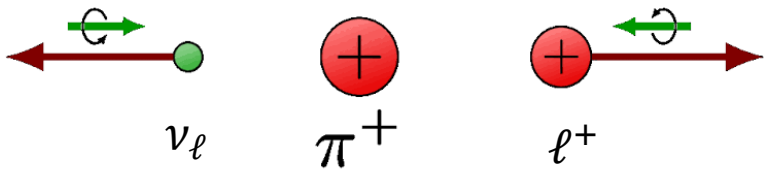
$$\boldsymbol{\rho = \frac{|\vec{p}|}{8\pi m_\pi^2}, with\ |\vec{p}| = \frac{m_\pi^2 - m_\ell^2}{2m_\pi}}$$

The «helicity suppression» in pion decay

The decay width reads as (see any text-book):

$$\Gamma(\pi \to \ell \nu_\ell) = \frac{G^2}{8\pi} f_\pi^2 m_\pi \boldsymbol{m_\ell^2} \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 ,$$

which favors the final state with the largest lepton mass (in this case, the muon).
This is a consequence of *angular momentum* (j_z) *conservation* and the *structure of the vector current*, which implies *helicity conservation in the ultrarelativistic case*

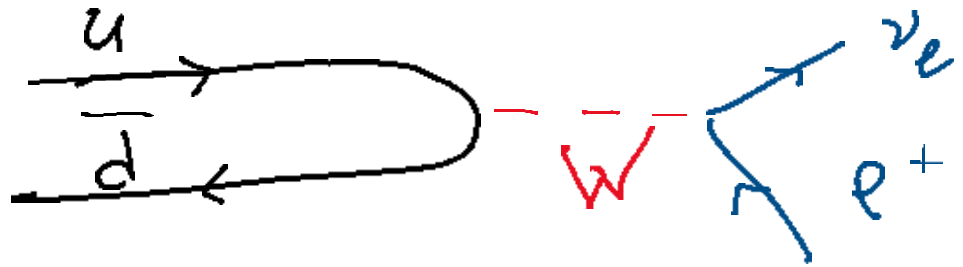


The picture represents a sketch of the decay in the pion rest frame.
To conserve (j_z) the charged lepton is produced with the wrong helicity.
This is not forbidden, but it is depressed by a factor $\propto (m_\ell/m_\pi)^2$, which favors decay to a muon
A synthetic proof follows.
A detailed discussion can be found at the link below:

https://pdg.web.cern.ch/pdg/2020/reviews/rpp2020-rev-pseudoscalar-meson-decay-cons.pdf

Derivation of the helicity suppression

$$\sigma = \frac{1}{2m_\pi} \cdot |\mathcal{M}|^2 \cdot \rho$$



$\rho = \frac{1}{8\pi m_\pi^2} \frac{m_\pi^2 - m_e^2}{2m_\pi}$ is the phase space factor

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\pi^\lambda J_{\lambda,e}$$

$J_{\lambda,e} = \bar{\psi}_\nu \gamma_\lambda \frac{1-\gamma^5}{2} \psi_e$ is the lepton current (Dirac)

$J_\pi^\lambda = f(\pi) p_\pi^\lambda$ is the only possible expression for the pion current

$= f(\pi)(p_e^\lambda + p_\nu^\lambda)$ 4momentum conservation

Contracting the currents

$$J^\mu_\pi J_{\ell e} = f_\pi (p_\ell^\mu + p_\nu^\mu)\, \bar\psi_\nu \gamma_\mu \frac{1-\gamma^5}{2} \psi_e$$

$$= f_\pi \left[\bar\psi_\nu \gamma_\mu p_e^\mu \frac{1-\gamma^5}{2} \psi_\ell + \bar\psi_\nu \gamma_\mu p_\nu^\mu \frac{1-\gamma^5}{2} \psi_e \right]$$

Dirac equation $\begin{cases} \gamma_\mu p_e^\mu \psi_e = m_e \psi_e \\ \bar\psi_\nu \gamma_\mu p_\nu^\mu = m_\nu \bar\psi_\nu \simeq 0 \end{cases}$

$$q^\mu J_\mu = f_\pi m_\ell \left[\bar\psi_\nu \psi_e - \bar\psi_\nu \gamma^5 \psi_e \right]$$

by contracting the spinors and taking the square:

$$|M|^2 \propto f_\pi^2 m_\ell^2 |\vec p| = f_\pi^2 \frac{m_\pi^2 - m_\nu^2}{m_\pi} \cdot m_e^2$$

→ expresses the HELICITY SUPPRESSION

Leptonic decays of charged pion and kaon

	π^+	K^+
Mass (MeV)	139,6	493,7
Pseudoscalar constant f	130,2	155,7
Lifetime PDG τ (ns)	26,0	12,4
Phase space ρ_e	1	1
Phase space ρ_μ	0,434	0,95
$Br(e\nu_e)$ PDG	$1{,}23 \times 10^{-4}$	$1{,}60 \times 10^{-5}$
$Br(\mu\nu_\mu)$ PDG	1	0,636
Br (e/μ) PDG	$1{,}23 \times 10^{-4}$	$2{,}52 \times 10^{-5}$
Phase space e/μ	2,3	1,05
Br(e/μ) Computed	1,26	$2{,}49 \times 10^{-5}$
Lifetime Computed (μ), ns	25,1	0,65
$R = \tau\,(Computed)/\tau\,(PDG)$	**0.985**	**0.052**

Phase space:

$$\rho_{ell} = \left(1 - \frac{m_\ell^2}{m_h^2}\right),$$

Partial width:

$$\Gamma_\ell = \Gamma(h \to \ell\nu_\ell) = \frac{G^2}{8\pi} f_h^2 m_h \boldsymbol{m_\ell^2} \left(1 - \frac{m_\ell^2}{m_h^2}\right)^2$$

Total width:

$$\Gamma = \Gamma_\ell/B\,r(\ell\nu_\ell)$$

Lifetime:

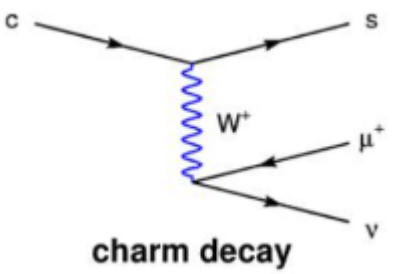
$$\tau = \hbar/\Gamma = \hbar\, Br\,(\ell\nu_\ell)/\Gamma_\ell$$

The ratios of Branching Ratios is consistent the measurements (PDG)

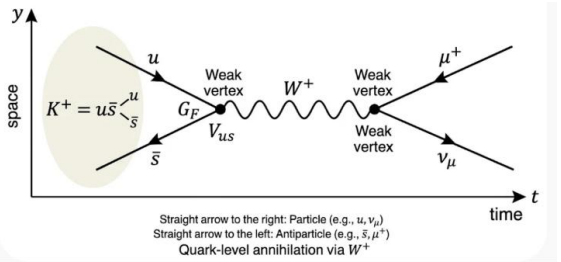
The computed lifetimes are **slightly** (π) or **very largely** (K)different from the PDG values

The Cabibbo angle

- Left-handed quarks couple with the W boson in pairs: $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$
  - Therefore, the s and the b, lighter than their partners c and t, should be stable,
  - … and the annihilation $K^+ \to \mu^+ \bar{\nu}_\mu$ should not take place



- This puzzle was solved by Nicola Cabibbo (when only u,d,s were supposed to exist), proposing that the u pairs with a linear superposition of d and s: $\binom{u}{d'} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_c \end{pmatrix}$



- We must recompute the rates as

$$\Gamma(\pi \to \ell\nu_\ell) = \frac{G^2}{8\pi} f_\pi^2 m_\pi \boldsymbol{m_\ell^2} \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 \boldsymbol{\cos^2\theta_c}$$

$$\Gamma(K \to \ell\nu_\ell) = \frac{G^2}{8\pi} f_K^2 m_K \boldsymbol{m_\ell^2} \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2 \boldsymbol{\sin^2\theta_c}$$

$$\boldsymbol{tg^2\theta_c} = \frac{\tau_K^{Computed}/\tau_K^{PDG}}{\tau_\pi^{Computed}/\tau_\pi^{PDG}},$$

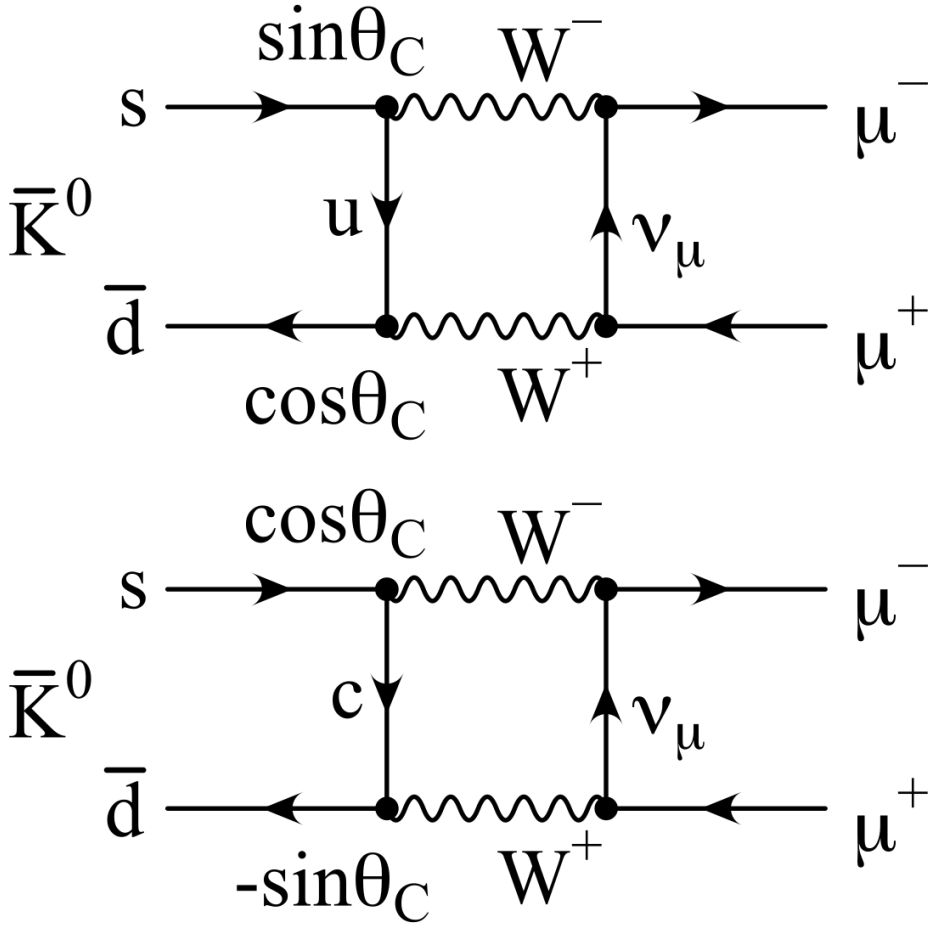
$$\boldsymbol{\theta_c = 0.226\ (\simeq 13^0)}$$

(see previous slide)

In facts, Cabibbo introduced this rotation to explain inconsistencies between nuclear β decays and the muon decay. We forced the historical perspective for sake of clarity.

The GIM mechanism

- The coupling of u quark s allows for the decay $\bar{K}^0 \rightarrow \mu^+\mu^-$, as depicted in the top diagram, which however diverges
- In 1970 Glashow Iliopoulos and Maiani showed that the divergence could be canceled by introducing a new quark *c*harm, with complementary couplings to d and s (bottom plot)
- The sum of the two diagrams allowed a prediction for the c mass of about 1.5 GeV
  - $\Gamma(\bar{K}^0 \rightarrow \mu^+\mu^-) \propto \cos^2\theta_c \sin^2\theta_c \times[\frac{m_c^2}{m_W^2} - \frac{m_u^2}{m_W^2}]$



The Cabibbo Kobayashi Maskawa matrix

- In 1964 Cronin and Fitch observed CP violation in the decay $K_L \to \pi^+\pi^-$
  - K_L is a CP-odd state decaying to three-pions final state
  - The two-pions final state was observed with a frequency of about 2 per mille
  - While it was well known that charged weak currents fully violate P and C symmetries, the combined CP symmetry was expected to be respected also by WI
  - Superweak theories were proposed to explain this tiny effect
- In 1970 Kobayashi and Maskawa showed that extending the number of quark families to three, a non-trivial phase, accounting for CP violation, could be naturally embedded in the Standard Model.
  - The c quark had not yet been observed!
- The 3x3 matrix which extends the Cabibbo mixing to the case of three quark families is called the CKM matrix.