Dr. Ripalta Amoruso

Tutor meetings:

Starting from FRIDAY 24th of September

16:30-18:30

Same time/place for the following weeks

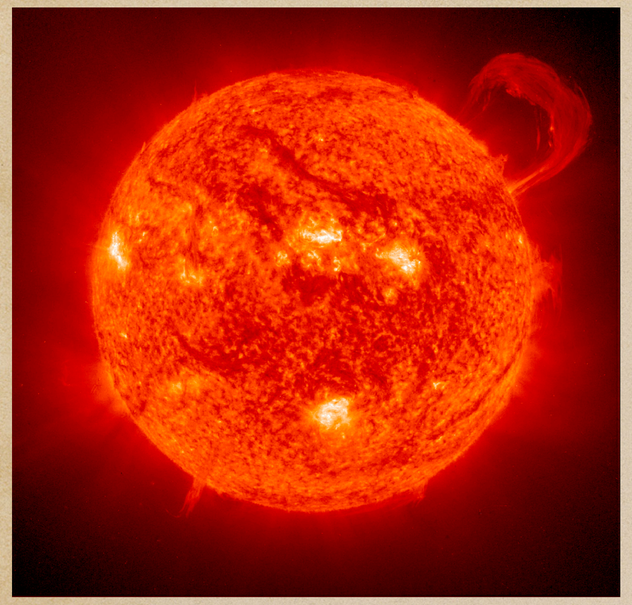
Stars &

Stellar evolution

Topic 5

Material from
Chap. 9, 11, 12, 15 "Fundamental Astronomy" edition, by H. Karttunen, H. Oja, M. Poutanen, K. J. Donner

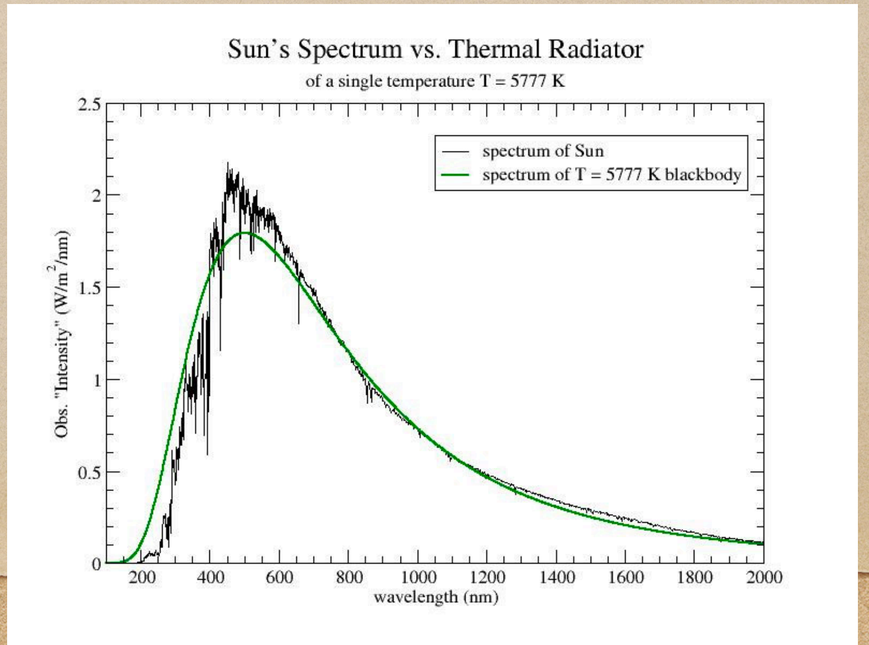
The stars



The stars are in first approximation spheres of gas (or plasma, that is, ionized gas because it is very hot)

In their nucleus thermonuclear reactions produce energy (typically burning H in He)

In the first approximation, their spectral energy distributions can be approximated with those of black bodies of different temperatures



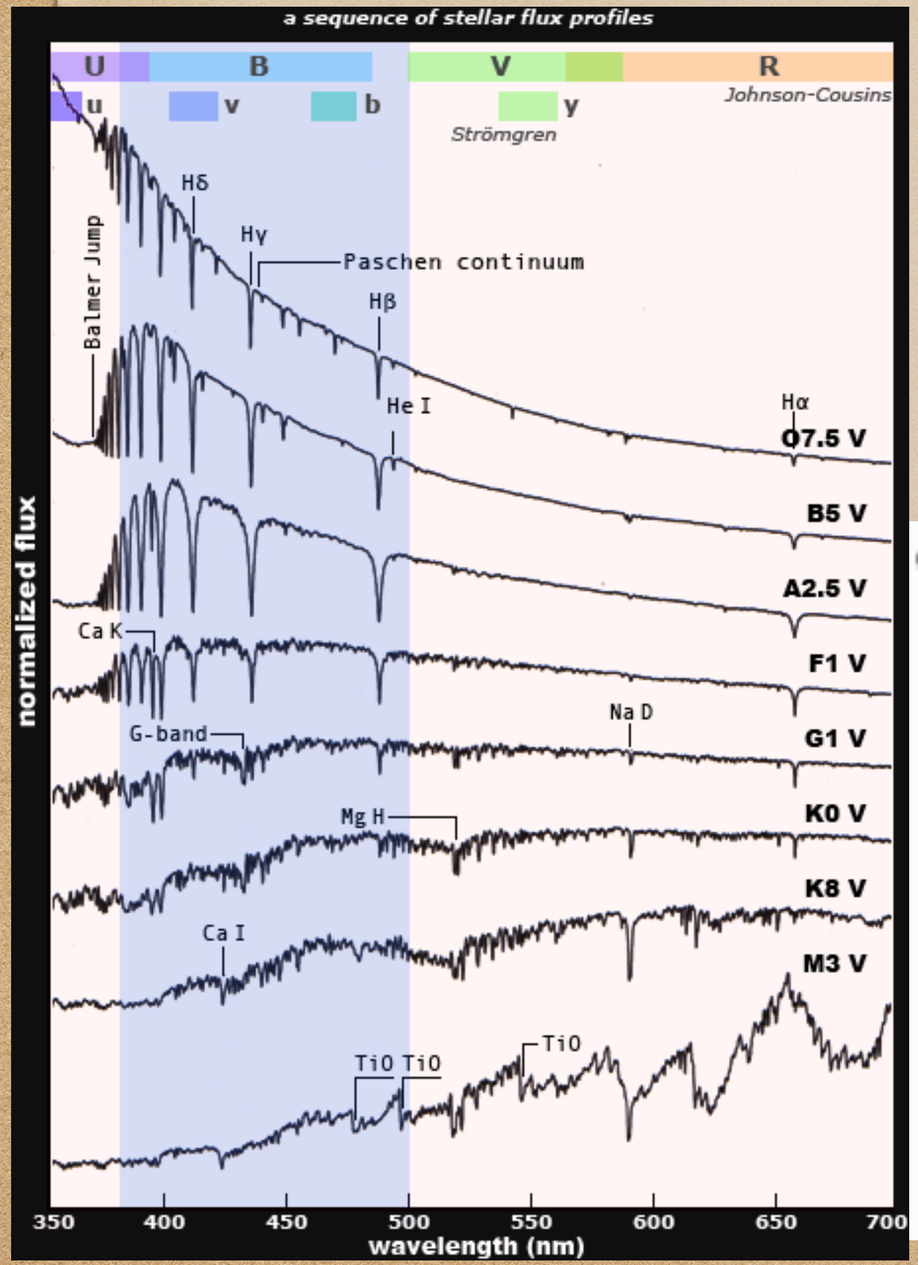
Spectral classes

The spectra of the stars can be classified according to the spectral lines present:
Harvard spectral classification

Class	Temperatura (Kelvin)	Colore convenzionale	Colore apparente	Massa (masse solari)
O	≥ 33 000 K	blu	blu	≥ 16 $M_\odot$
B	10 000–33 000 K	azzurro	blu chiaro	2,1–16 $M_\odot$
A	7 500–10 000 K	bianco	azzurro	1,4–2,1 $M_\odot$
F	6 000–7 500 K	bianco-giallo	bianco	1,04–1,4 $M_\odot$
G	5 200–6 000 K	giallo	bianco-giallo	0,8–1,04 $M_\odot$
K	3 700–5 200 K	arancione	giallo-arancione	0,45–0,8 $M_\odot$
M	≤ 3 700 K	rosso	arancio-rosso	0,08-0,45 $M_\odot$

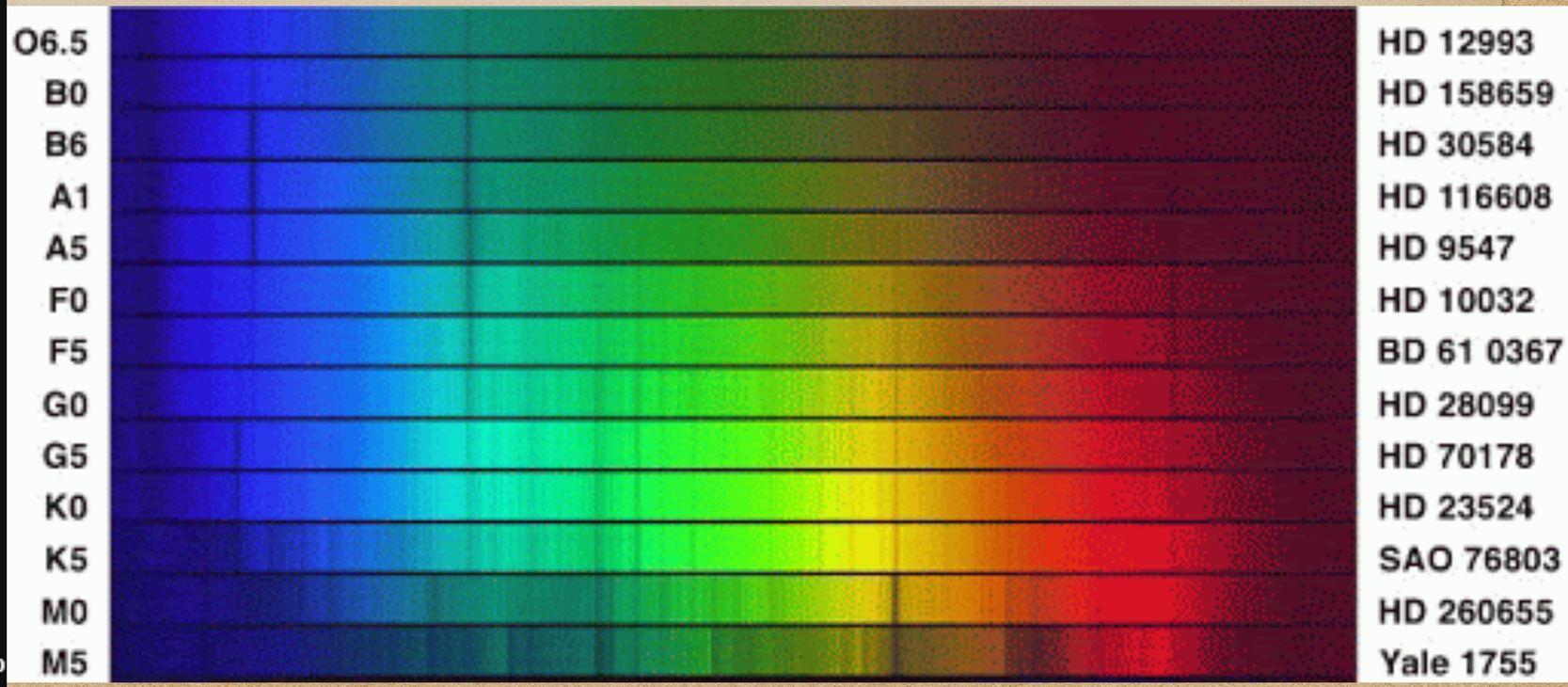
In practice, each spectral type corresponds to a defined temperature range

Spectral classes



What changes?

- Spectrum shape (black body)
- Different lines, with different intensity
- H: strong in types O, B, A, F, almost absent in G, K, M
- High ionization lines: only in O, B
- K, M: deep absorption bands



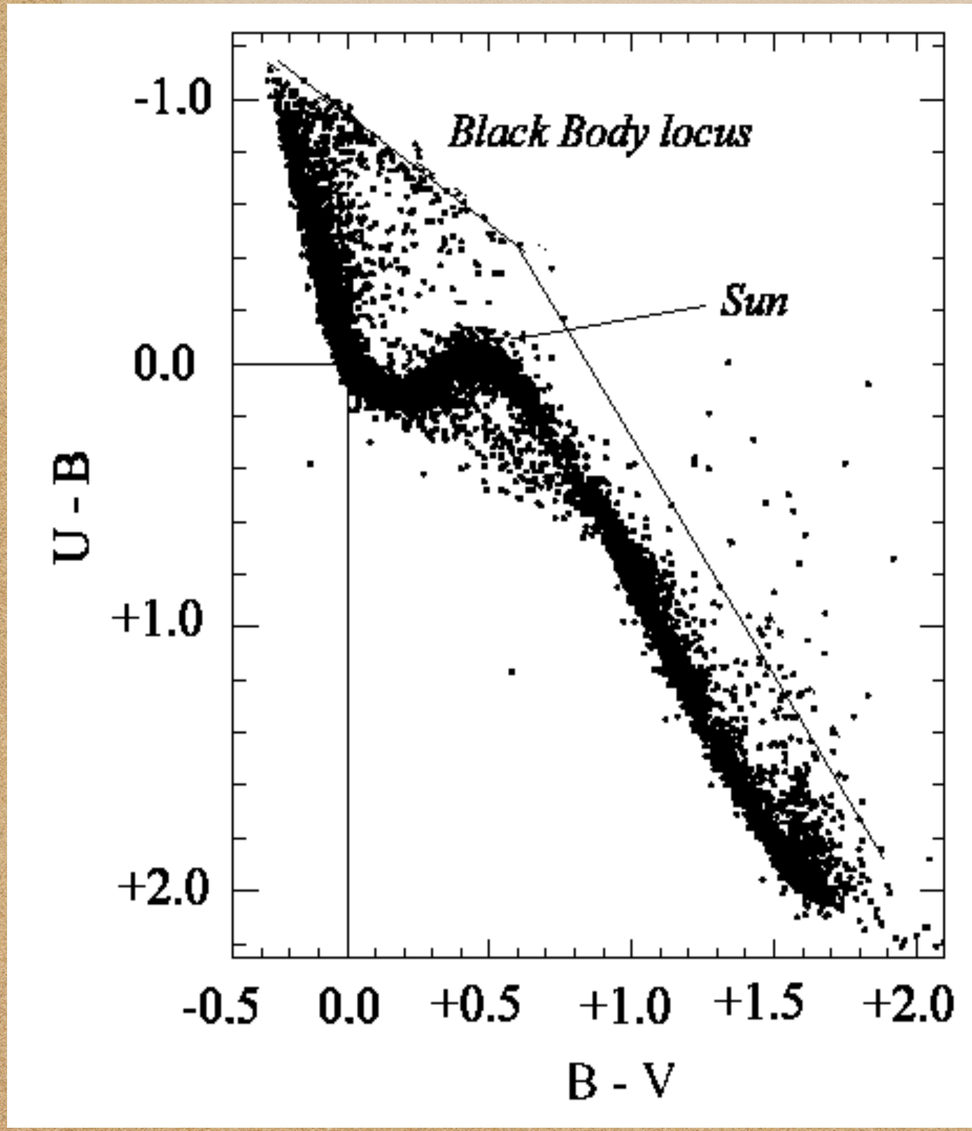
The properties of a star are determined by its radius and temperature:

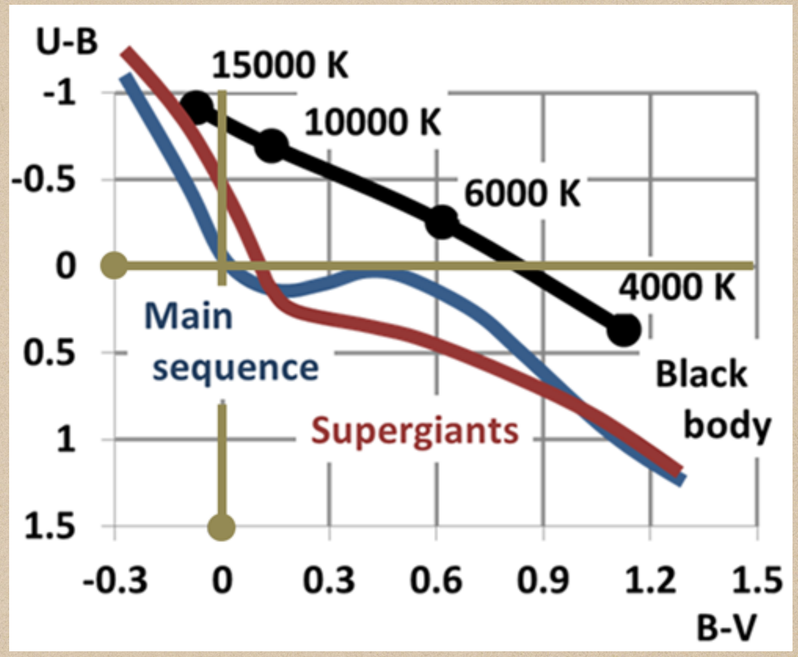
$$L = 4\pi R^2 \sigma T^4$$

However, stars are not exactly black bodies. It is defined as effective temperature T_{eff} of a star like the temperature that a black body of radius R should have to radiate the same luminosity as the star:

$$\sigma T_{eff}^4 \equiv \frac{L}{4\pi R^2}$$

Color-color diagram for bright stars M<+7

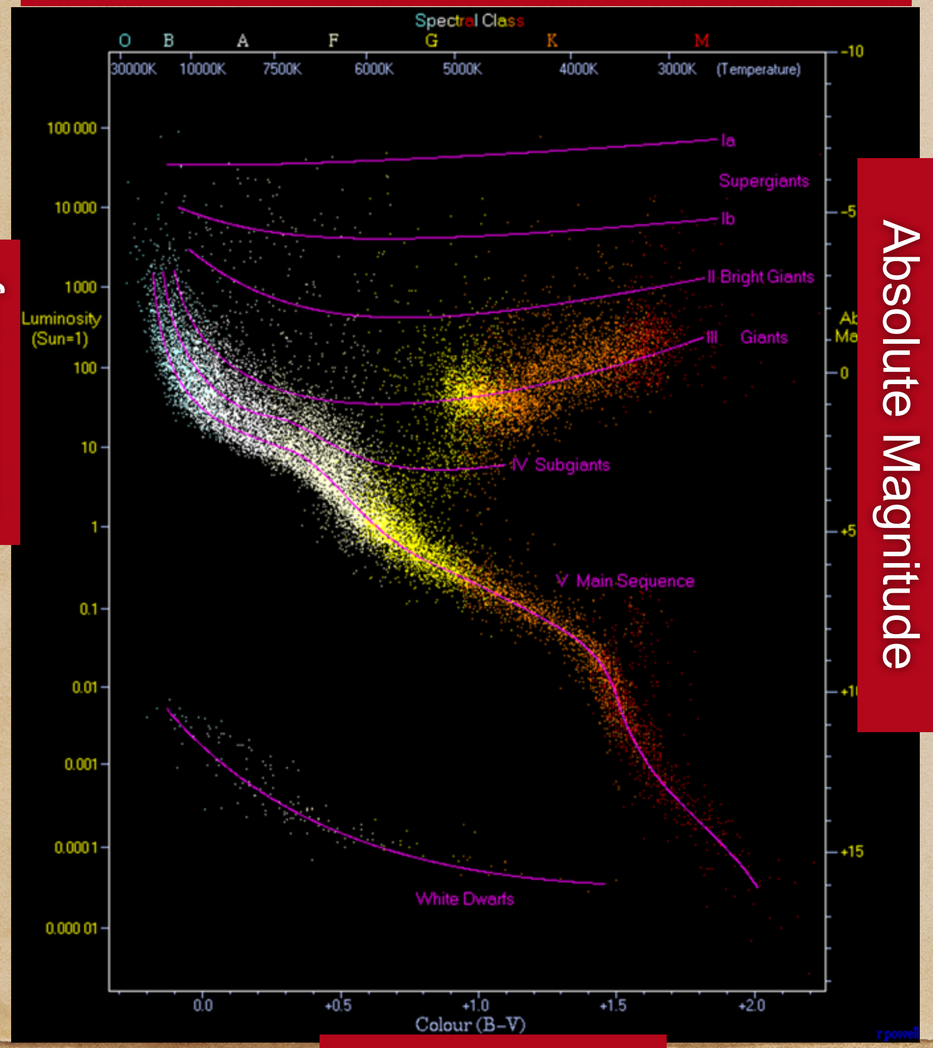




The stars occupy a well-defined band called t***he main sequence*** that differs from the location of the black body especially in the area of stars like Vega (again, ***in reality, stars are not black bodies***).

Hertzsprung-Russell diagram

Temperature (or spectral class)



Luminosity

Absolute Magnitude

Colour (B-V)

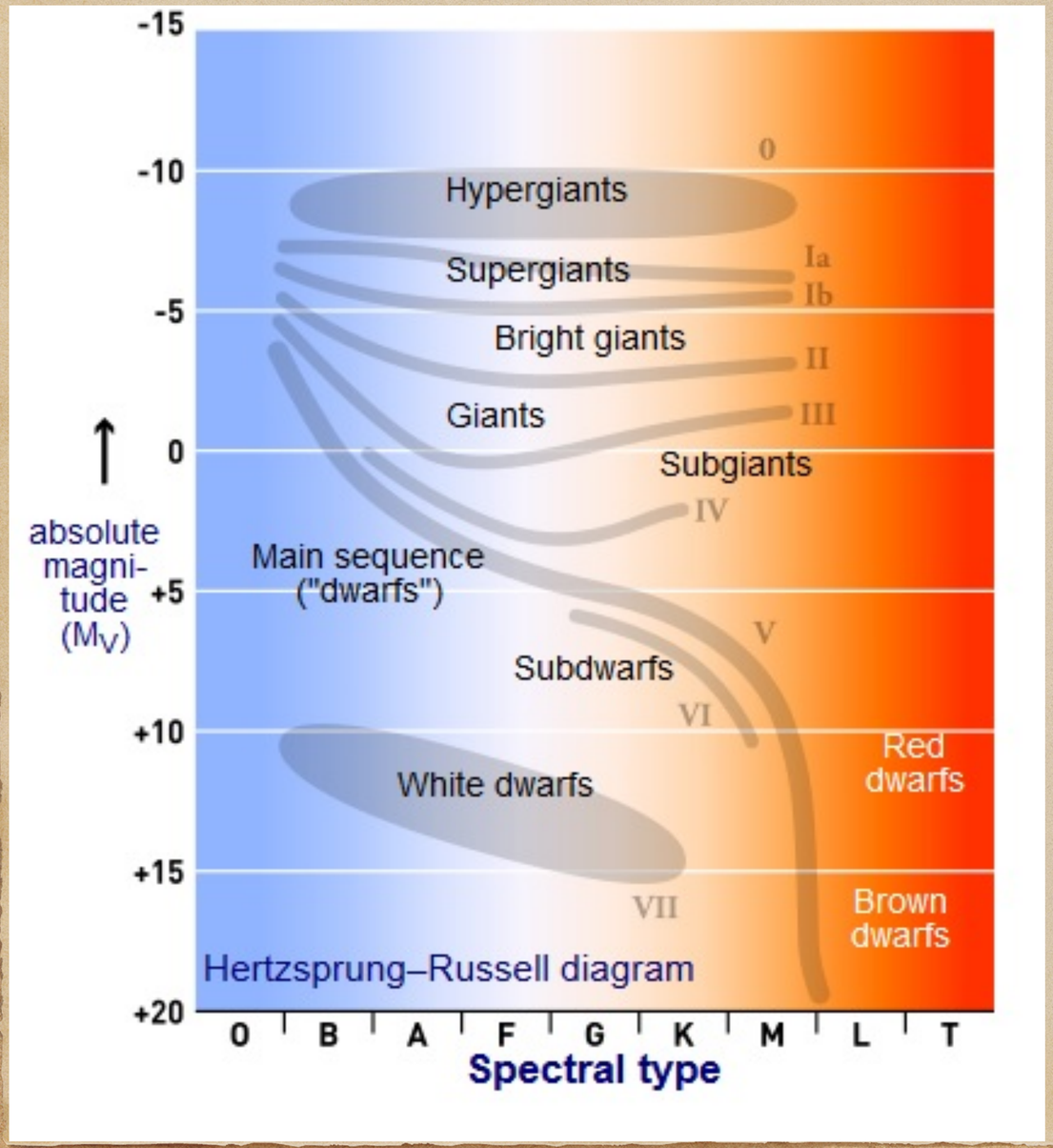
Fundamental diagram:

- Colour vs absolute magnitude
- Temperature vs luminosity

Colour B-V is an estimate of the ***color temperature*** T_c (defined so that a temperature BB T_c would have the same colour index).

The absolute magnitude determines the luminosity of the star (and therefore its radius)

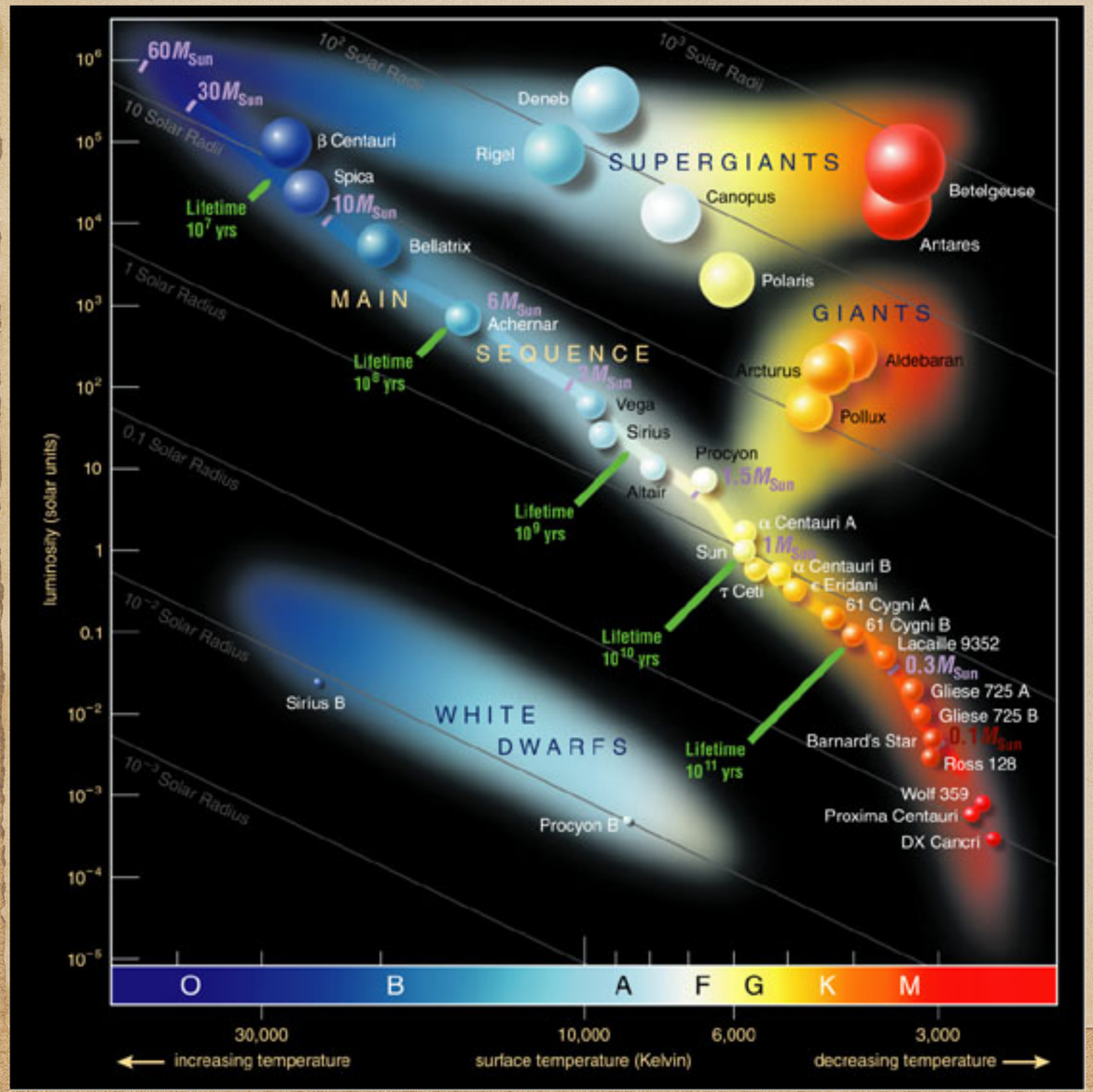
Hertzsprung-Russell diagram



Luminosity classes:

I Supergiants
II Bright Giants
III Giants
IV Subgiants
V Main sequence stars
VI Sub-dwarfs
VII White Dwarfs

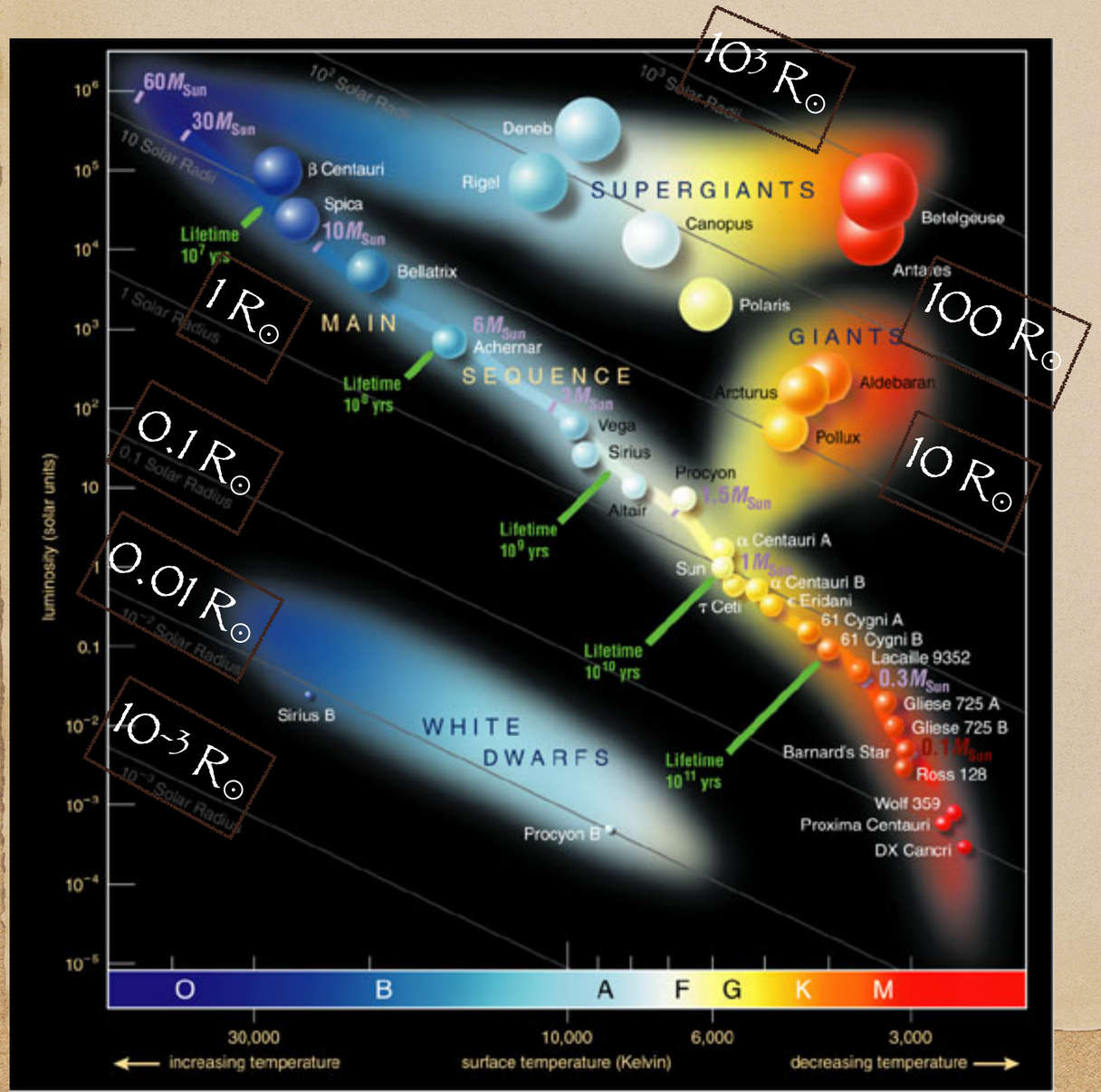
Hertzsprung-Russell diagram



The stars do not randomly fill the H-R diagram but are grouped into 4 main groups:

1. *Main sequence*
2. *Super giants*
3. *Giants*
4. *White dwarfs*

Hertzsprung-Russell diagram



Loci of constant radius:

$$L = \sigma T^4 \cdot 4\pi R^2$$

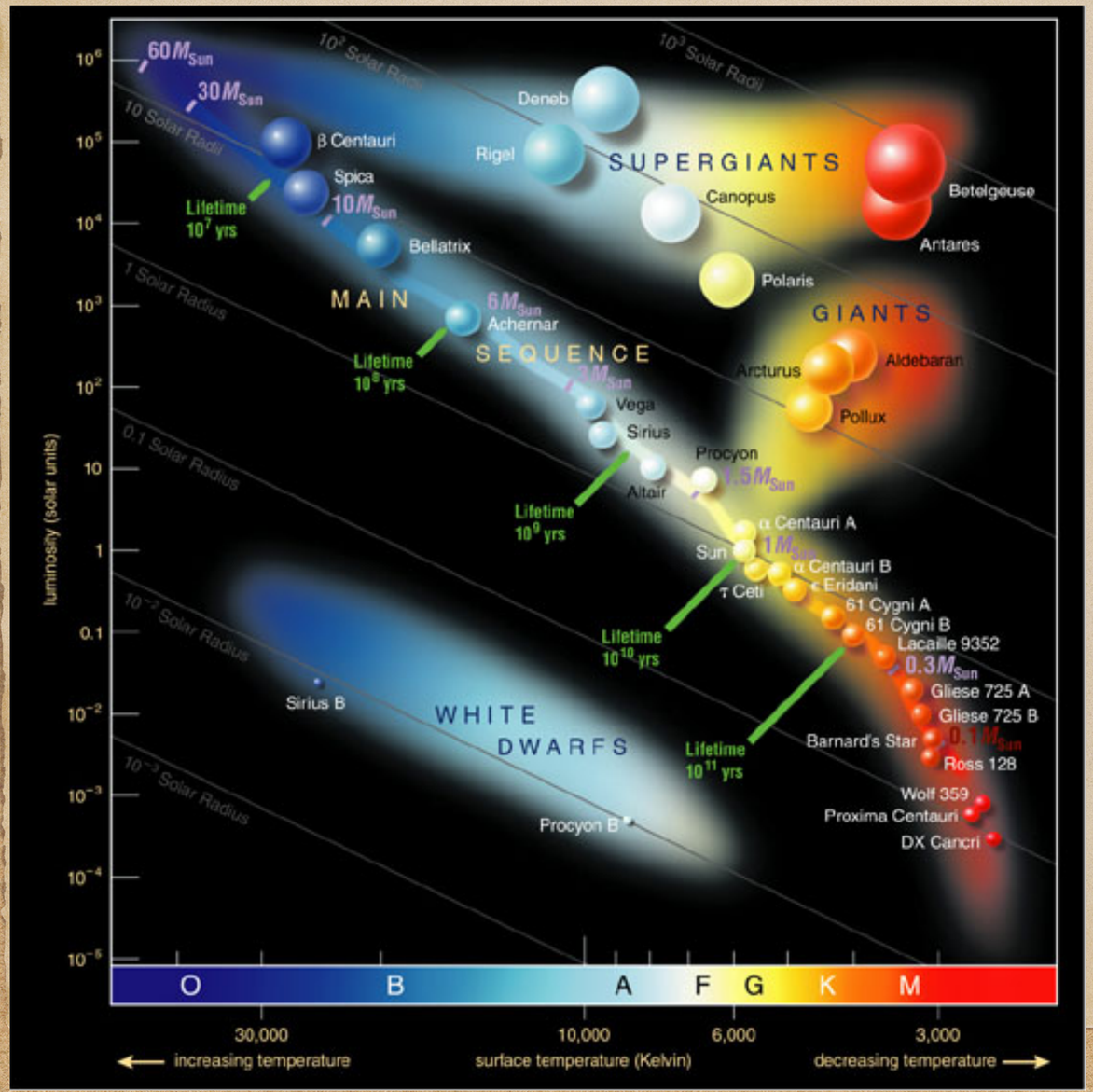
$$\log L = 4 \log T + 2 \log R + c$$

$$\log R = \frac{\log L}{2} - 2 \log T - c$$

Diagonals in the HR diagram:

- The radius grows as L grows
- The radius decreases as T grows

Main sequence



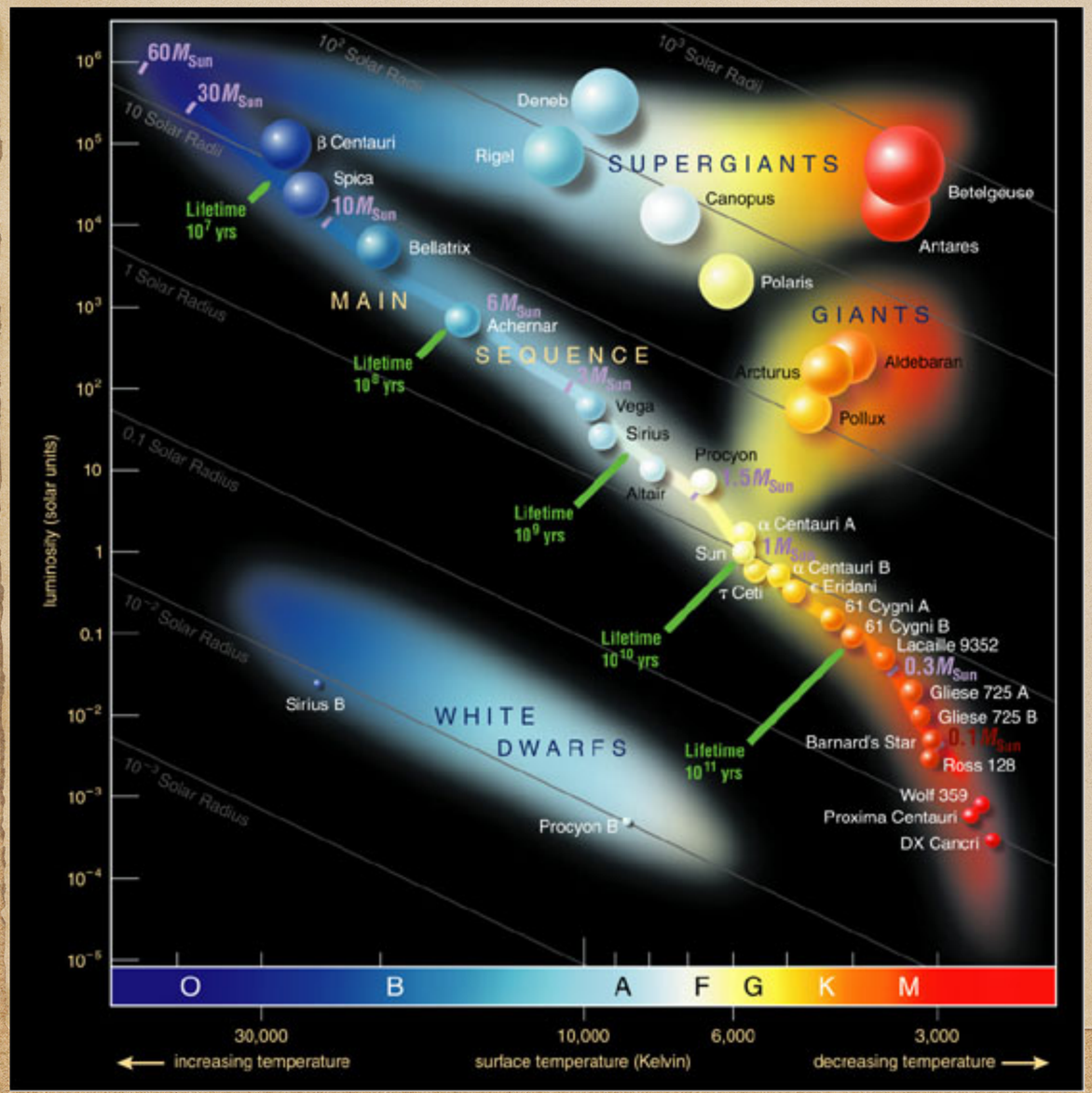
All the stars along the main sequence are burning H into He in their nuclei.

It extends from hot and very massive stars (top left) to low mass and cold stars (bottom right)

They have very different radii (oblique lines in the figure), therefore very different surface gravity:

$$g = \frac{GM}{R^2}$$

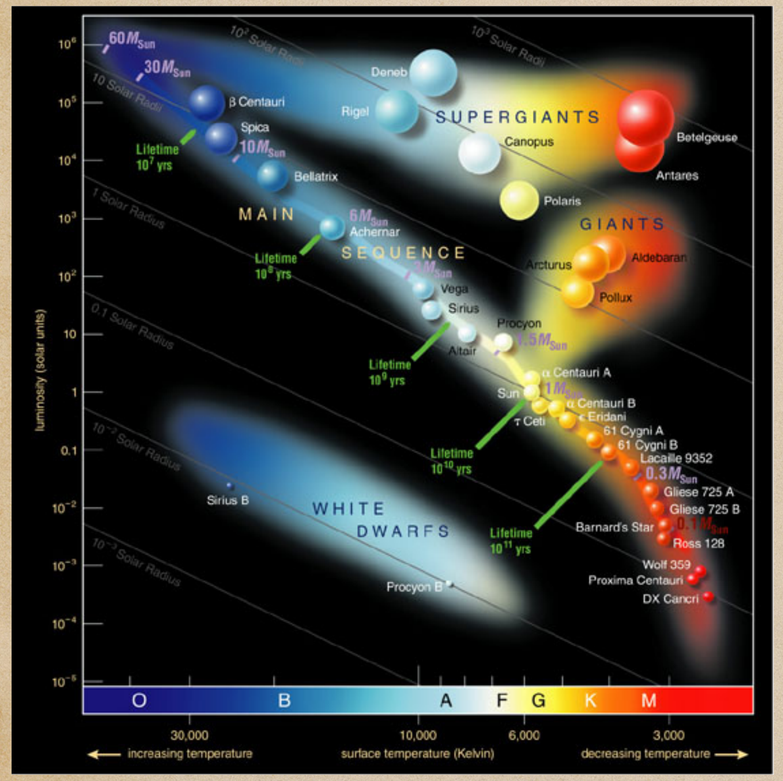
Main sequence



If the distance of a star, and therefore its luminosity, is known and if in addition its **surface gravity is obtained from the width of the line in a spectroscopic observation**, we obtain the stellar mass from these parameters. It turns out that for main sequence stars:

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{3.5}$$

Main sequence phase duration



The main sequence stars burn at their center H in He

When in the center all H turns into He the thermonuclear reactions stop and this phase ends

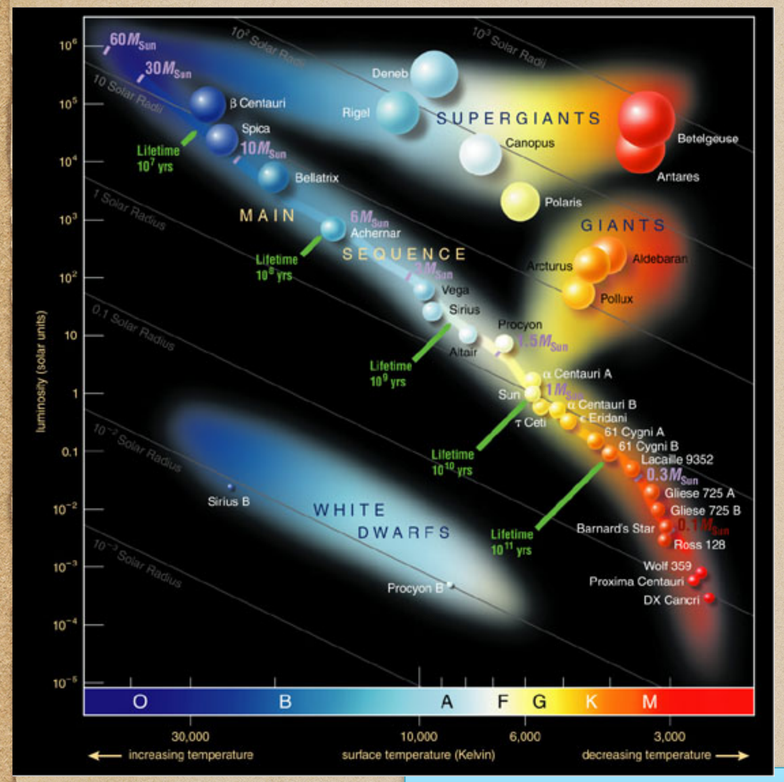
The total energy produced at this stage is:

$$E_{MS} = 0.1 \times Mc^2 \times 0.007$$

Total fraction of burned H

thermonuclear reaction efficiency

Main sequence phase duration



So the duration of the MS phase will be :

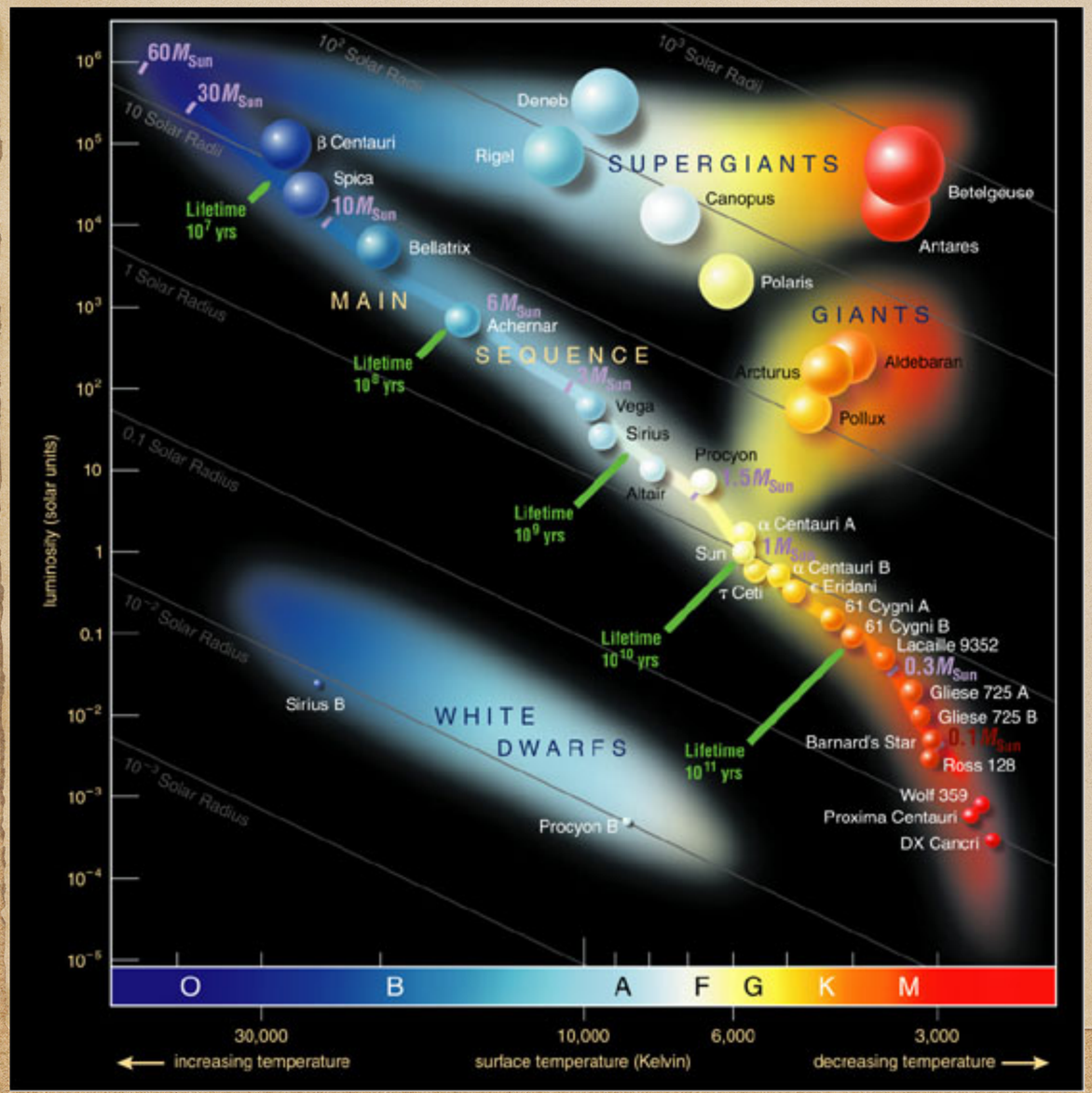
$$t_{MS} = \frac{E_{MS}}{L} \approx 8 \times 10^9 \frac{M / M_\odot}{L / L_\odot} yr \approx$$

$$\approx 8 \times 10^9 \left(\frac{M}{M_\odot} \right)^{-2.5}$$

The main sequence life is inversely proportional to the mass elevated to 2.5

Less massive stars live much longer, that is, they spend more time on the main sequence, burning hydrogen

Main sequence phase duration

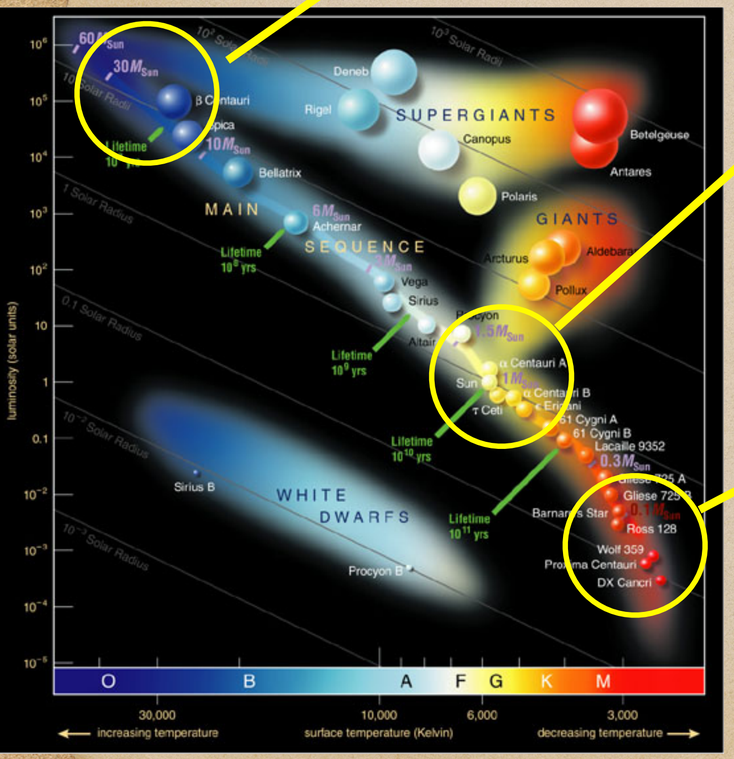


The stellar mass determines both the surface temperature T and the luminosity since it is the key factor in the speed of the burning star H.

The massive stars start their life with a larger Hydrogen reservoir but burns it very quickly (high luminosity)!

Main sequence phase duration

O stars (30-60 $M_\odot$): they spend a few million years in MS

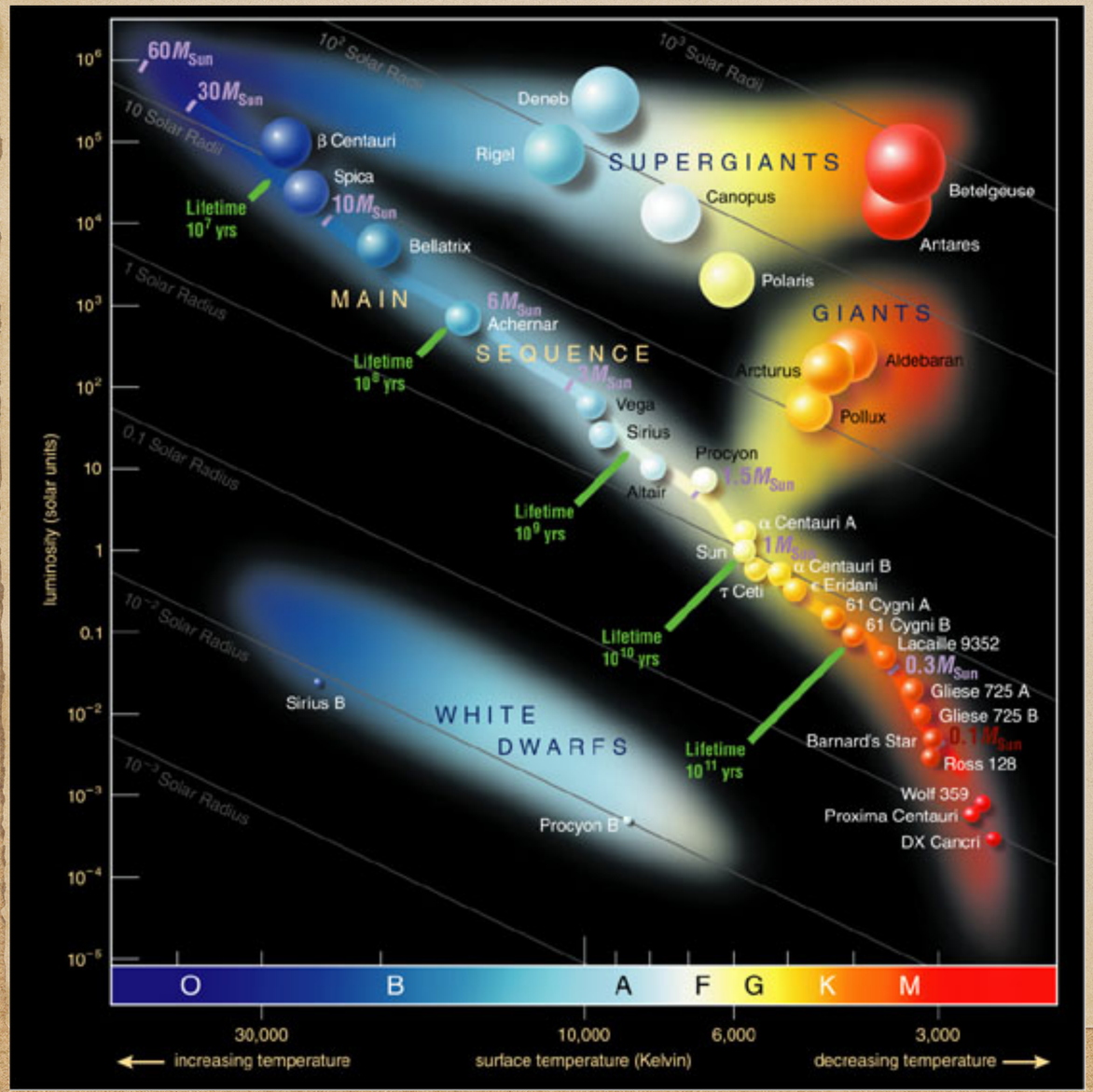


Sun (1 $M_\odot$): it passes in MS 8-10 billion years

Stars with M<0.7$M_\odot$: They pass in MS>13.5 billion years (age of the Universe). They are all still in MS!!!

Mass $M_\odot$	T_{MS} [yr]
0.1	$2.5 \cdot 10^{12}$
0.5	$4.5 \cdot 10^{10}$
1.0	$8 \cdot 10^{9}$
1.25	$4.5 \cdot 10^{9}$
1.5	$3 \cdot 10^{9}$
3	$5 \cdot 10^{8}$
5	$1.4 \cdot 10^{8}$
9	$3.3 \cdot 10^{7}$
15	$9.2 \cdot 10^{6}$
25	$2.5 \cdot 10^{6}$

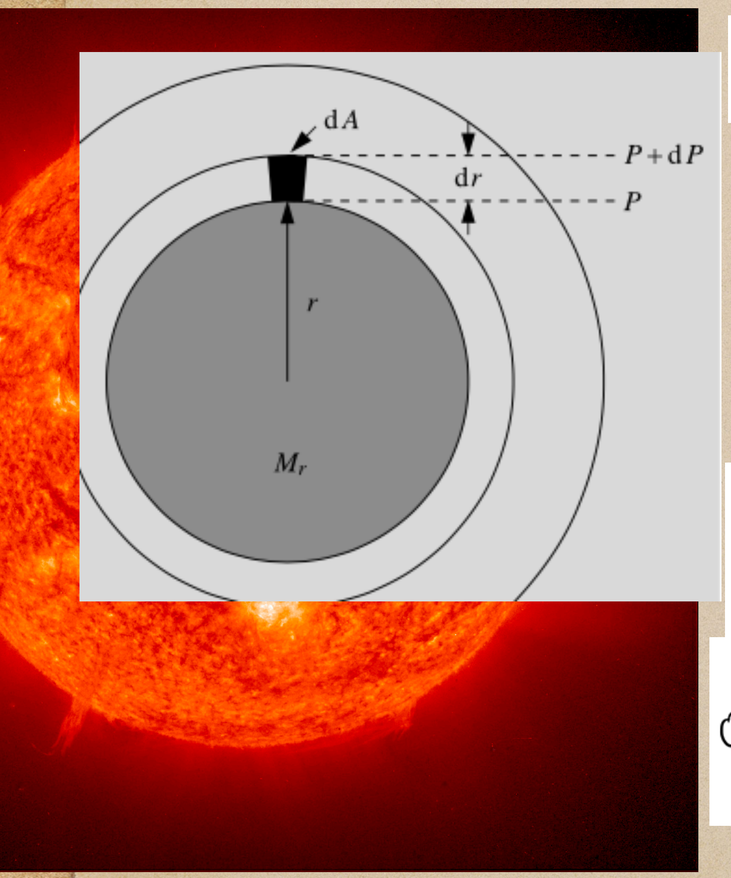
Hertzsprung-Russell diagram



1. Main sequence
2. *Supergiants*
3. *Giants*
4. *White dwarfs*

Equations of the stellar structure

Stars (like planets) are in **hydrostatic equilibrium:**



$dV = dA \cdot dr$ → cilindric volume at distance r from the center.

$dm = \rho\, dA\, dr$ → mass volume

$$dF_g = -\frac{G M_r\, dm}{r^2} = -\frac{G M_r\, \rho}{r^2} dA\, dr$$ → gravitational force on dV element

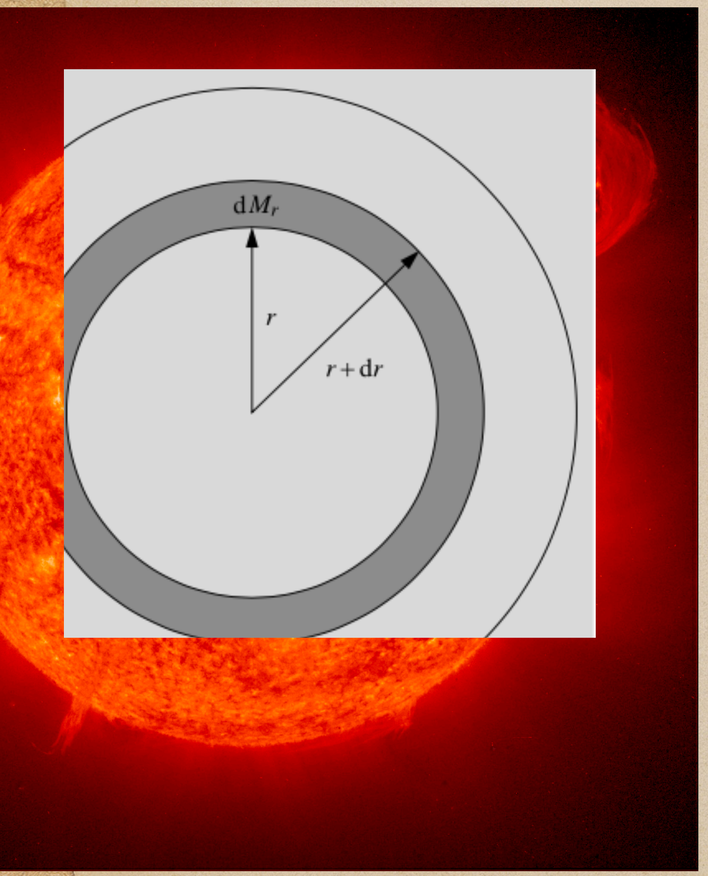
$$dF_p = P\,dA - (P+dP)\,dA = -dP\,dA$$ → pressure force on dV element

$$dF_g + dF_p = 0 \qquad -\frac{G M_r\, \rho}{r^2} dA\, dr - dP\,dA = 0$$ → equilibrium

$$\frac{dP}{dr} = -\frac{G M_r\, \rho}{r^2}$$

1. Hydrostatic equilibrium equation

Equations of the stellar structure



Shell volume x density

$$dM_r = 4\pi r^2 \rho dr$$

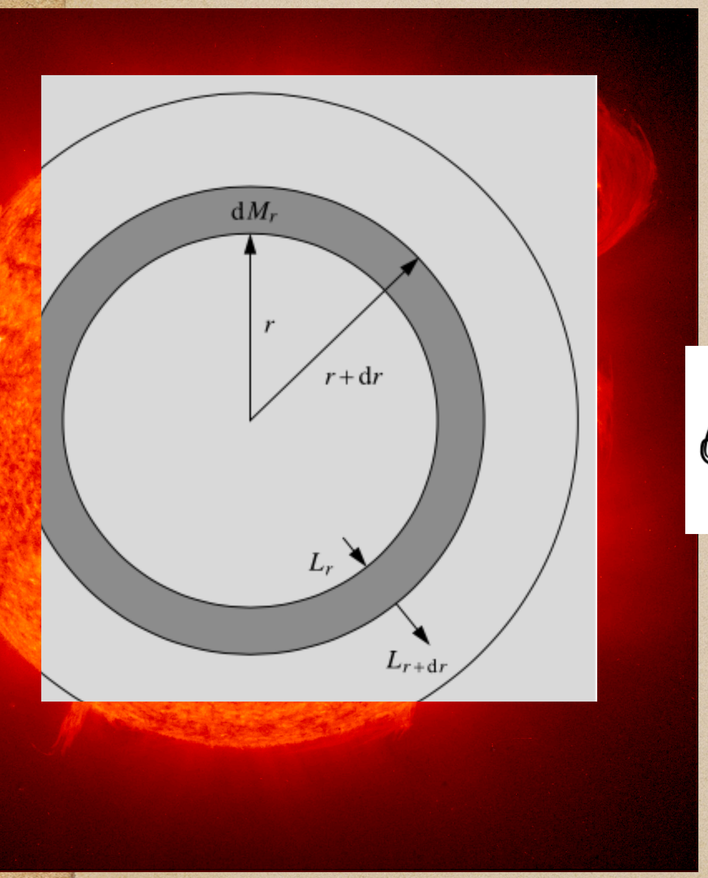
→ Mass contained in a thick shell dr at a distance r from the center

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

2. Mass continuity equation

Equations of the stellar structure

The energy produced by the star must be transported to the surface and emitted



L_r → Energy flowing through the sphere of radius r per unit of time

Energy production coefficient=energy release per unit time and mass

$$dL_r = L_{r+dr} - L_r = \varepsilon dM_r = 4\pi r^2 \rho \varepsilon dr$$

→ Energy produced in the shell dr

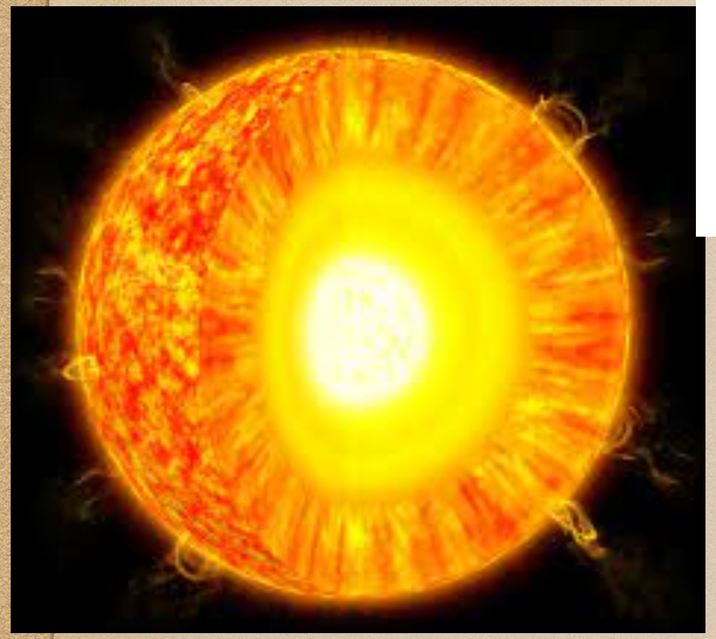
$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

3. Energy conservation equation

Energy is produced essentially in the core

The rate at which energy is produced depends on the distance to the centre. Essentially all of the energy radiated by the star is produced in the hot and dense core. In the outer layers the energy production is negligible and L_r is almost constant.

Equations of the stellar structure



$$\frac{dT}{dr}$$

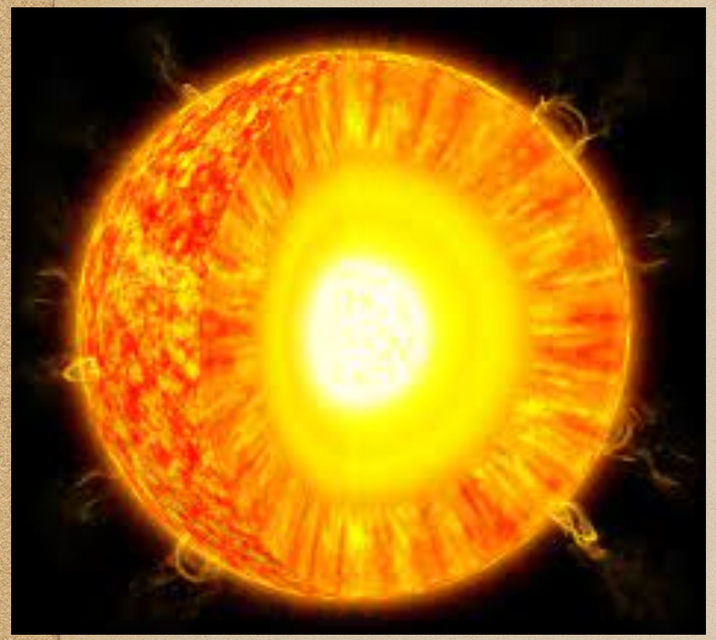
→ Temperature gradient depending on the distance from the center

The shape of this gradient depends on the energy transport mechanism:

1. Conduction
2. Convection
3. Radiation

1. Requires free electrons that carry energy; inefficient in normal stars, where electrons move little; efficient only in compact stars (white dwarfs or neutron stars)	2. It is due to macroscopic movements of material: hot "bubbles" rise, cold ones go down; a net flow of heat upwards is obtained	3. The transport takes place through electromagnetic waves that transport energy

Equations of the stellar structure



$\frac{dT}{dr}$ → Temperature gradient depending on the distance from the center

4. Temperatura gradient

$$\frac{dT}{dr} = -\frac{3}{4ac} \cdot \frac{K\rho}{T^3} \cdot \frac{L_r}{4\pi r^2}$$

→ radiative gradient

One or the other is used depending on which is the least steep; a region becomes convective if the radiative gradient exceeds the convective one

$a = \frac{4\sigma}{c}$

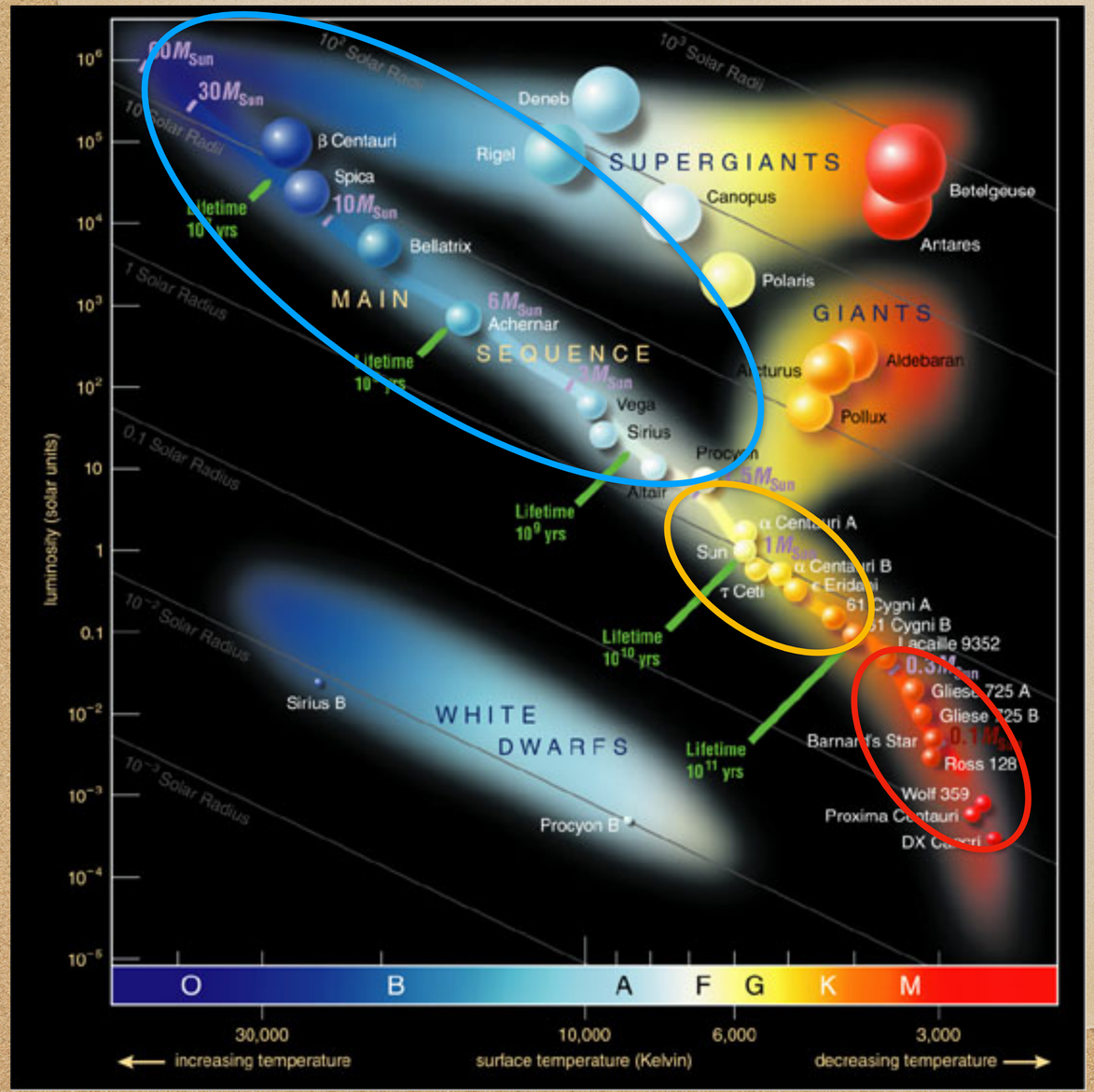
K → Absorption coefficient

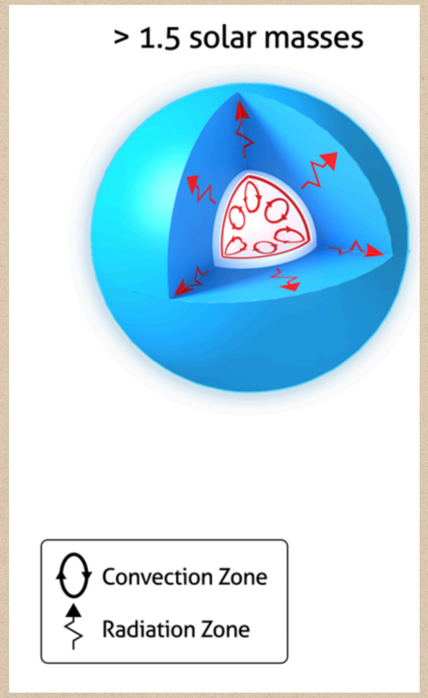
$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}$$

→ convective gradient

γ → Adiabatic exponent

Stellar radiative transport





M > 1.5M⊙:

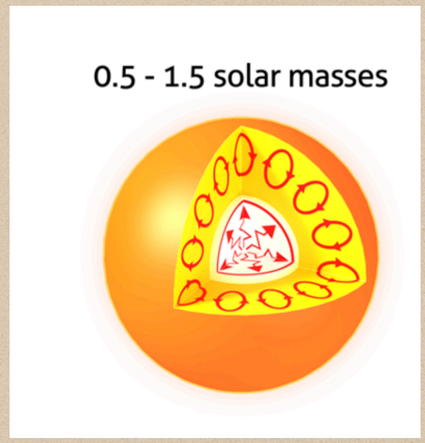
Convective core,

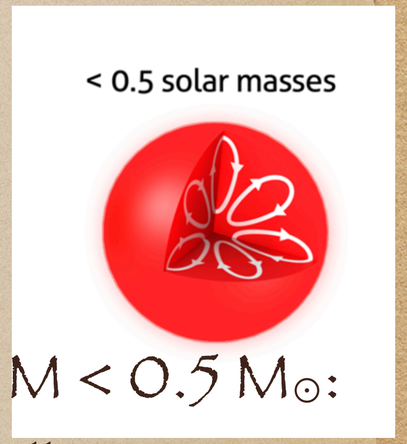
radiative external

0.5 M⊙ < M < 1.5M⊙:

Radiative core,

convective external





M < 0.5 M⊙:

All convective

Energy sources

Temperatures in the center of stars can reach millions of degrees

→ sufficient to fuel thermonuclear fusion reactions

(N.B.: The minimum mass of a celestial body to ignite fusion reactions in the nucleus is 0.08 $M_\odot$)

In fusion, lighter elements are burned into heavier elements; the products have a total mass smaller than the initial one; the difference is released in the form of energy: $E = mc^2$

The mass of the nucleus is smaller than the sum of the mass of the nucleons (protons and neutrons)

This difference in mass, transformed into energy, is called binding energy (for nucleone):

$$Q = \frac{1}{A}\left(Zm_{\mathrm{p}} + Nm_{\mathrm{n}} - m(Z, N)\right)c^2$$

$m_{\mathrm{p}} =$ proton mass,

$m_{\mathrm{n}} =$ neutron mass,

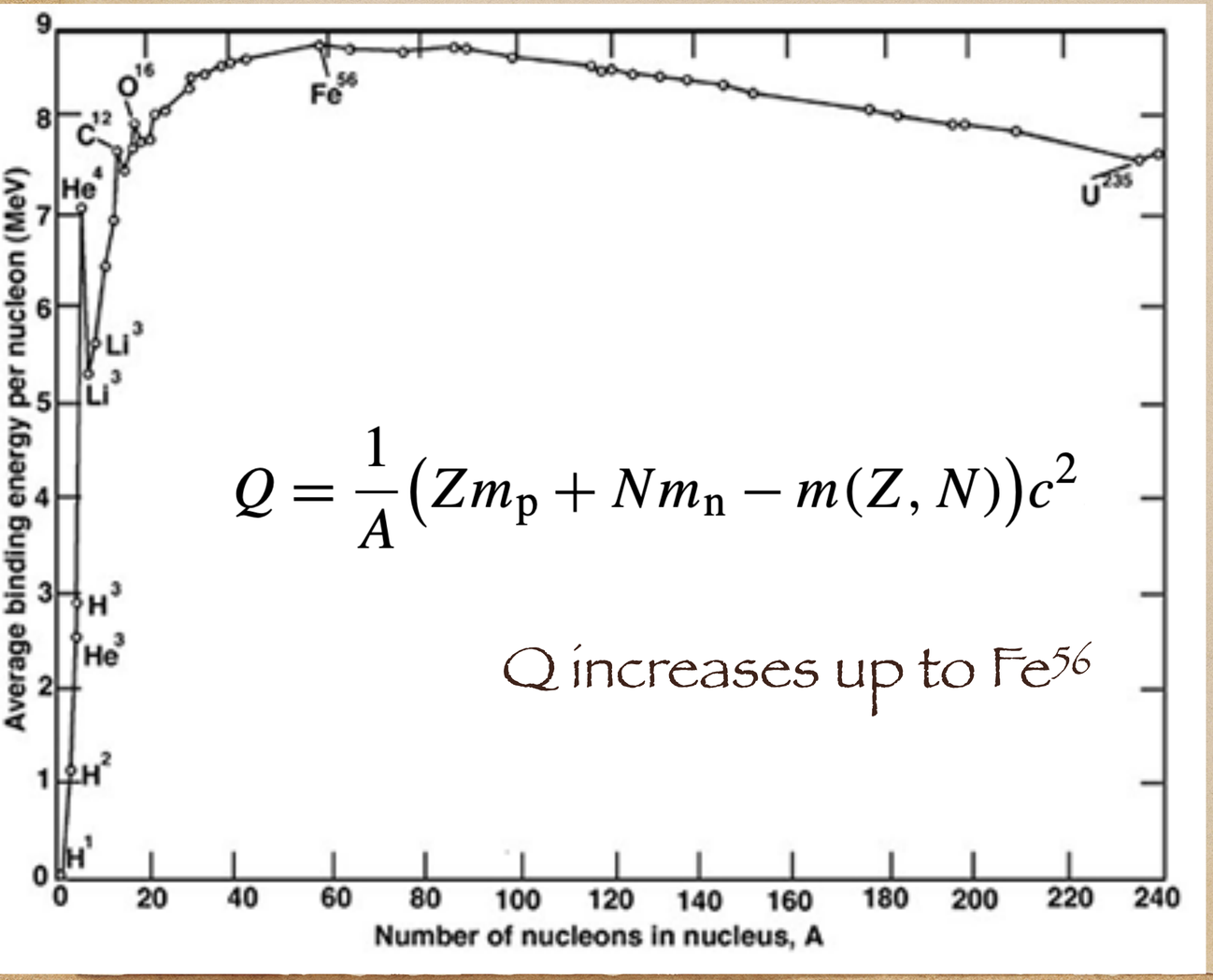
$Z =$ nuclear charge $=$ atomic number,

$N =$ neutron number,

$A = Z + N =$ atomic weight,

$m(Z, N) =$ mass of the nucleus.

Energy sources



$$Q = \frac{1}{A}\left(Zm_{\mathrm{p}} + Nm_{\mathrm{n}} - m(Z,N)\right)c^2$$

Q increases up to Fe^{56}

Fusion of 4 nuclei of H in a nucleus of He:

$$4p \rightarrow He$$

$$4 \times 1.672 \cdot 10^{-27}\mathrm{kg} \rightarrow 6.688 \cdot 10^{-27}\mathrm{kg}$$

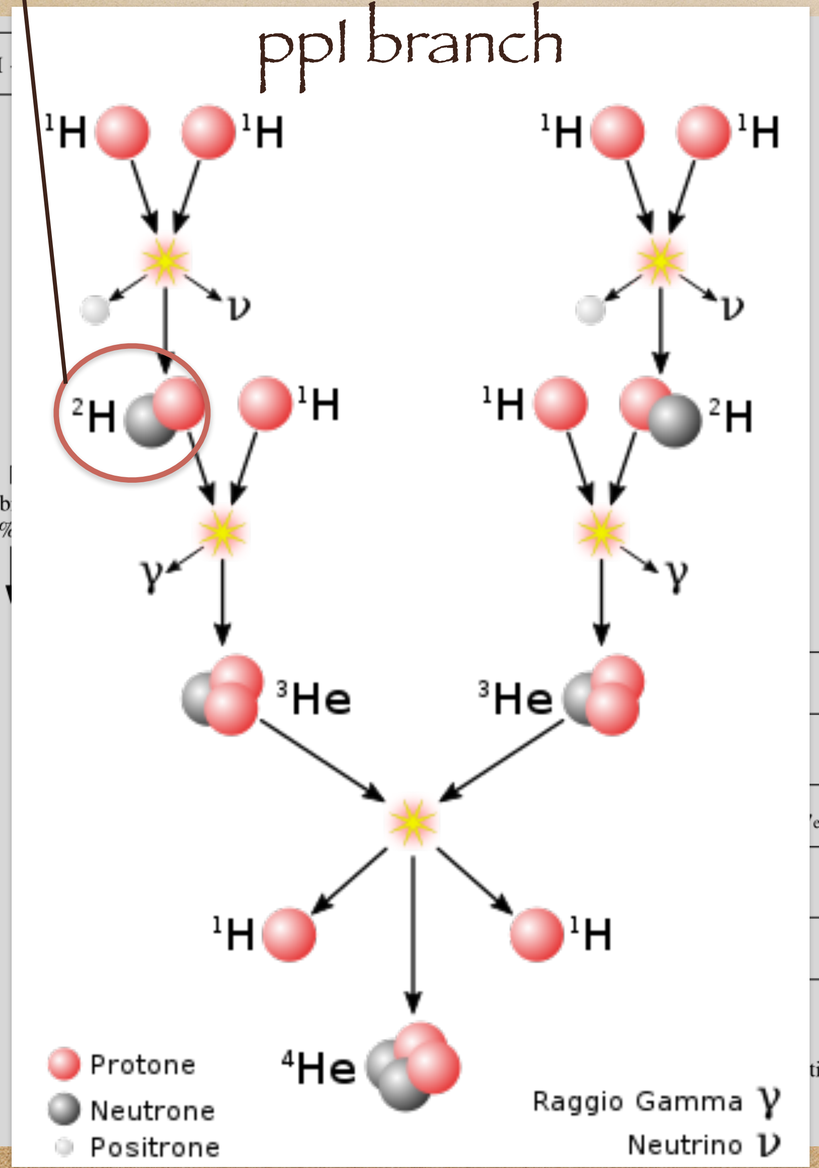
$$(6.688 - 6.644) \cdot 10^{-27}\mathrm{kg} = 4.6 \cdot 10^{-29}\mathrm{kg}$$

$$E \approx mc^2 \rightarrow E \approx 4.14 \cdot 10^{-12}\mathrm{J}$$

Burned mass 4.6/(6.688·100)≈0.7%;

For 1 kg of H, $E \approx 6.4 \cdot 10^{14}\mathrm{J}$

Proton - proton chain



Main channel in stars with $M < 1.5\ M_\odot$ ($T < 20 \cdot 10^6$ K)

- Average collision time p+p: 10^{10} yr (Otherwise the Sun would have run out a long time ago)
- The neutrino produced in the first reaction escapes from the Sun and takes energy away
- The positron is instantly annihilated with an electron
- 91% Of the Sun's energy occurs through the ppI branch (most likely branch)
- The second reaction is much faster than the first: very low deuterium content in the Sun

⚛️ Step 1 — Proton–proton fusion

Two protons (^{1}H + ^{1}H) fuse to form:

- **Deuterium (^{2}H)** — one proton becomes a neutron via beta$^+$ decay
- **A positron (e^+)**
- **A neutrino (ν)**

$$^1H + ^1H \rightarrow ^2H + e^+ + \nu_e$$

This step is *slow*, and it's what limits the overall rate of fusion in the Sun.

⚛️ Step 2 — Formation of Helium-3

The deuterium nucleus then fuses with another proton to form:

- **Helium-3 (^{3}He)**
- **A gamma ray (γ)**

$$^2H + ^1H \rightarrow ^3He + \gamma$$

⚛️ Step 3 — Formation of Helium-4

Finally, two **Helium-3 nuclei** fuse together to produce:

- **Helium-4 (^{4}He)**
- **Two protons (^{1}H)** that are returned to the plasma

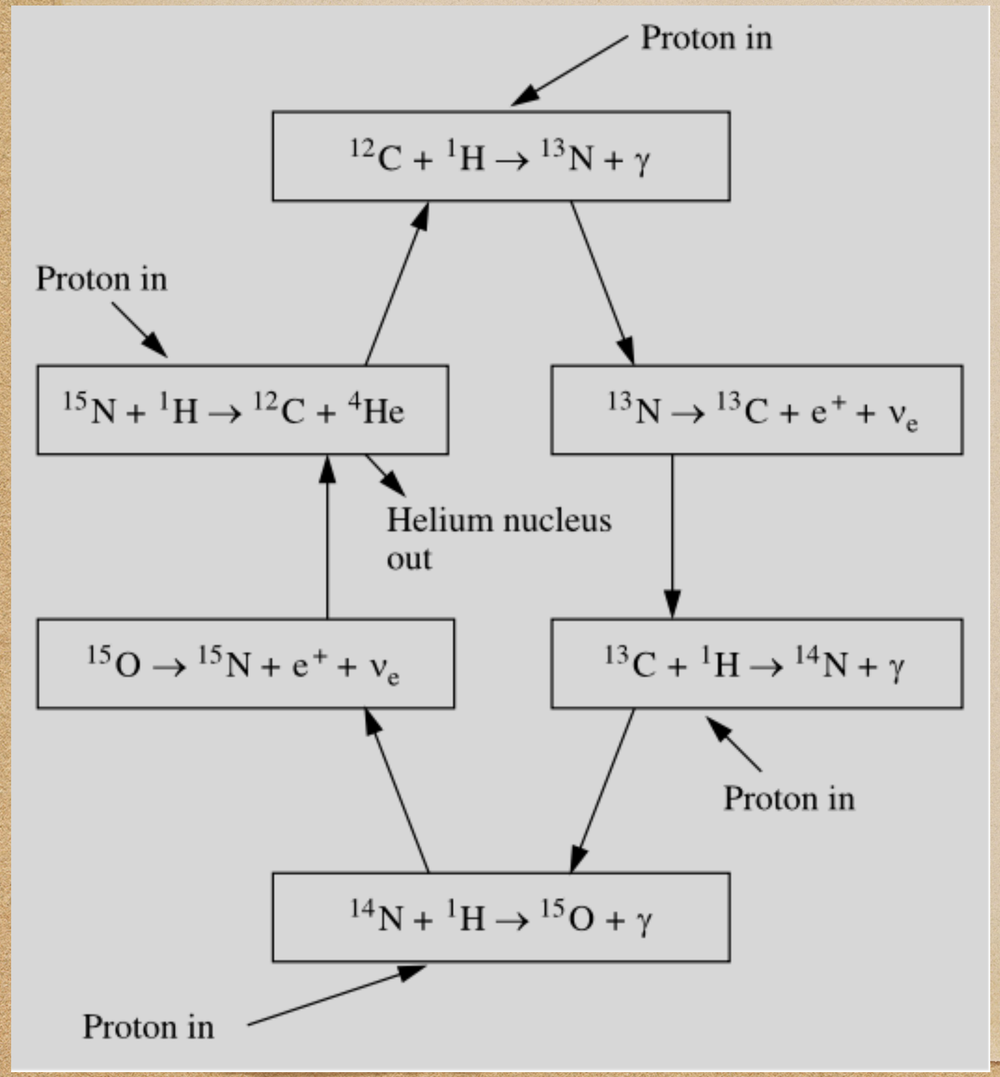
$$^3He + ^3He \rightarrow ^4He + 2(^1H)$$

🔥 Net reaction (overall result):

$$4(^1H) \rightarrow ^4He + 2e^+ + 2\nu_e + \text{energy } (\gamma \text{ rays})$$

This process **releases about 26.7 MeV of energy**, which sustains the Sun's luminosity.

CNO chain



Main channel in stars with $M > 1.5\ M_\odot$

$(T > 20 \cdot 10^6\ K)$

- Carbon, Nitrogen and Oxygen act as catalysts
- It is basically a cycle, with the entry of 4 protons
- $^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$ It is the slowest and determines the speed of the channel (~ a million years)
- Produces slightly less energy, because more energy is extracted from neutrinos

$$4({}^{1}H) \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + \text{energy}$$

Other reactions

α particle : nucleus He, 2p+2n

Triple-α process

$$(1) \quad {}^4\mathrm{He} + {}^4\mathrm{He} \leftrightarrow {}^8\mathrm{Be},$$

$$(2) \quad {}^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma.$$

- Requires the almost simultaneous collision of 3 α particles
- $T > 10^8$ K

Carbon burning

$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg} + \gamma \\ &\rightarrow {}^{23}\mathrm{Na} + {}^{1}\mathrm{H} \\ &\rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{23}\mathrm{Mg} + \mathrm{n} \\ &\rightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He}. \end{aligned}$$

- $T > 5\text{-}8 \cdot 10^8$ K

Oxigen burning

$$\begin{aligned} {}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{32}\mathrm{S} + \gamma \\ &\rightarrow {}^{31}\mathrm{P} + {}^{1}\mathrm{H} \\ &\rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{31}\mathrm{S} + \mathrm{n} \\ &\rightarrow {}^{24}\mathrm{Mg} + 2\,{}^{4}\mathrm{He}. \end{aligned}$$

- $T > 10^9$ K

Elements heavier than Iron need energy:

- They are not formed by thermonuclear reactions
- They are formed only by capturing neutrons in the final stages of stellar evolution

Silicon Burning

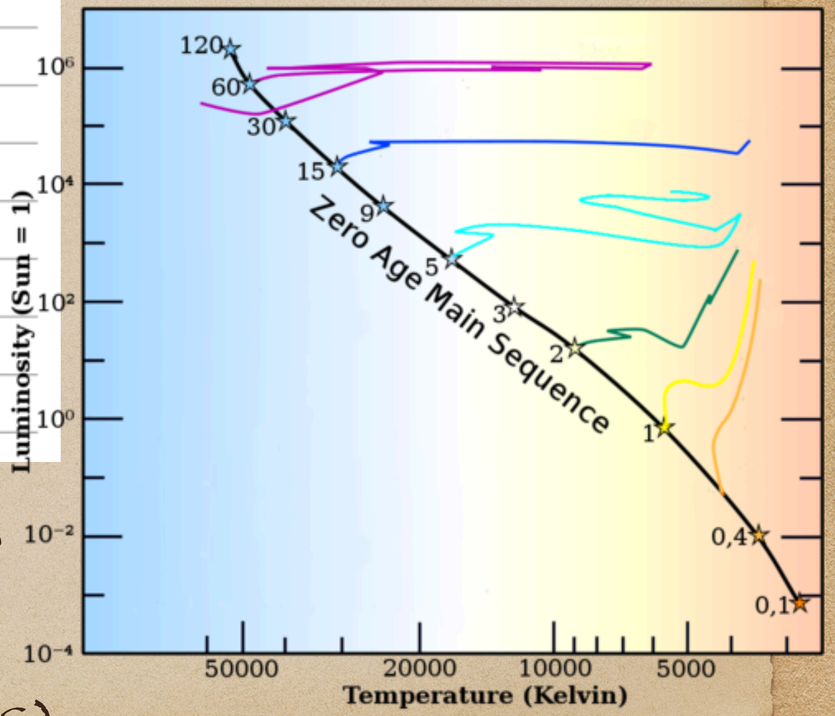
$$\begin{aligned} {}^{28}\mathrm{Si} + {}^{28}\mathrm{Si} &\rightarrow {}^{56}\mathrm{Ni} + \gamma, \\ {}^{56}\mathrm{Ni} &\rightarrow {}^{56}\mathrm{Fe} + 2\,\mathrm{e}^{+} + 2\,\nu_{\mathrm{e}}. \end{aligned}$$

Stellar models

By solving the 4 differential equations of the stellar structure, fixed the mass and the chemical composition, a stellar theoretical model is obtained

Table 11.1 Properties of zero age main sequence stars. (T_c = central temperature; ρ_c = central density; M_{ci} = relative mass of convective interior; M_{ce} = relative mass of convective envelope)

M [$M_\odot$]	R [$R_\odot$]	L [$L_\odot$]	T_e [K]	T_c [10^6 K]	ρ_c [kg/m^3]	M_{ci} [M]	M_{ce} [M]
30	6.6	140,000	44,000	36	3000	0.60	0
15	4.7	21,000	32,000	34	6200	0.39	0
9	3.5	4500	26,000	31	7900	0.26	0
5	2.2	630	20,000	27	26,000	0.22	0
3	1.7	93	14,000	24	42,000	0.18	0
1.5	1.2	5.4	8100	19	95,000	0.06	0
1.0	0.87	0.74	5800	14	89,000	0	0.01
0.5	0.44	0.038	3900	9.1	78,000	0	0.41



- As soon as they are formed, the stars have a homogeneous composition;
- They are arranged on the Zero Age Main Sequence (ZAMS): location of the points in the HR diagram of stars with age 0

Stellar evolution

Evolutionary times

3 evolutionary times determine the evolution of the stars

Nuclear scale time

Time in which a star radiates all the energy produced by thermonuclear reactions

$$t_n \approx \frac{0.007 \times 0.1\, Mc^2}{L}$$

- 0.007: Efficiency
- 0.1: Mass fraction in the nucleus
- Mc^2: Energy emitted
- L: Give the main sequence time

Thermal scale time

Energy transport time from the center to the surface (time to radiate all the energy if the thermonuclear reactions were turned off). From the virial theorem, kinetic energy = 0.5 potential

$$t_t \approx \frac{0.5\, GM^2/R}{L}$$

Dynamic scale time

Star collapse time if the pressure fades (free fall time from the surface):

$$t_d = \frac{2\pi}{2}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}}.$$

Evolutionary times

3 evolutionary times determine the evolution of the stars

Nuclear scale time

Time in which a star radiates all the energy produced by thermonuclear reactions

Mass fraction in the nucleus

Energy emitted

Efficiency

Give the main sequence time

Sun: 10.000 million years

Thermal scale time

Energy transport time from the center to the surface (time to radiate all the energy if the therm ned off). From the virial theorem, kinetic energy = 0.5 potential

$$t_t \approx \frac{0.5\,GM^2/R}{L}$$

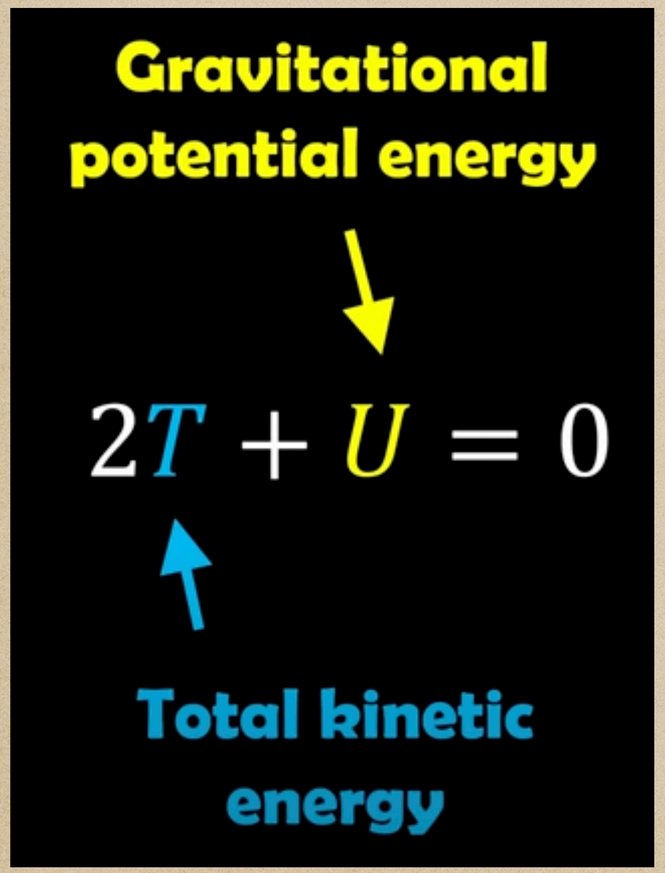
Sun: 20 million years

Dynamic scale time

Star collapse time if the pressure fades (rface):

$$t_d = \frac{2\pi}{2}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}}.$$

Sun: Half an hour


Gravitational potential energy
$2T + U = 0$
Total kinetic energy

Virial Theorem is useful for the study of **stars**, **galaxies** & **clusters**







Virial Theorem is useful for the study of stars, galaxies & clusters



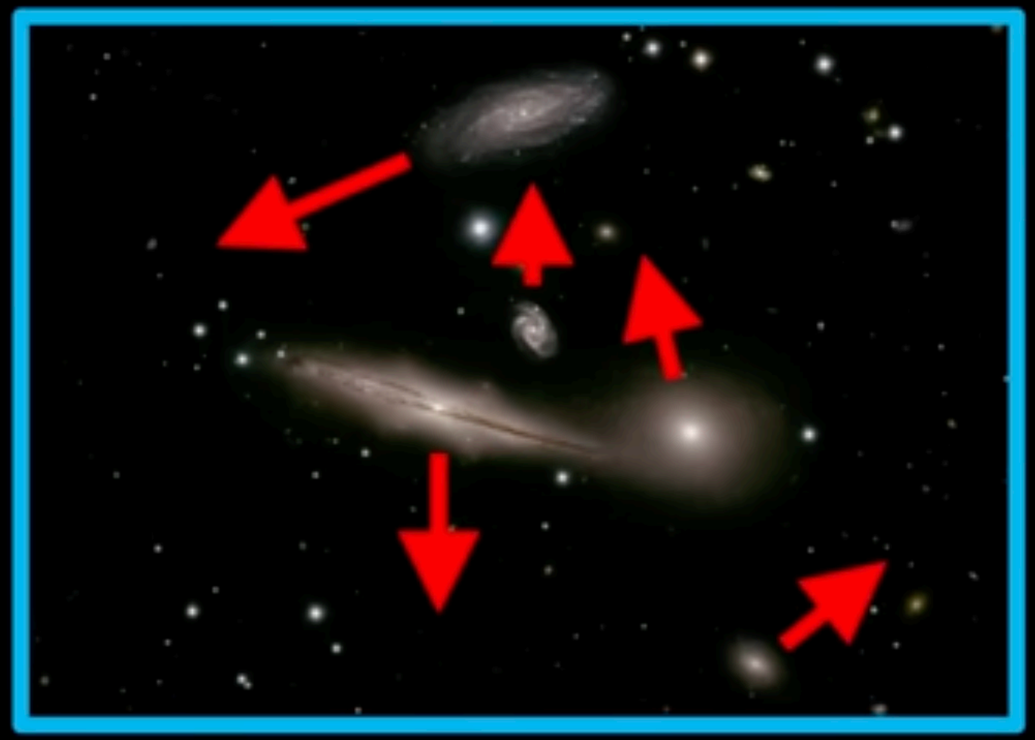
Gravitational potential energy

$$2T + U = 0$$

Total kinetic energy



The **Virial Theorem** in astrophysics relates the total kinetic energy (T) & gravitational potential energy (U) of a system in equilibrium



$$2T + U = 0$$

If the system is in equilibrium

$$T = -\frac{1}{2}U$$

Mass of body i

Velocity of body i

$$T = \sum_{i}^{n} \frac{1}{2} m_i v_i^2$$

Total kinetic energy of a n-body system

Mass of body i

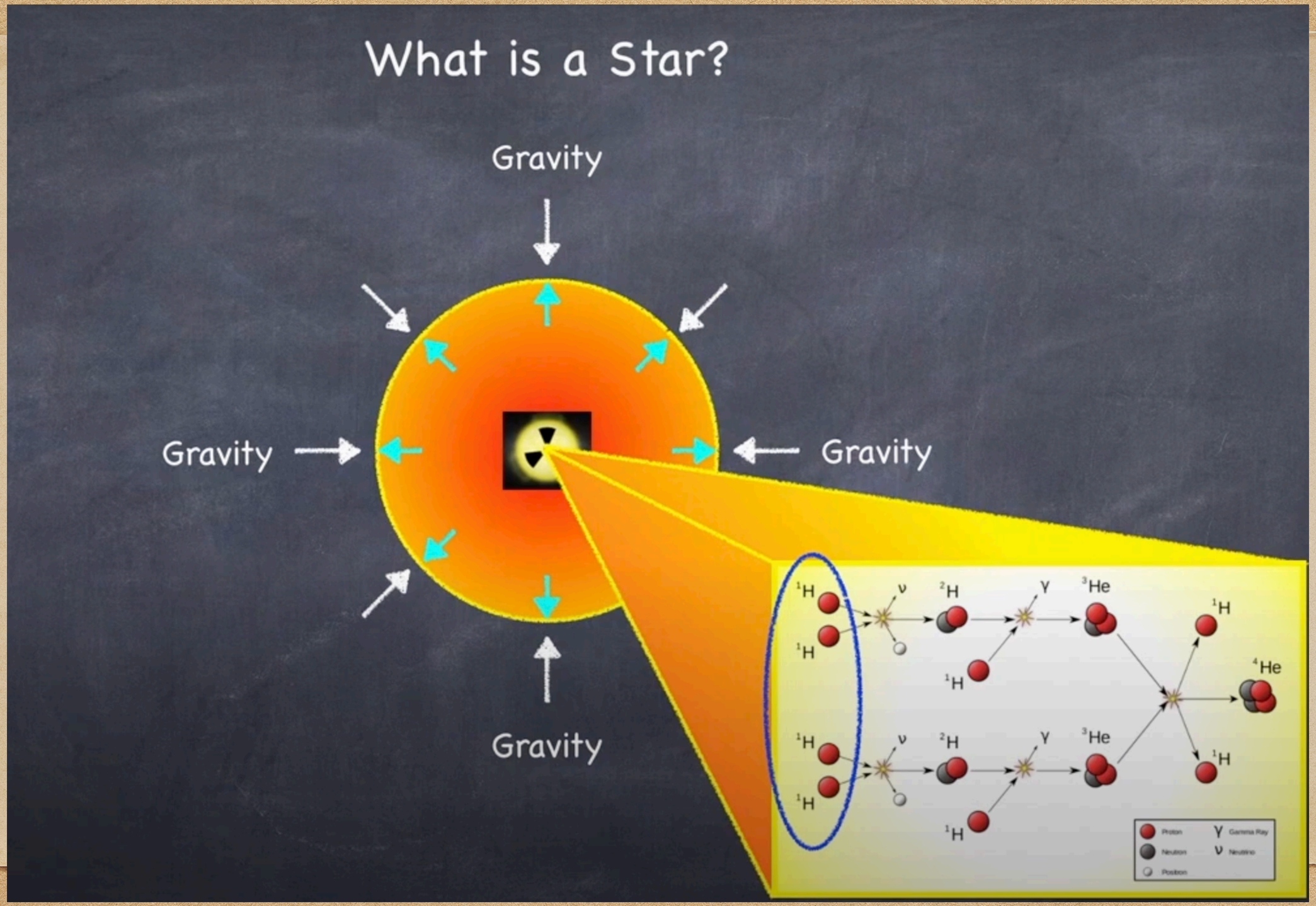
Mass of body j

$$U = -G \sum_{i<j}^{n} \frac{m_i m_j}{r_{ij}}$$

Distance between body i & j

Total potential energy is found by summing over all pairs

What is a Star?



Star Formation

Jeans Mass from small perturbation theory

Self-gravitating gas cloud...



Ideally from an infinite distribution of gas:
At which scale does it collapse?
==> let's assume an average constant density within the cloud

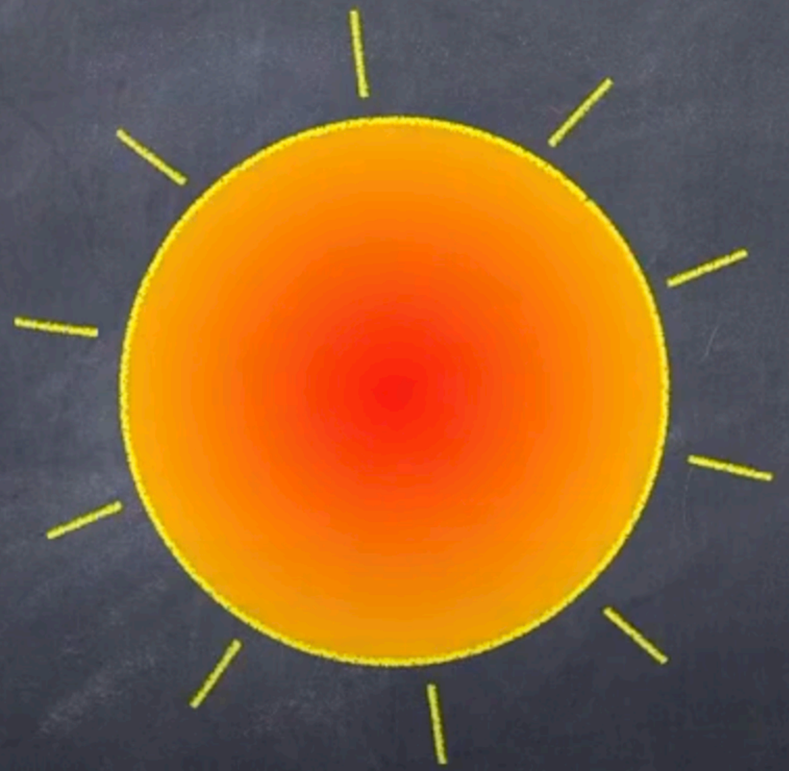
Star Formation

Self-gravitating gas cloud...

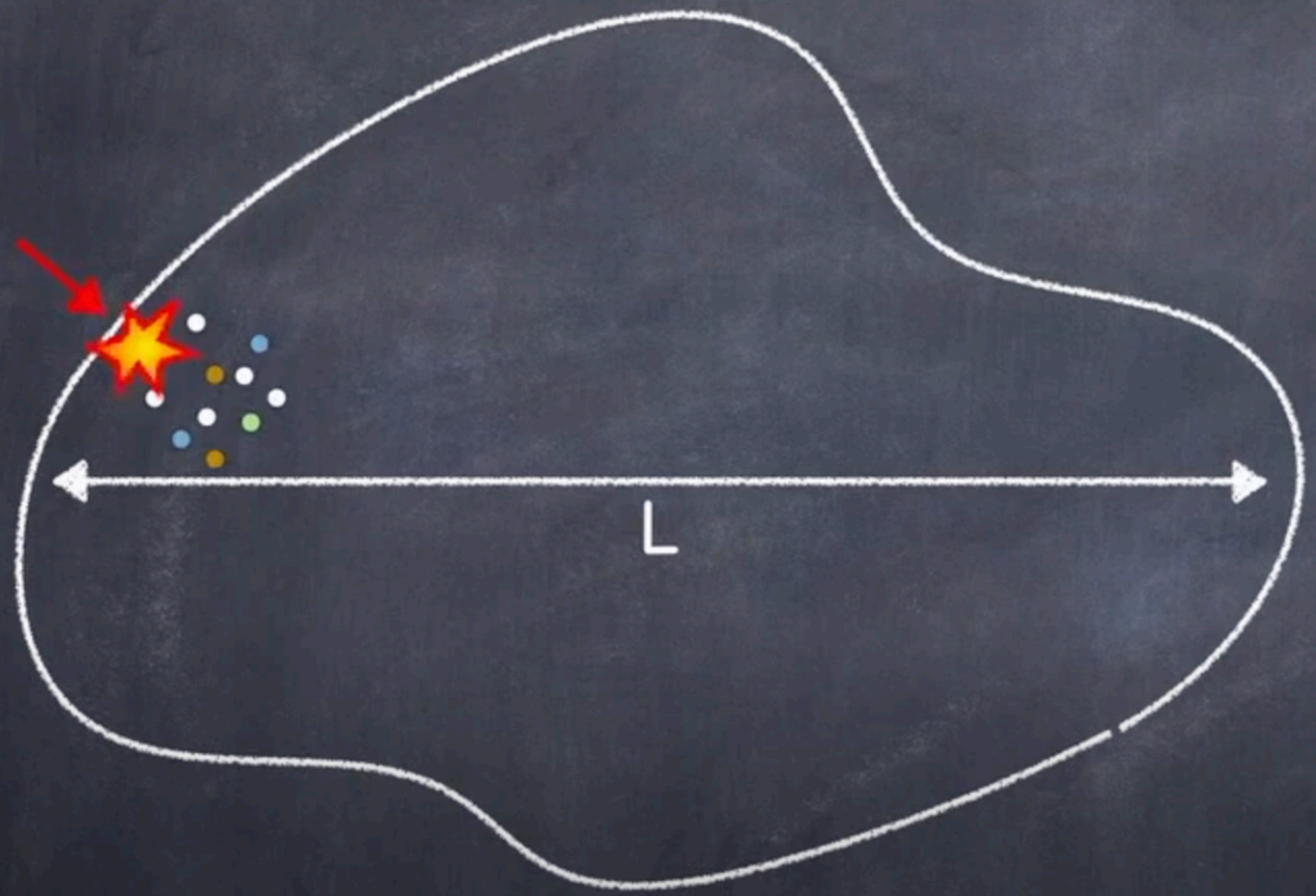


collapses...

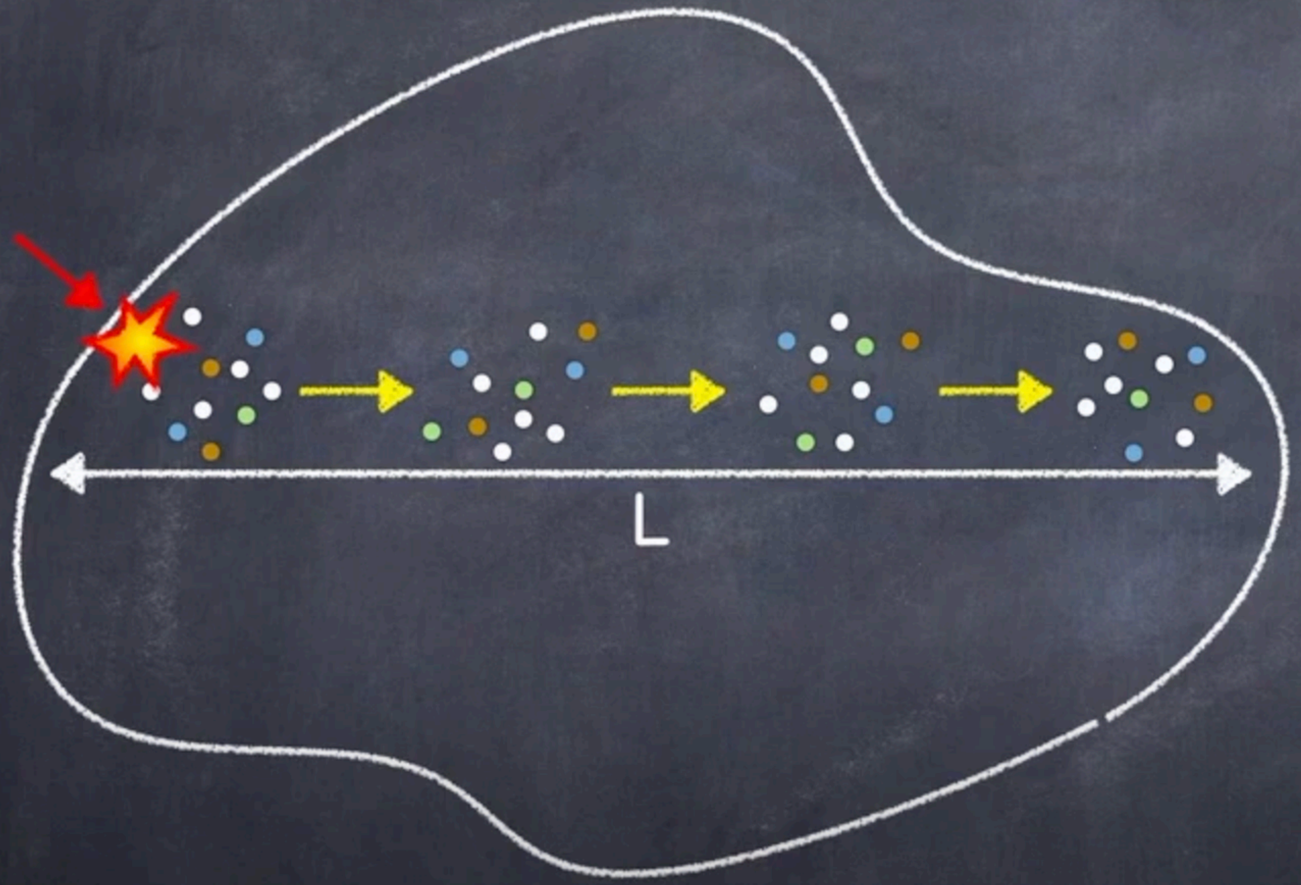
heats up until nuclear fusion.



When does a gas cloud contract?



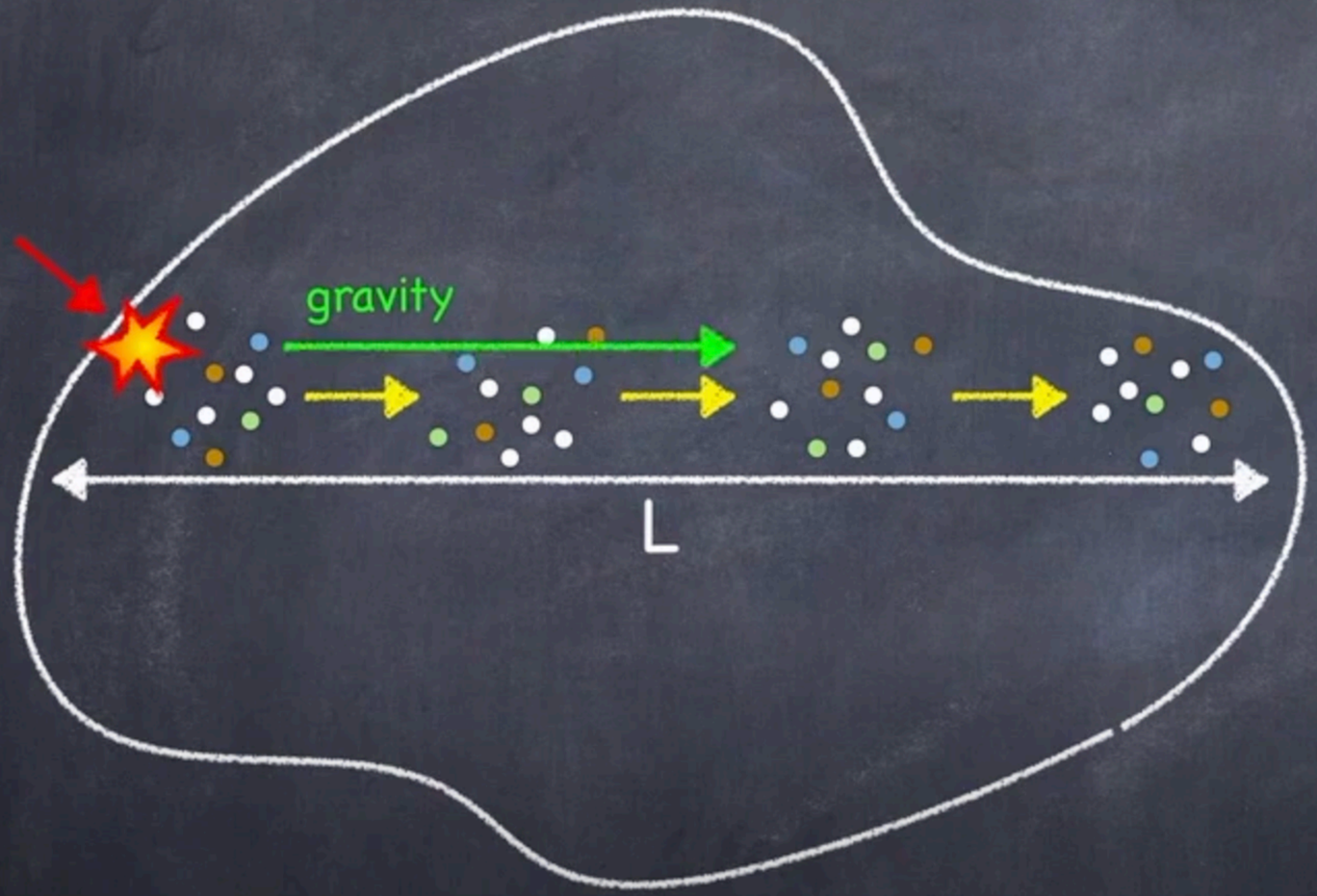
When does a gas cloud contract?



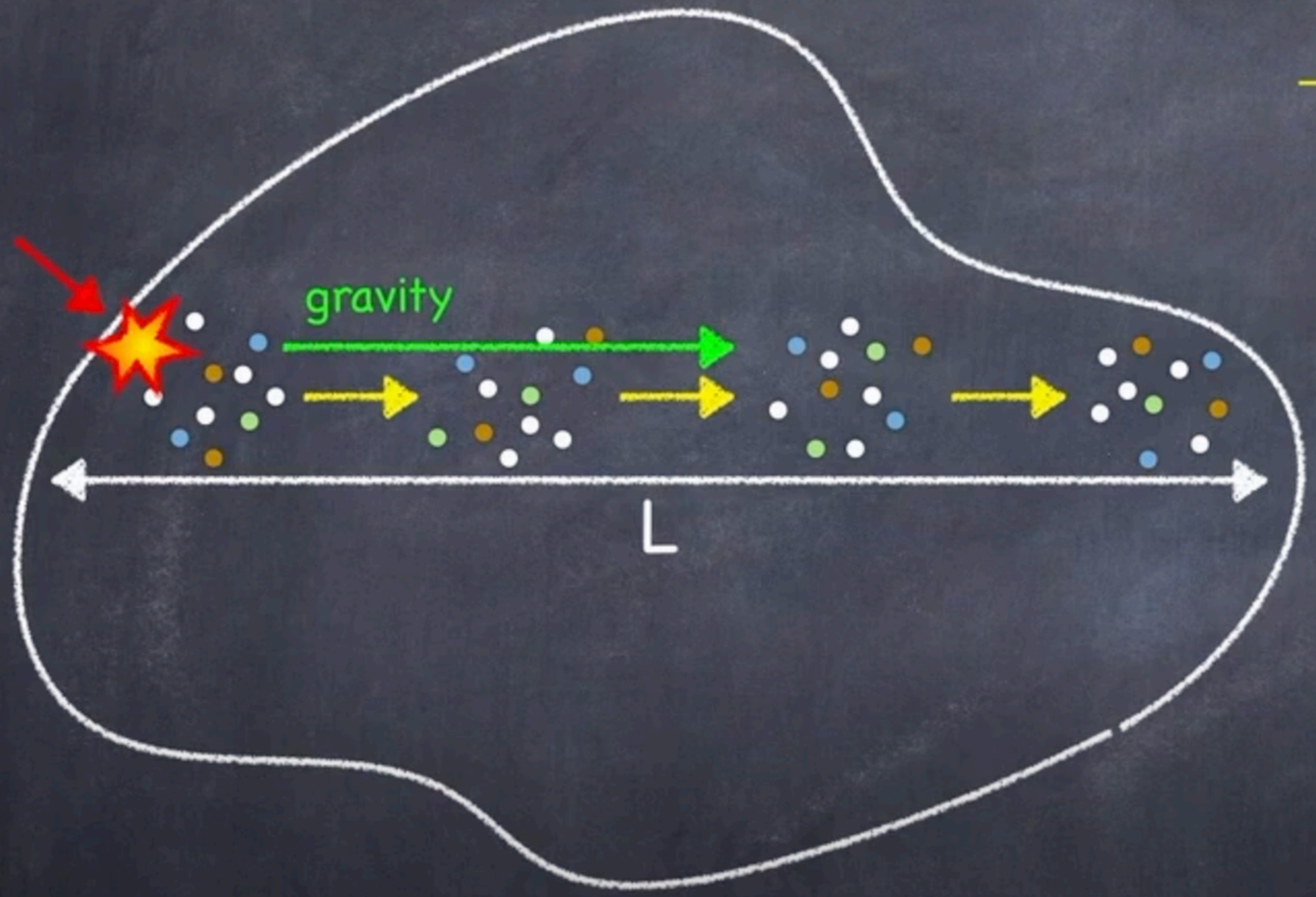
When does a gas cloud contract?

Sound crossing time

$$t_s = \frac{L}{c_s}$$



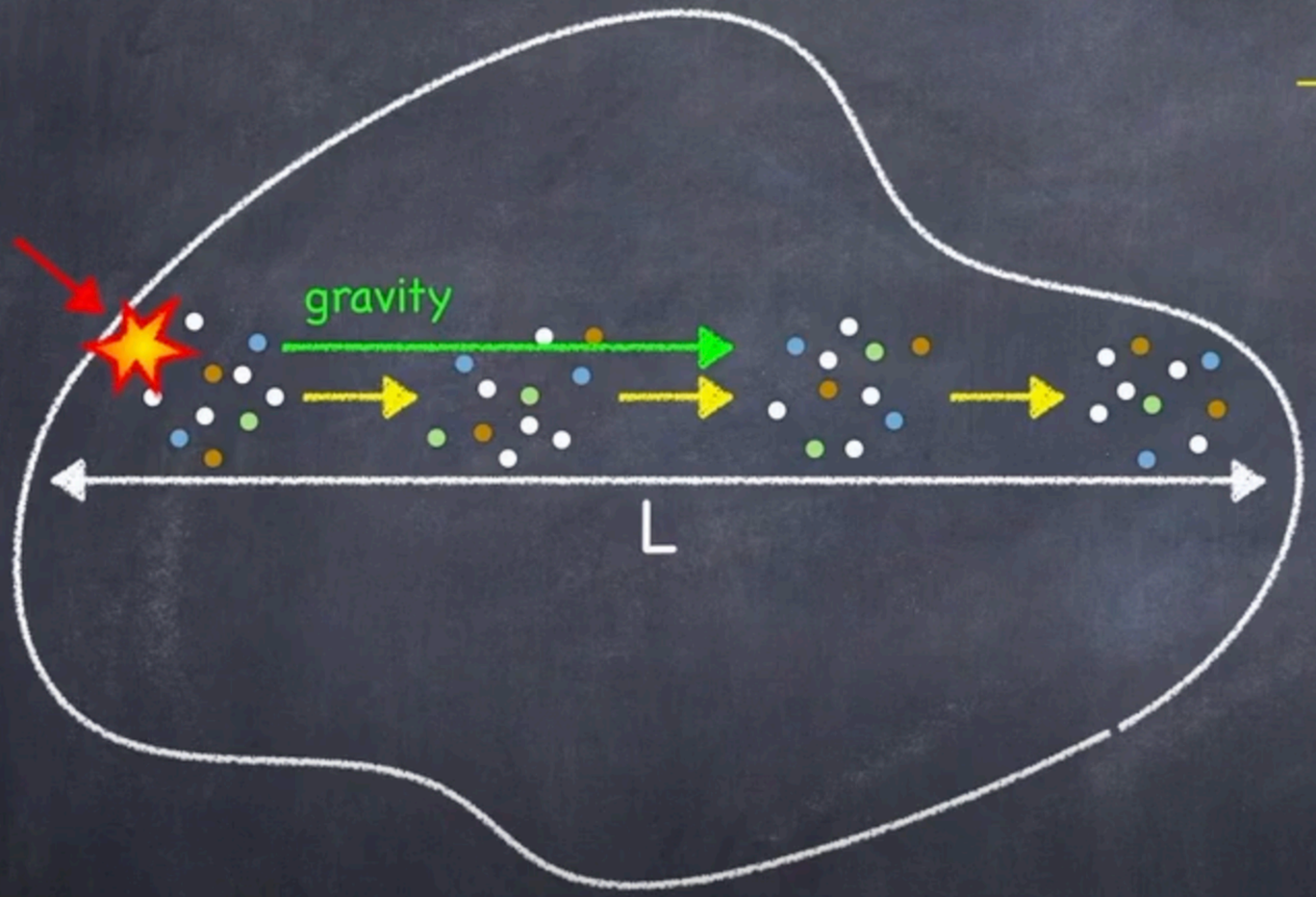
When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$ Conservation of energy

When does a gas cloud contract?



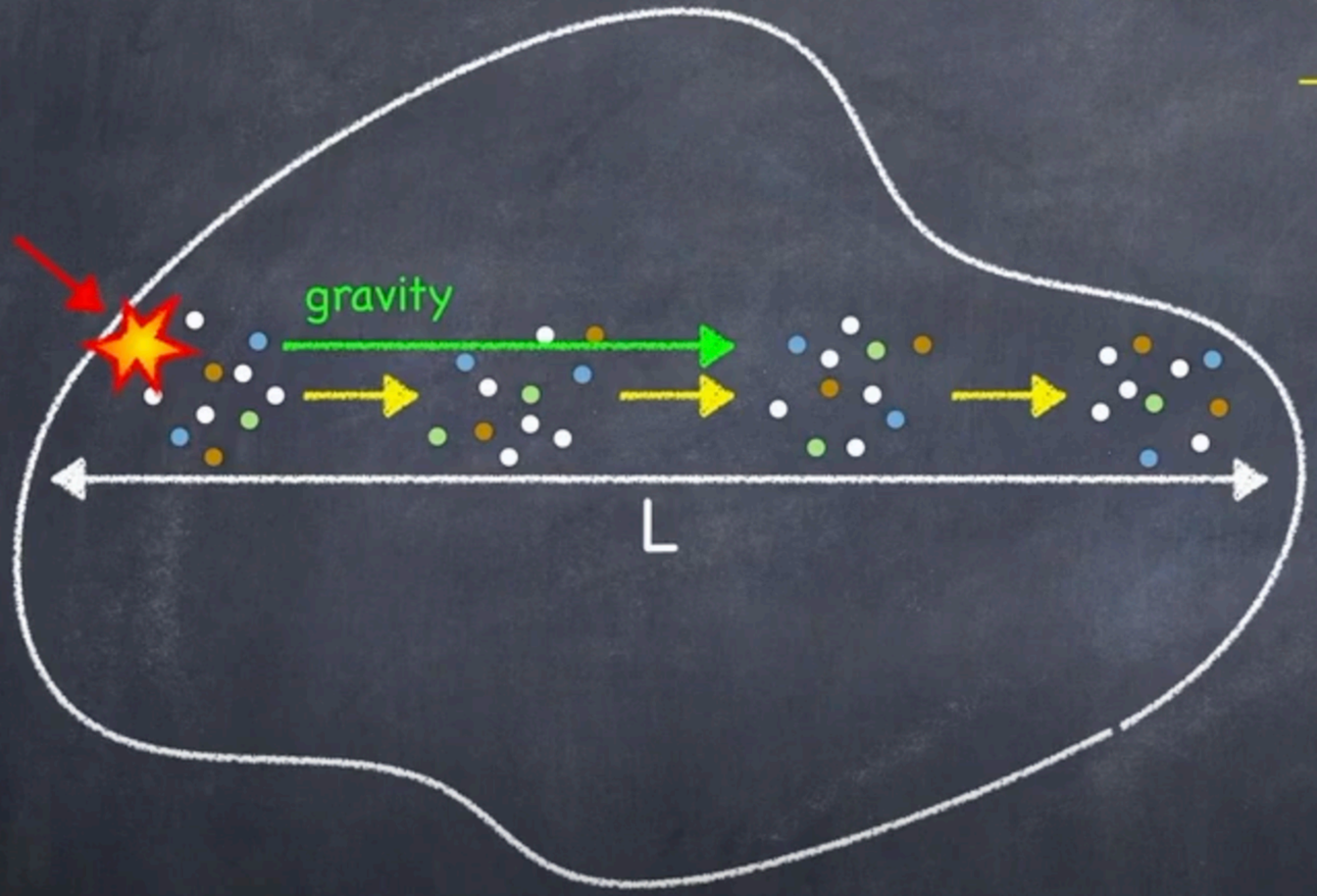
$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

~Bound system

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$

When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

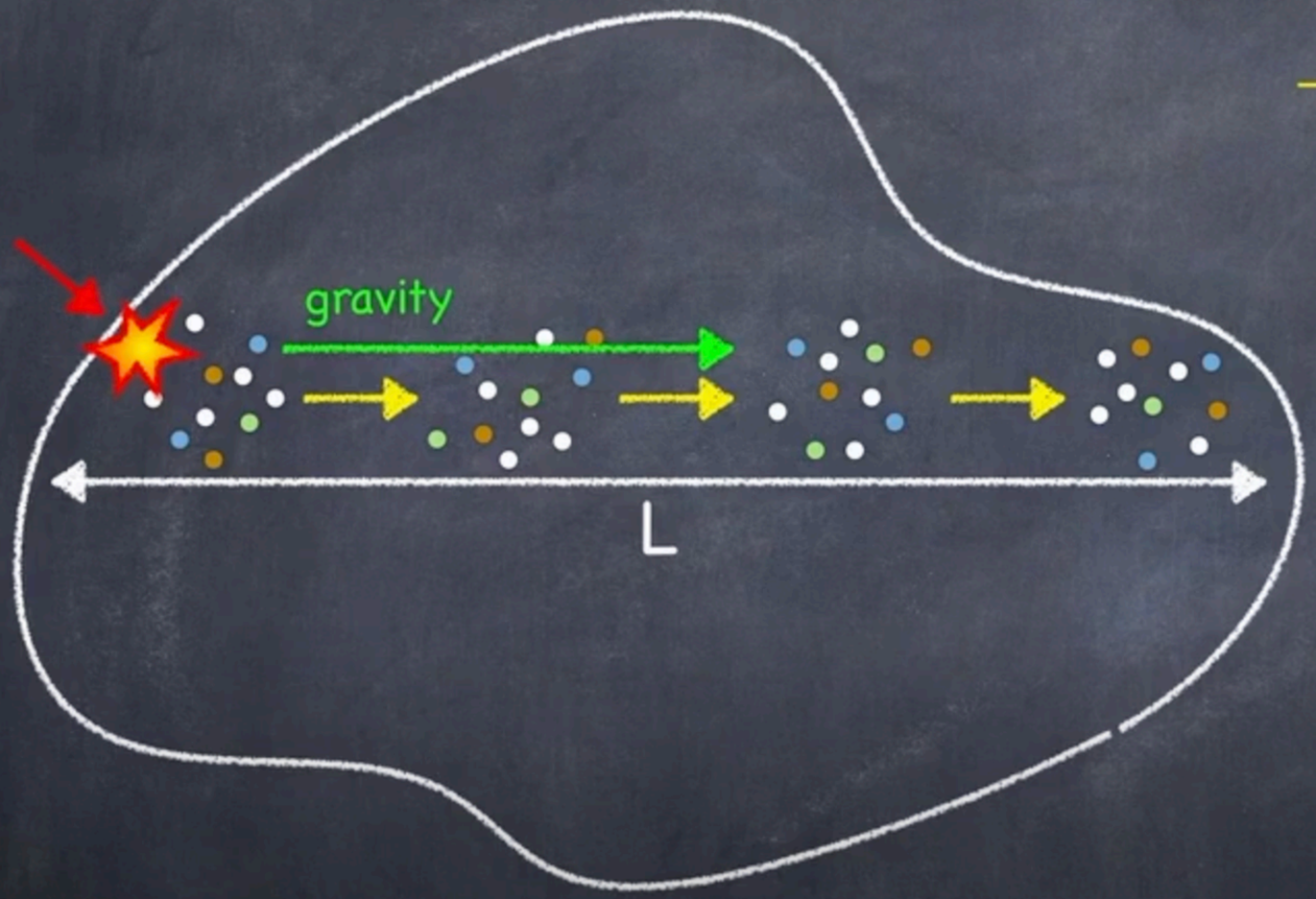
$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$

$$\int_{L/2}^{0} \sqrt{r} \cdot dr \approx -\int_{0}^{t_{ff}} \sqrt{2GM} \cdot dt$$

in-falling

When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$

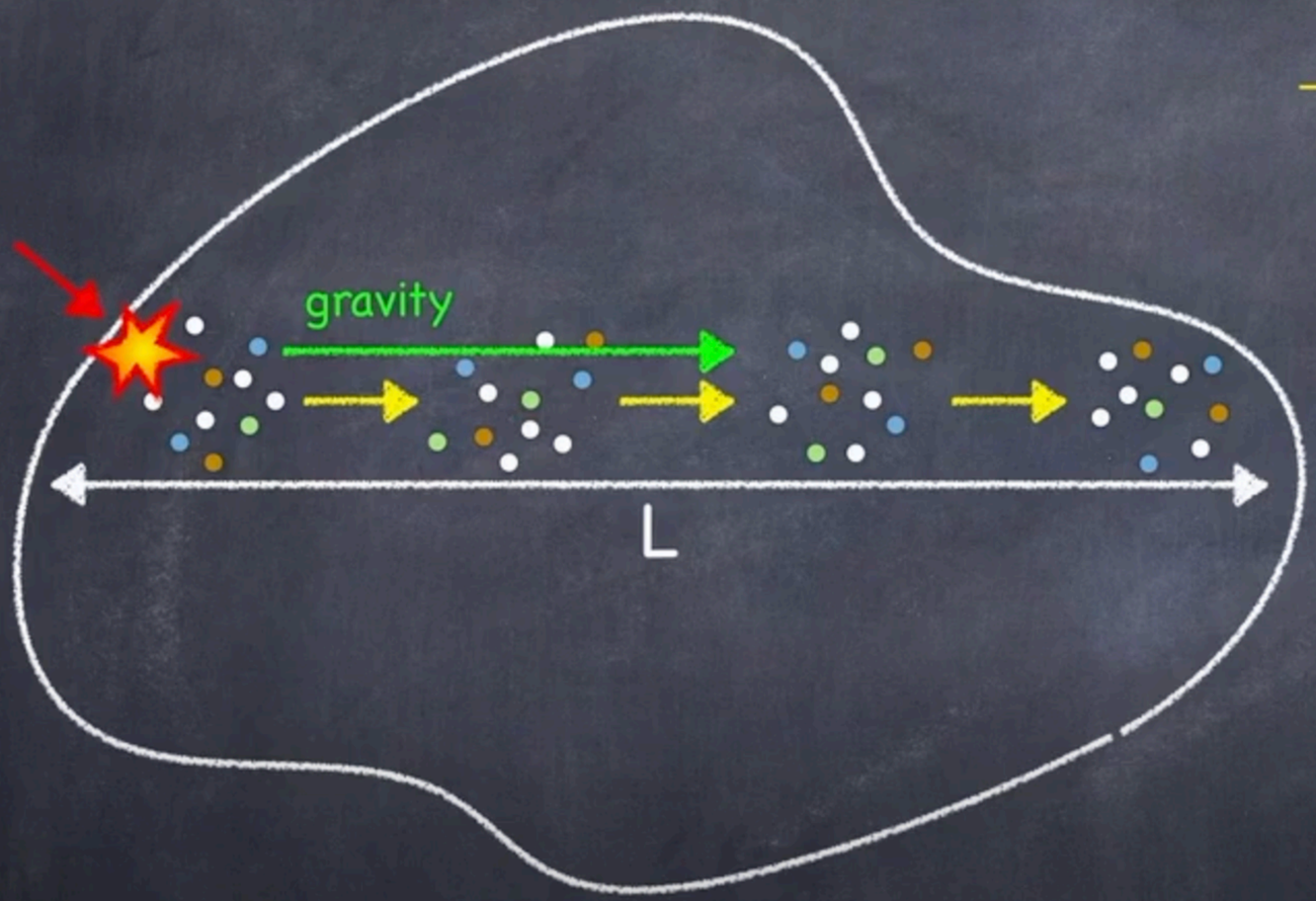
$$\int_{L/2}^{0} \sqrt{r} \cdot dr \approx -\int_{0}^{t_{ff}} \sqrt{2GM} \cdot dt$$

in-falling

$$\frac{2}{3} \cdot \left[0 - (L/2)^{3/2}\right] \approx -\sqrt{2GM} \cdot t_{ff}$$

$$\rightarrow t_{ff} \approx \frac{2}{3} \cdot \sqrt{\frac{L^3}{16GM}}$$

When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

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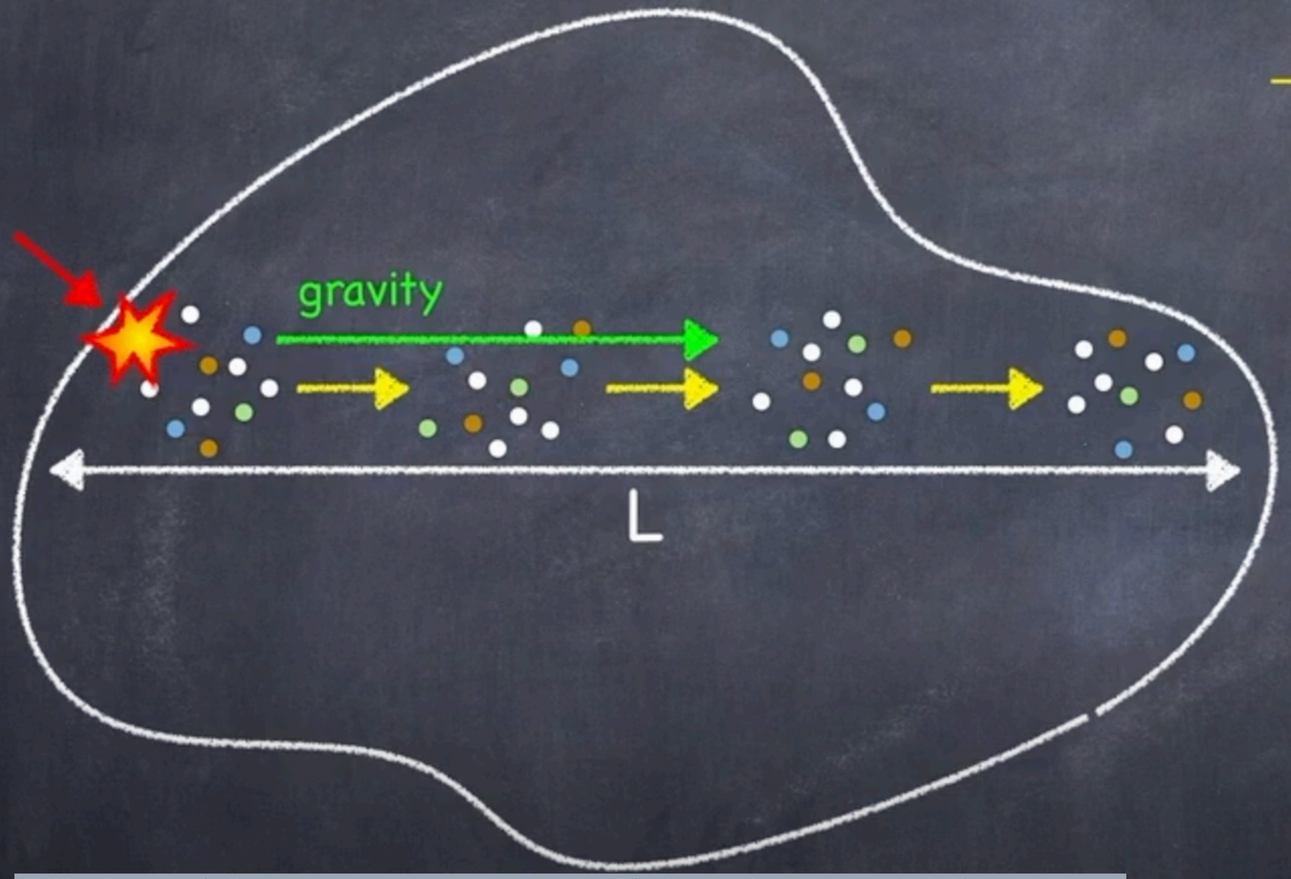
in-falling

$$\frac{2}{3} \cdot \left[0 - (L/2)^{3/2}\right] \approx -\sqrt{2GM} \cdot t_{ff}$$

$$\rightarrow t_{ff} \approx \frac{2}{3} \cdot \sqrt{\frac{L^3}{16GM}} \approx \frac{1}{\sqrt{G\rho}}$$

$$\rightarrow t_s = t_{ff} \rightarrow \frac{L}{c_s} = \frac{1}{\sqrt{G\rho}}$$

When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$

$$\int_{L/2}^{0} \sqrt{r} \cdot dr \approx -\int_{0}^{t_{ff}} \sqrt{2GM} \cdot dt$$

in-falling

$$\frac{2}{3} \cdot \left[0 - (L/2)^{3/2}\right] \approx -\sqrt{2GM} \cdot t_{ff}$$

$$\rightarrow t_{ff} \approx \frac{2}{3} \cdot \sqrt{\frac{L^3}{16GM}} \approx \frac{1}{\sqrt{G\rho}}$$

$$\rightarrow t_s = t_{ff} \rightarrow \frac{L}{c_s} = \frac{1}{\sqrt{G\rho}} \rightarrow L_J = \frac{c_s}{\sqrt{G\rho}}$$

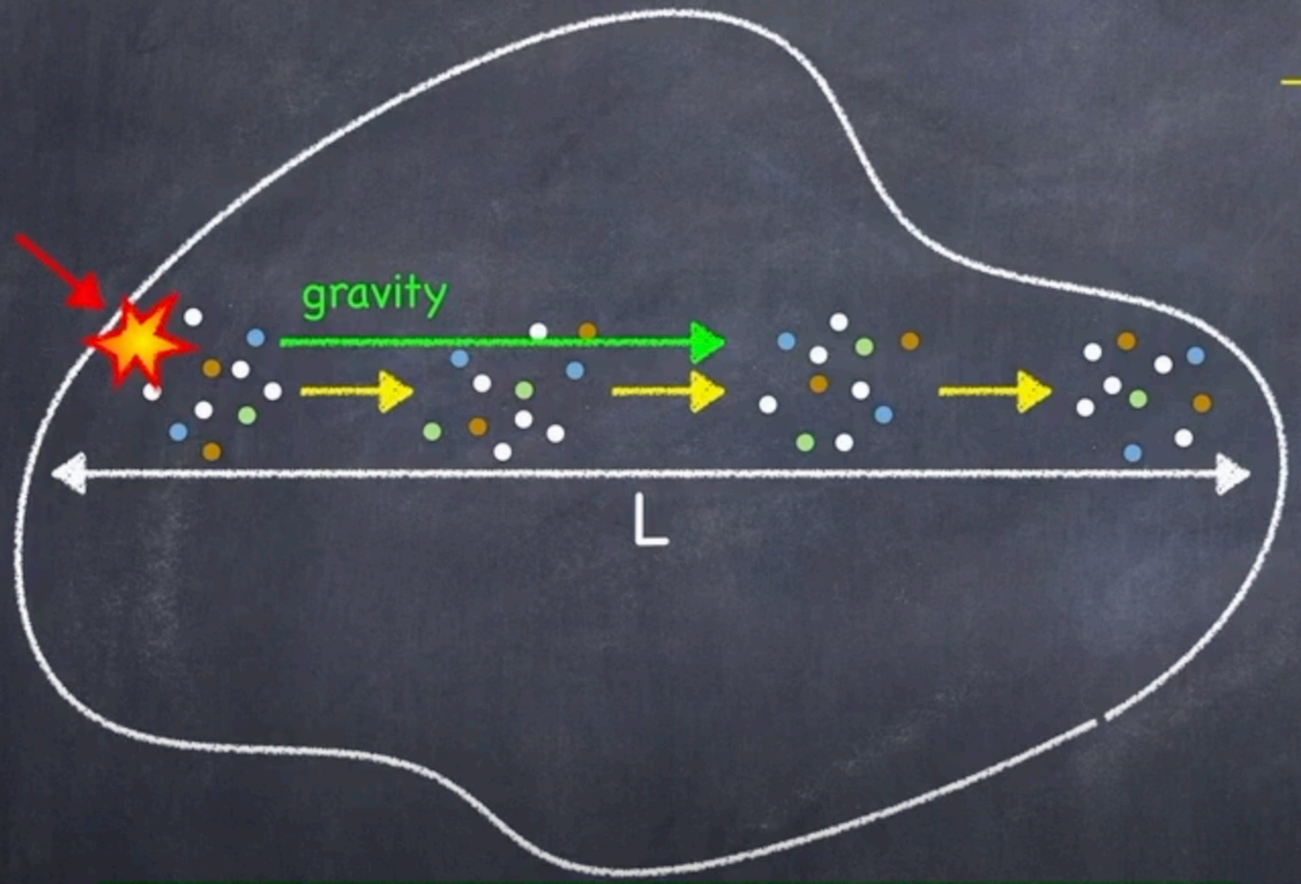
"Jeans Length"



James Jeans

If a self-gravitating cloud is too large,
It will become unstable

When does a gas cloud contract?



$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $\quad E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$

$$\int_{L/2}^{0} \sqrt{r} \cdot dr \approx -\int_{0}^{t_{ff}} \sqrt{2GM} \cdot dt$$

in-falling

$$\frac{2}{3} \cdot \left[0 - (L/2)^{3/2}\right] \approx -\sqrt{2GM} \cdot t_{ff}$$

$$\rightarrow t_{ff} \approx \frac{2}{3} \cdot \sqrt{\frac{L^3}{16GM}} \approx \frac{1}{\sqrt{G\rho}}$$

$$\rightarrow t_s = t_{ff} \rightarrow \frac{L}{c_s} = \frac{1}{\sqrt{G\rho}} \rightarrow L_J = \frac{c_s}{\sqrt{G\rho}}$$

"Jeans Length"



James Jeans

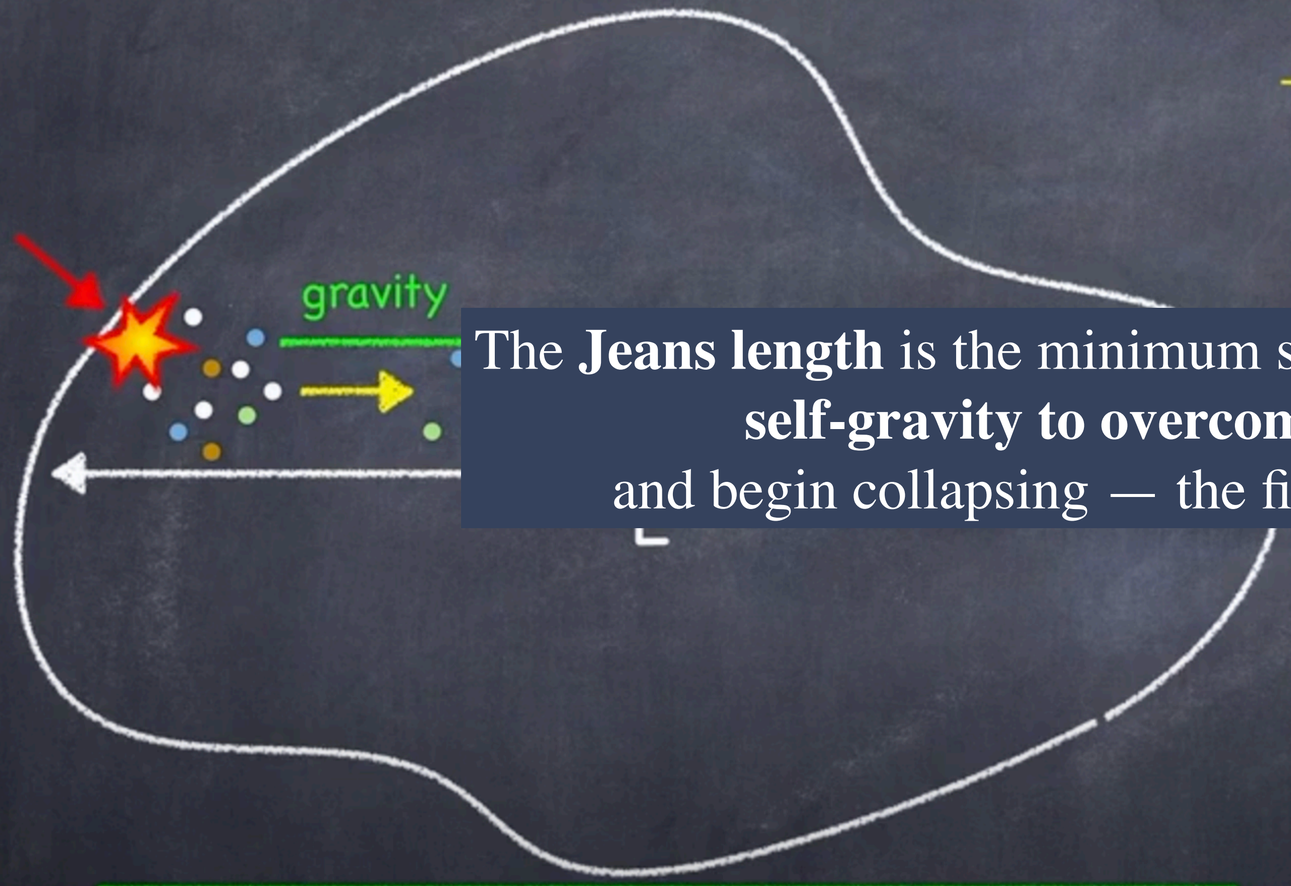
"Jeans Mass" $\rightarrow M_J = \rho \frac{4\pi}{3}\left(\frac{1}{2}L_J\right)^3 = \frac{\pi}{6}\frac{c_s^3}{\sqrt{G^3\rho}}$

When does a gas cloud contract?

$$t_s = \frac{L}{c_s} \rightarrow \text{Unstable if } t_s > t_{ff}$$

$\rightarrow t_{ff}$? $E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} \approx 0$

$$\left(\frac{dr}{dt}\right)^2 \approx \frac{2GM}{r}$$





James Jeans

The **Jeans length** is the minimum size a gas cloud must have for its **self-gravity to overcome internal pressure** and begin collapsing — the first step in **star formation**.

lling

$$\frac{2}{3}\cdot\left[0-(L/2)^{3/2}\right] \approx -\sqrt{2GM}\cdot t_{ff}$$

$$\rightarrow t_{ff} \approx \frac{2}{3}\cdot\sqrt{\frac{L^3}{16GM}} \approx \frac{1}{\sqrt{G\rho}}$$

$$\rightarrow t_s = t_{ff} \rightarrow \frac{L}{c_s} = \frac{1}{\sqrt{G\rho}} \rightarrow L_J = \frac{c_s}{\sqrt{G\rho}}$$

"Jeans Length"

"Jeans Mass" $\rightarrow M_J = \rho\frac{4\pi}{3}\left(\frac{1}{2}L_J\right)^3 = \frac{\pi}{6}\frac{c_s^3}{\sqrt{G^3\rho}}$

How it goes with the Temperature?

$$M_J = \rho \frac{4\pi}{3} \left(\frac{1}{2} L_J\right)^3 = \frac{\pi}{6} \frac{c_s^3}{\sqrt{G^3 \rho}}$$

$$M_J \sim \frac{c_s^3}{G^{3/2} \rho^{1/2}}$$

Sound's speed

$$c_s = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S}$$

For an ideal gas

$$P = \frac{\rho}{\mu m_H} k_B T$$

proton mass

mean molecular weight

Isothermal case

$$\gamma = 1$$

$$c_s^2 = \gamma \frac{P}{\rho} = \gamma \frac{k_B T}{\mu m_H}$$

$$c_s^2 = \frac{k_B T}{\mu m_H}$$

$$M_J \propto T^{3/2} \rho^{-1/2}$$

P → **pressure** of the gas

ρ → **mass density** of the gas (mass per unit volume)

k_B → **Boltzmann constant** $(1.38 \times 10^{-23}\ \mathrm{J/K})$

T → **absolute temperature** of the gas (in kelvin)

m_H → **mass of a hydrogen atom** $(1.67 \times 10^{-27}\ \mathrm{kg})$

μ → **mean molecular weight**, i.e. the average number of atomic mass units per particle

Let's consider a spherical cloud with a given Rc radius ==> at which mass the cloud collapse?

Jeans Mass from energetic arguments

In equilibrium → **the Virial theorem tells us:** $2K + U = 0$

Kinetic energy

Potential energy

1) if **K > |U| / 2** then the cloud will **expand**
2) if **K = |U| / 2** then **nothing happens**
3) if **K < |U| / 2** then the cloud will **collapse**

For a cloud of temperature ***T***, and ***N*** being the number of particles the kinetic energy (***K***) is:

$$K = \frac{3}{2} N k_B T$$

We can also see that the mass of the cloud M_c is the product of the number of particles ***N*** by the mean molecular weight μ and mass of hydrogen m_H: $M_c = N m_H \mu$ and thus we can write the kinetic energy as:

$$K = \frac{3}{2} \frac{M_c k_B T}{\mu m_H}$$

Suppose we have a **spherical cloud** of matter that may form stars. The gravitational energy of a point mass dm at a distance r from the centre of the sphere is simply given by:

$$dU = -\frac{GM_r}{r} dm$$

where M_r is the mass within radius r. For a spherically symmetric cloud, consider a thin shell of thickness dr, with mass $dm = 4\pi r^2 \rho_r dr$, where ρ_r is the density at radius r:

$$dU = -\frac{GM_r 4\pi r^2 \rho_r}{r} dr$$

Integrating over the total cloud, i.e., integrating to radius R leads to:

$$U = -4\pi G \int_0^R M_r \rho_r r dr$$

The solution will depend on the forms of M_r and ρ_r which will likely be inter-related and in general may depend on radius r.

For simplicity, assume that the cloud we are looking at has (at the start) **roughly uniform density, equal to the average density**. The average density is simply the M_r divided by the volume of a sphere:

$$\rho_r \approx \frac{M_R}{\frac{4}{3}\pi R^3} \Rightarrow M_r \approx \rho_r V_r = \frac{4}{3}\pi r^3 \frac{M_R}{\frac{4}{3}\pi R^3} = \frac{r^3 M_R}{R^3}$$

We can finally integrate to obtain:

$$dU = -4\pi G \int_0^R \frac{r^3 M_R}{R^3} \frac{M_R}{\frac{4}{3}\pi R^3} r dr = -G M_R^{\,2} \int_0^R \frac{r^4}{R^6} dr \Rightarrow U = -\frac{3}{5}\frac{G M_R^{\,2}}{R}$$

Specifically if we name M_c as the total mass of the cloud, R_c as the radius of the cloud the potential gravitational energy of the cloud with approximately uniform density is:

$$U = -\frac{3}{5}\frac{G M_c^{\,2}}{R_c}$$

In order for a cloud to collapse, it must satisfy the condition:

$$\frac{6}{2}\frac{M_c k_B T}{\mu m_H} < \frac{3GM_c^{\,2}}{5R_c} \Leftrightarrow M_c > \frac{5k_B T R_c}{G\mu m_H}$$

The minimum mass needed to collapse depends on:

- Temperature (how hot it is)
- Radius (size)
- What it is made of (composition)

$$M_c \approx \frac{5k_B T R_c}{G\mu m_H}$$

M_c is called the **Jeans Mass M_J:** the **critical mass above which a cloud will collapse.** The equation can also be re-arranged to solve for **R_c** and calculate the **maximum size a cloud can have.**

In order for a cloud to collapse, it must satisfy the condition:

$$\frac{6}{2}\frac{M_c k_B T}{\mu m_H} < \frac{3GM_c^{\,2}}{5R_c} \Leftrightarrow M_c > \frac{5k_B T R_c}{G\mu m_H}$$

The minimum mass needed to collapse depends on:

- Temperature (how hot it is)
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$$M_c \approx \frac{5k_B T R_c}{G\mu m_H}$$

M_c is called the **Jeans Mass**, **which a cloud** [illegible] also be re-arranged to solve fo[illegible] and calculate the **maximum size a cloud can have.**

The **Jeans mass** represents the **minimum mass** that can overcome the internal gas pressure at a given temperature and density.

Collapse if:

tff<ts

L>Ljeans

M>Mjeans

Fgravity>Fpressure

Expansion if:

ts<tff

L<Ljeans

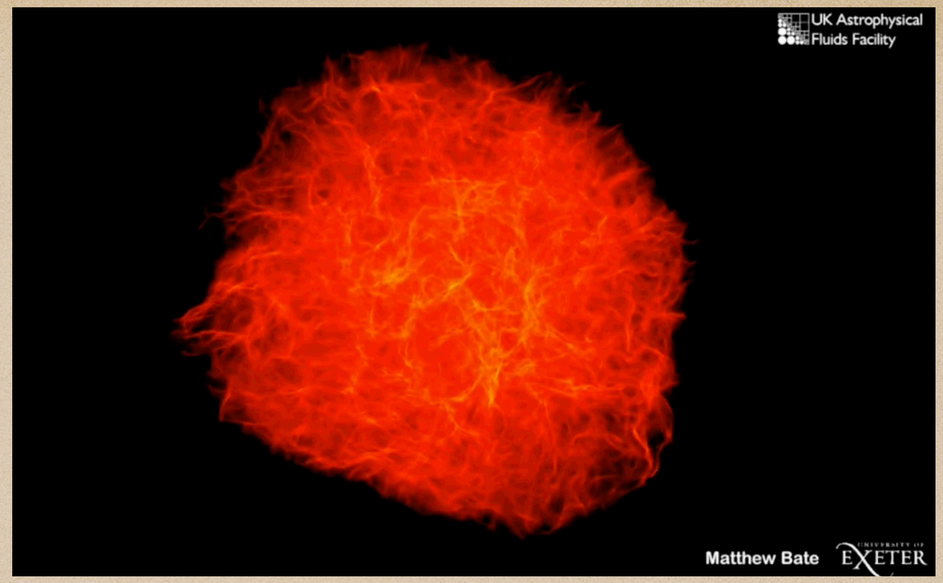
M<Mjeans

Fgravity<Fpressure

The **Jeans length** is the minimum size a gas cloud must have for its **self-gravity to overcome internal pressure** and begin collapsing — the first step in **star formation**.

The **Jeans mass** represents the **minimum mass** that can overcome the internal gas pressure at a given temperature and density.

The Jeans mass may be achieved in many sub-locations ==> hydro-sims



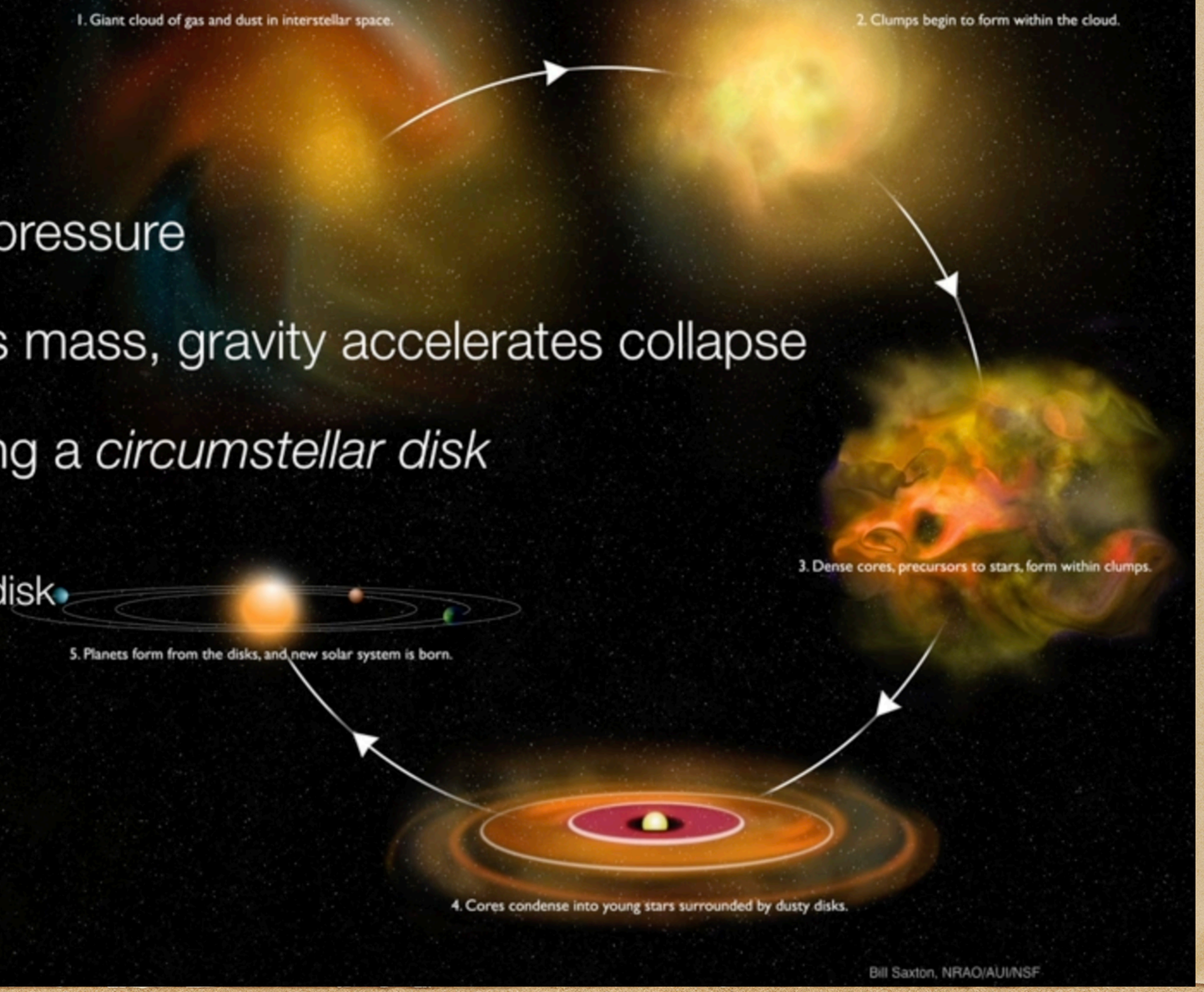
Cloud fragmentation

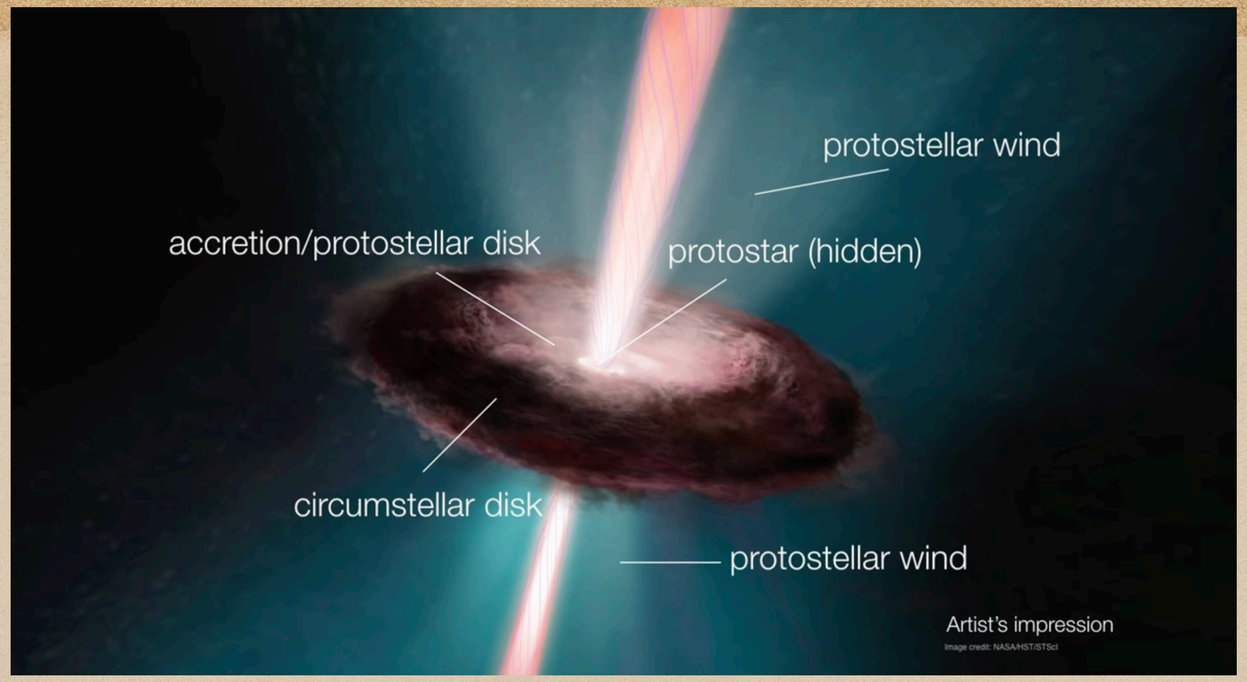




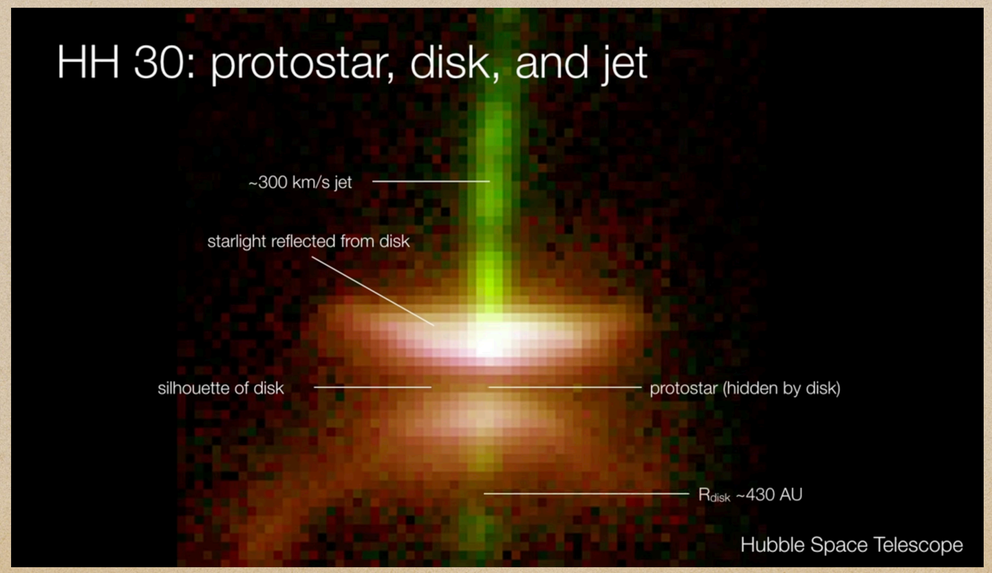
Formation cycle

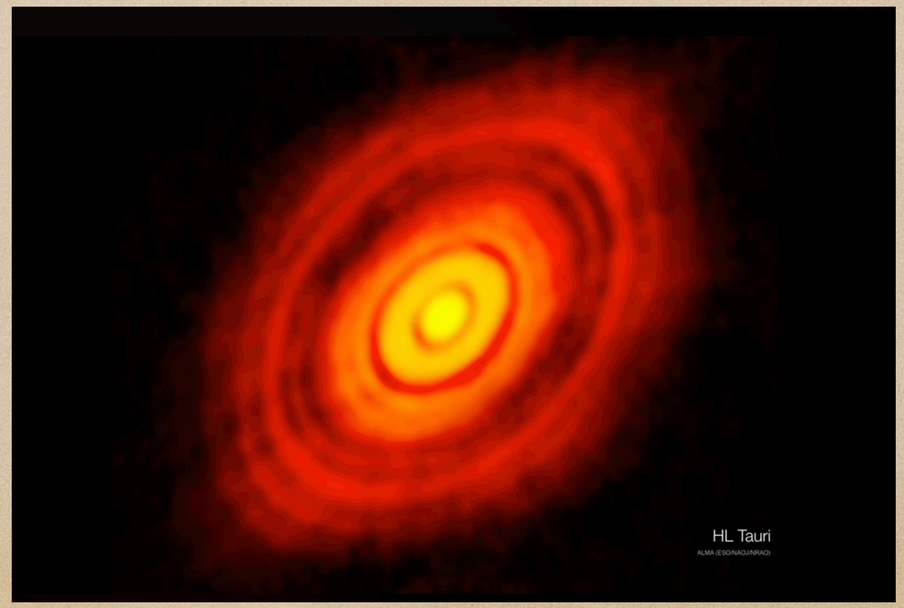
1. Pressure resists gravity
2. Dense clump forms, gravity > pressure
3. Core of clump (protostar) gains mass, gravity accelerates collapse
4. Rotation rate increases, creating a *circumstellar disk*
   - inner disk feeds protostar (*accretion* disk)
   - remaining disk flattens into *protoplanetary* disk
5. Planetary system forms
   - star “turns on” (H-burning in core)
   - winds blow away disk
   - planets sweep out orbits





Edge on



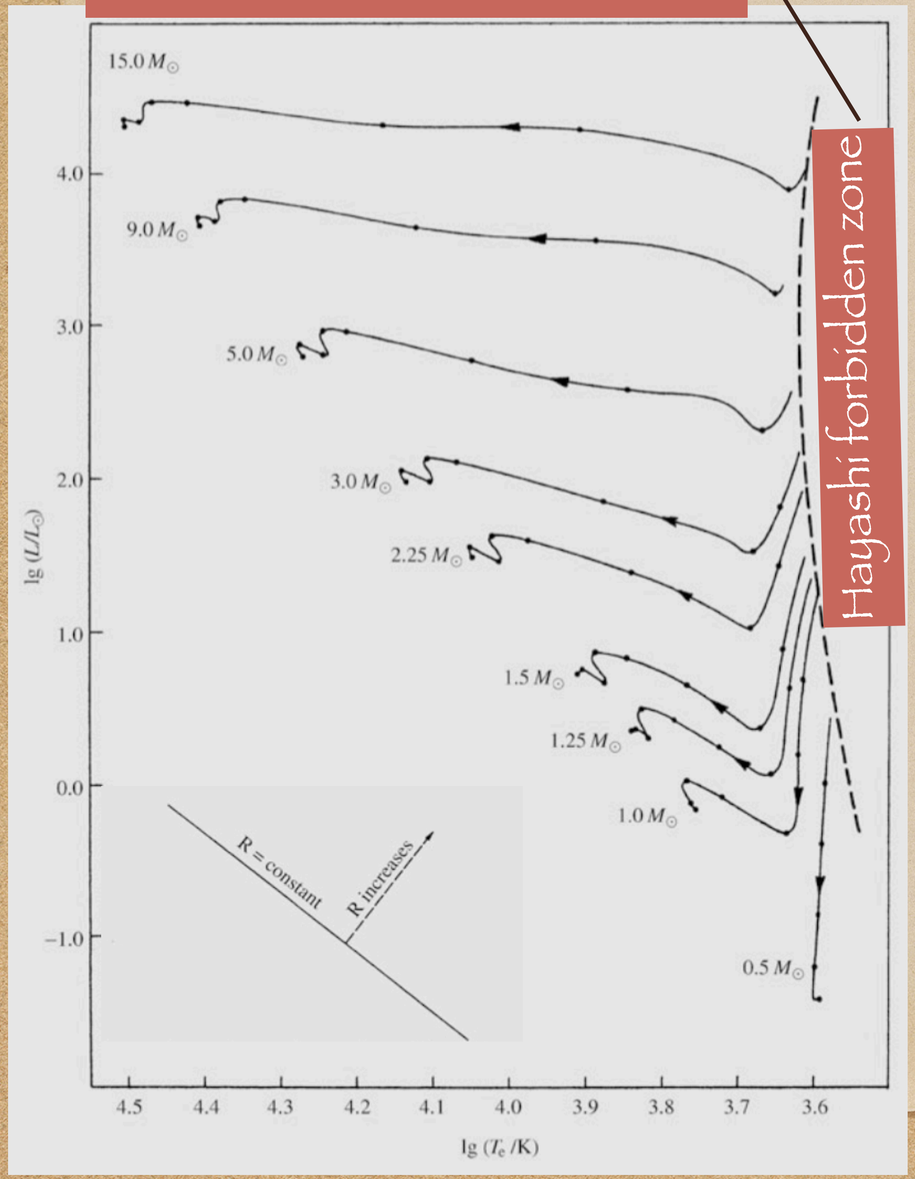


Face on

ALMA

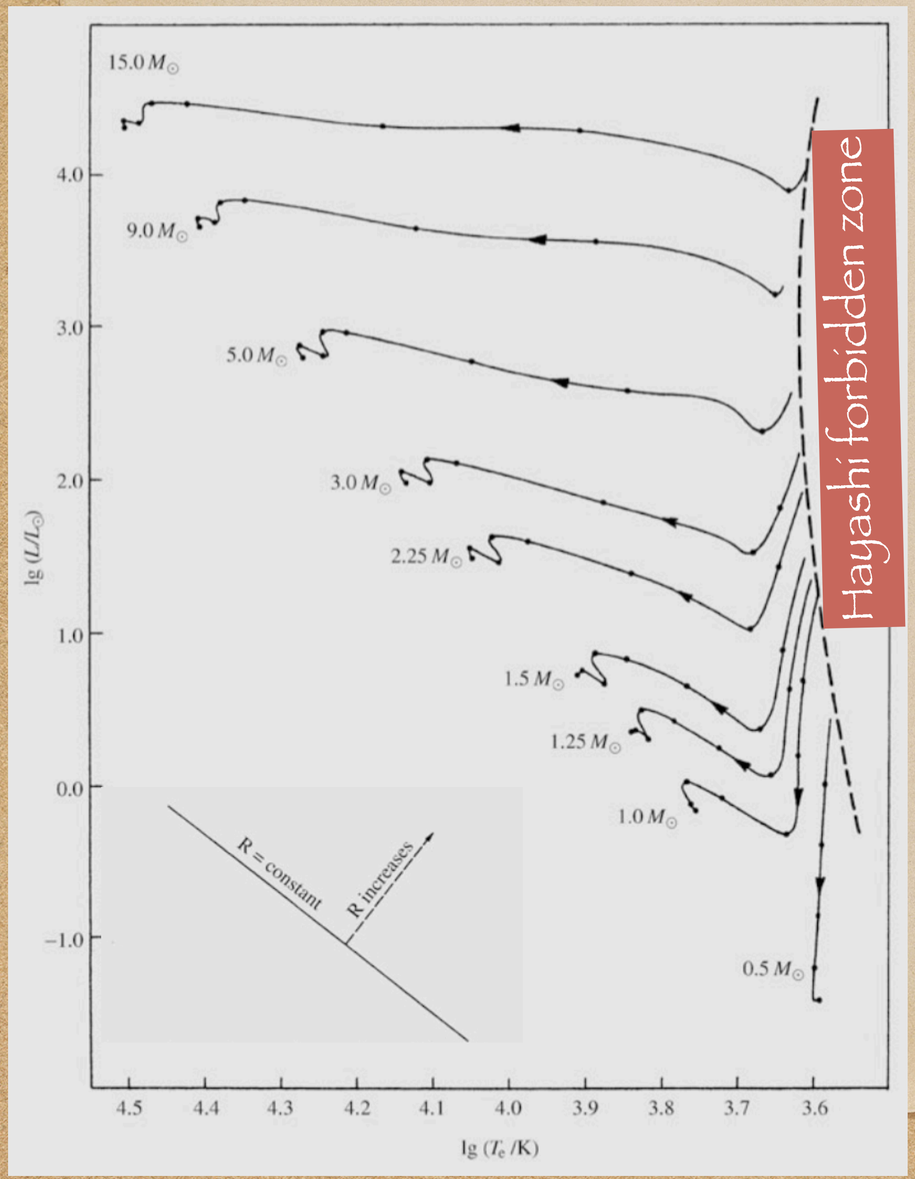
Contraction in Main sequence

No hydrostatic equilibrium
(T<4000K)



- During the contraction of the protostellar cloud, the potential energy is transformed into thermal energy and radiation
- Initially, radiation can escape freely
- Then the opacity (the gas's ability to absorb radiation) increases, the gas heats up, and the pressure from the underlying layers increases
- This happens more quickly in the center, so in the nucleus it slows down the contraction
- As the temperature increases, the molecules are dissolved into atoms, then the atoms are ionized
- So far, the luminosity is produced by the conversion of potential energy
- All this happens in a dynamic time, then...
- ...The star reaches a balance, it is made of plasma, entirely convective, not hot enough to ignite thermonuclear reactions→ Hayashi line

Contraction in Main sequence



- The next phase takes place over a longer time: thermal time

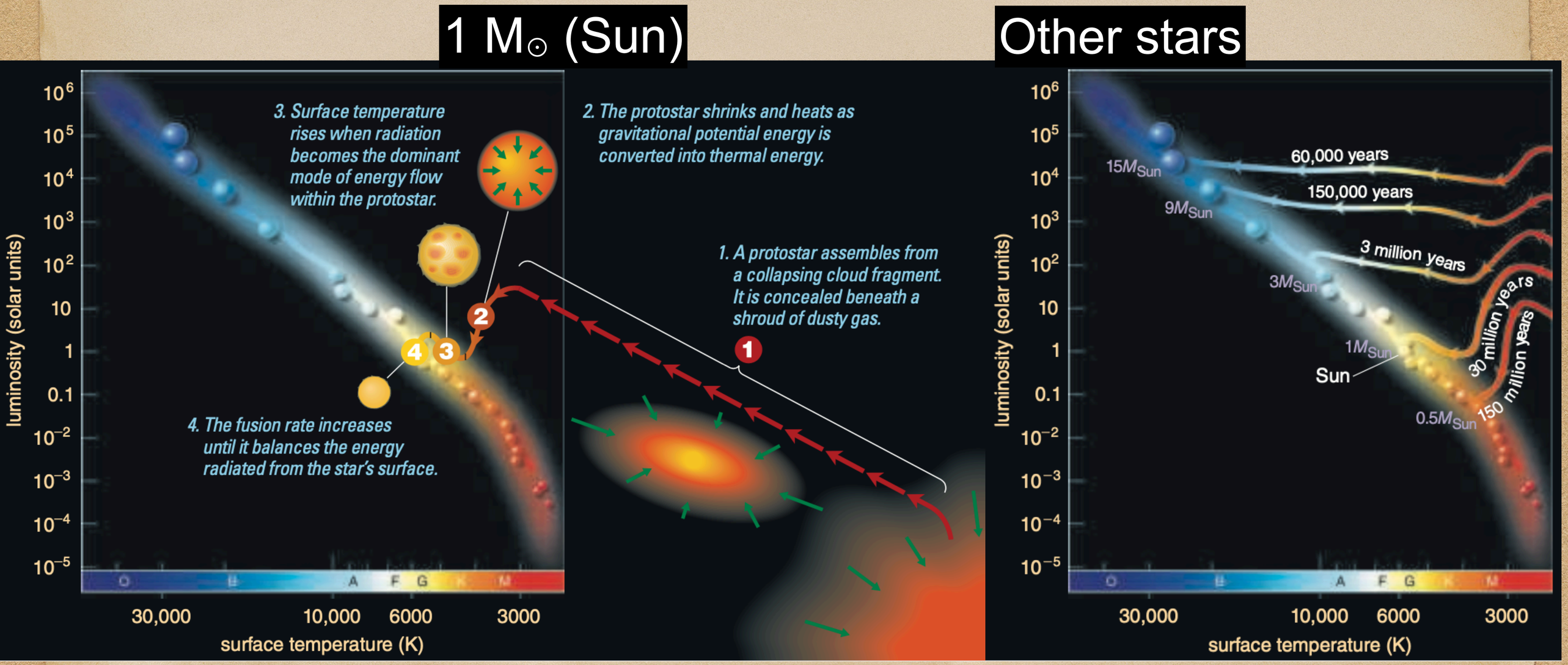
- The star contracts, moving vertically downwards → becomes less bright, the radius decreases

- The temperature increases at the center, the opacity decreases, the nucleus becomes radiative

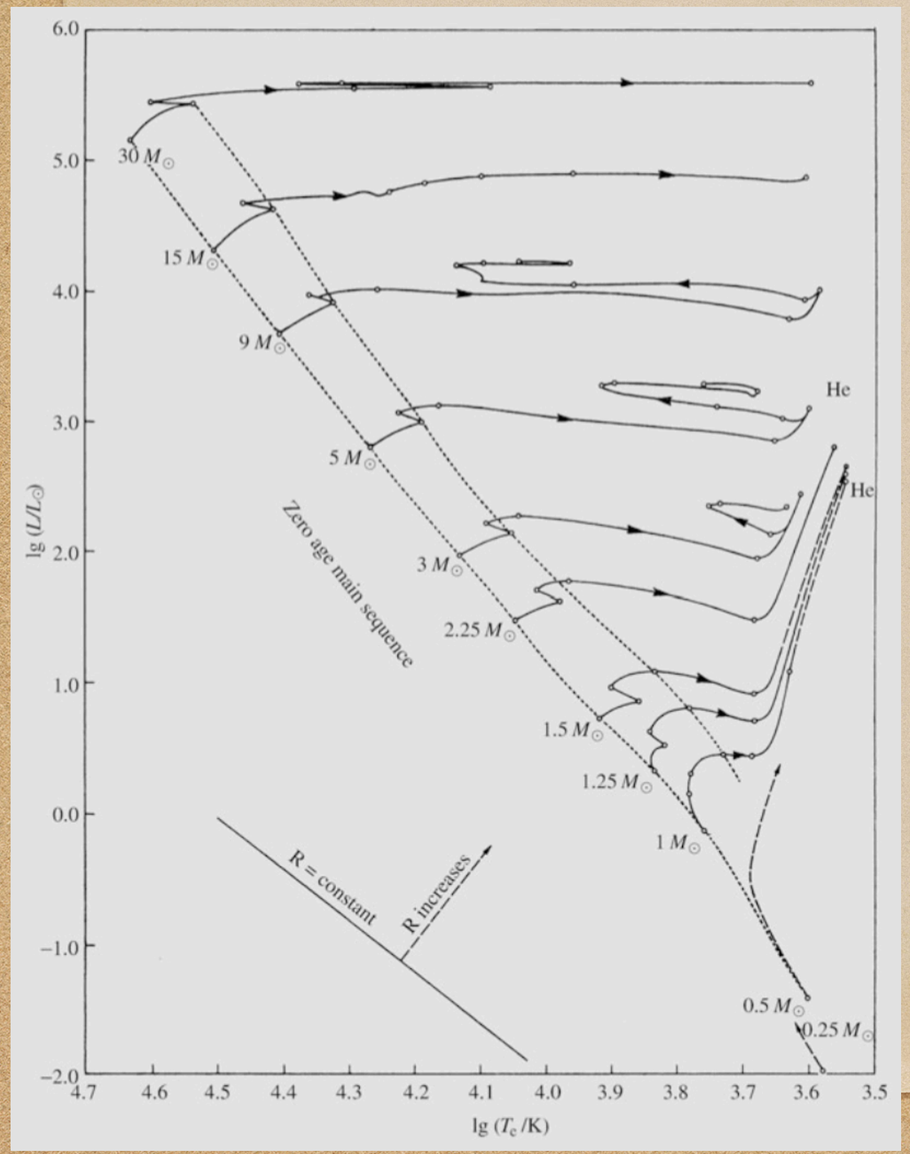
- The thermonuclear reactions in the center are ignited, the brightness increases → the star moves upwards to the left (for the most massive stars, this happens first, because being hotter the thermonuclear reactions in the center start earlier)

- This phase lasts from 60,000 yr for the most massive stars up to ~10^8 yr or the stars of 0.1 $M_\odot$

Pre-sequence stars



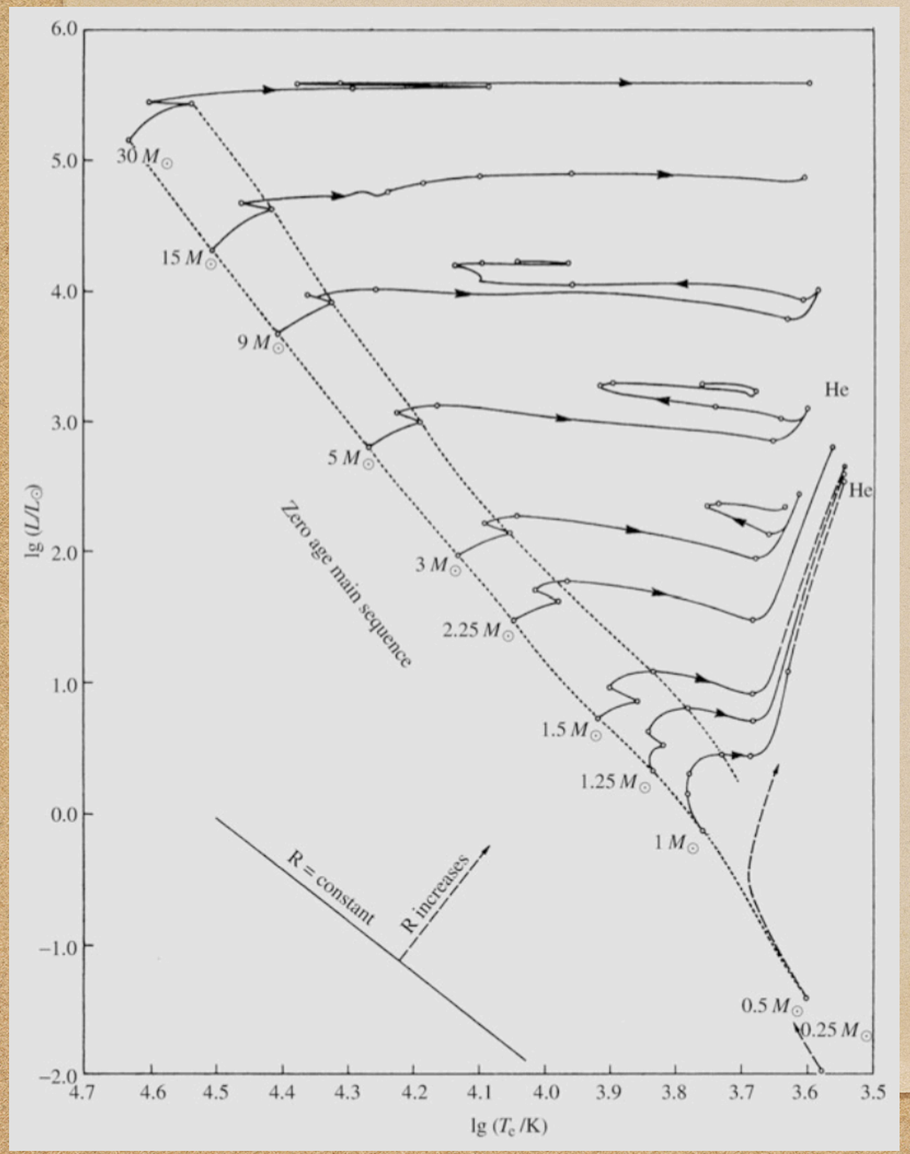
Main sequence phase



- It is the phase in which the main source of brightness of a star is the burning of hydrogen in helium in the nucleus
- It happens on a nuclear time scale
- It's the longest phase, that's why most of the stars are in main sequence
- There is an upper limit to the mass of a star: if the mass exceeds that limit, the radiation pressure from the lower layers prevents further gas from growing; theoretical calculation $M_{lim} \approx 120 M_\odot$ (however very rare)
- Limit lower than the mass of a star: below 0.08 $M_\odot$ The central temperature does not rise enough to ignite the burning of hydrogen→ brown dwarf

- Upper main sequence: CNO channel
- Lower main sequence: p+p chain

Post- Main sequence phases

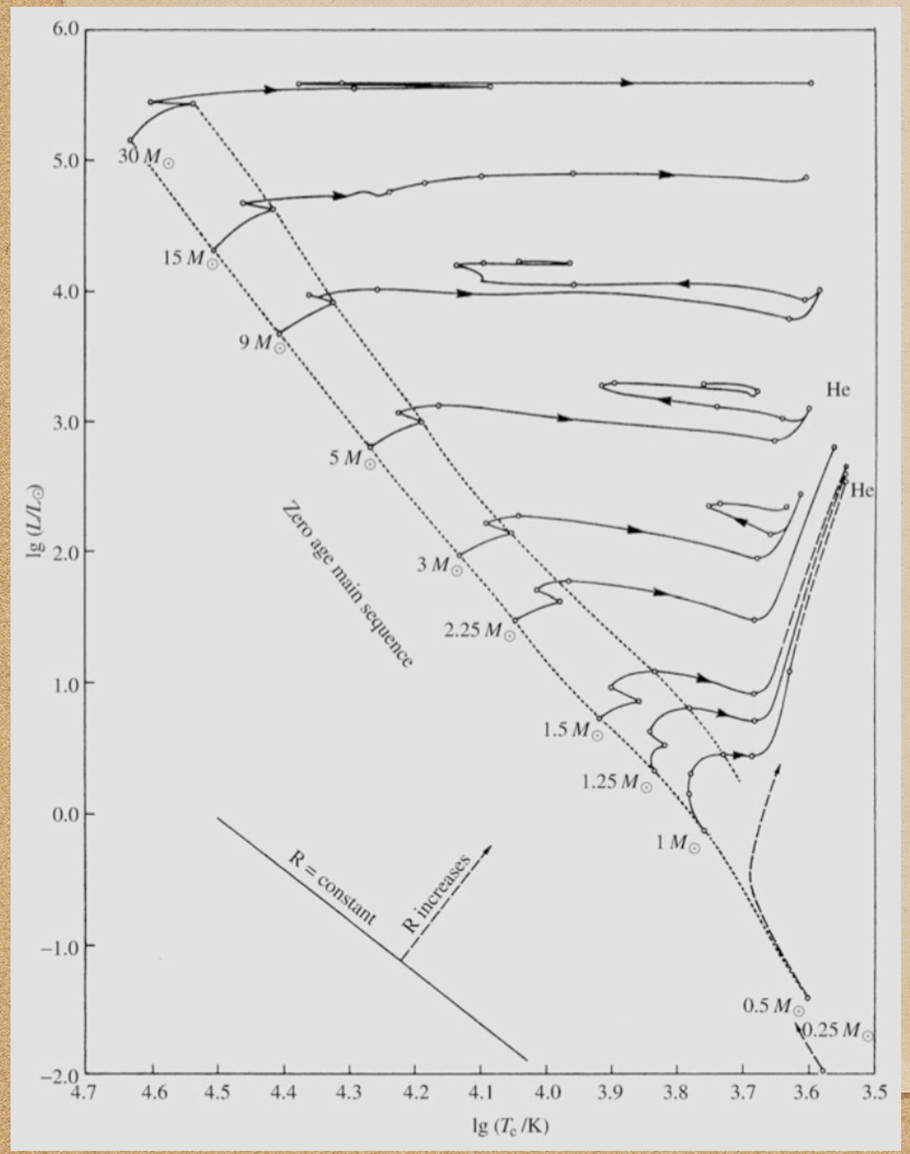


- The main sequence phase ends when the hydrogen in the center is exhausted: the star is no longer in equilibrium

- In the immediately following phase, the hydrogen is burned in a shell around the nucleus → subgiants branch (SGB)
  - Slow for small mass stars, fast for massive stars; i the burning of hydrogen in the shell increases the radius: the stars move to the right, reach the Hayashi line and rise it (their luminosity increases) → red giants branch (RGB)

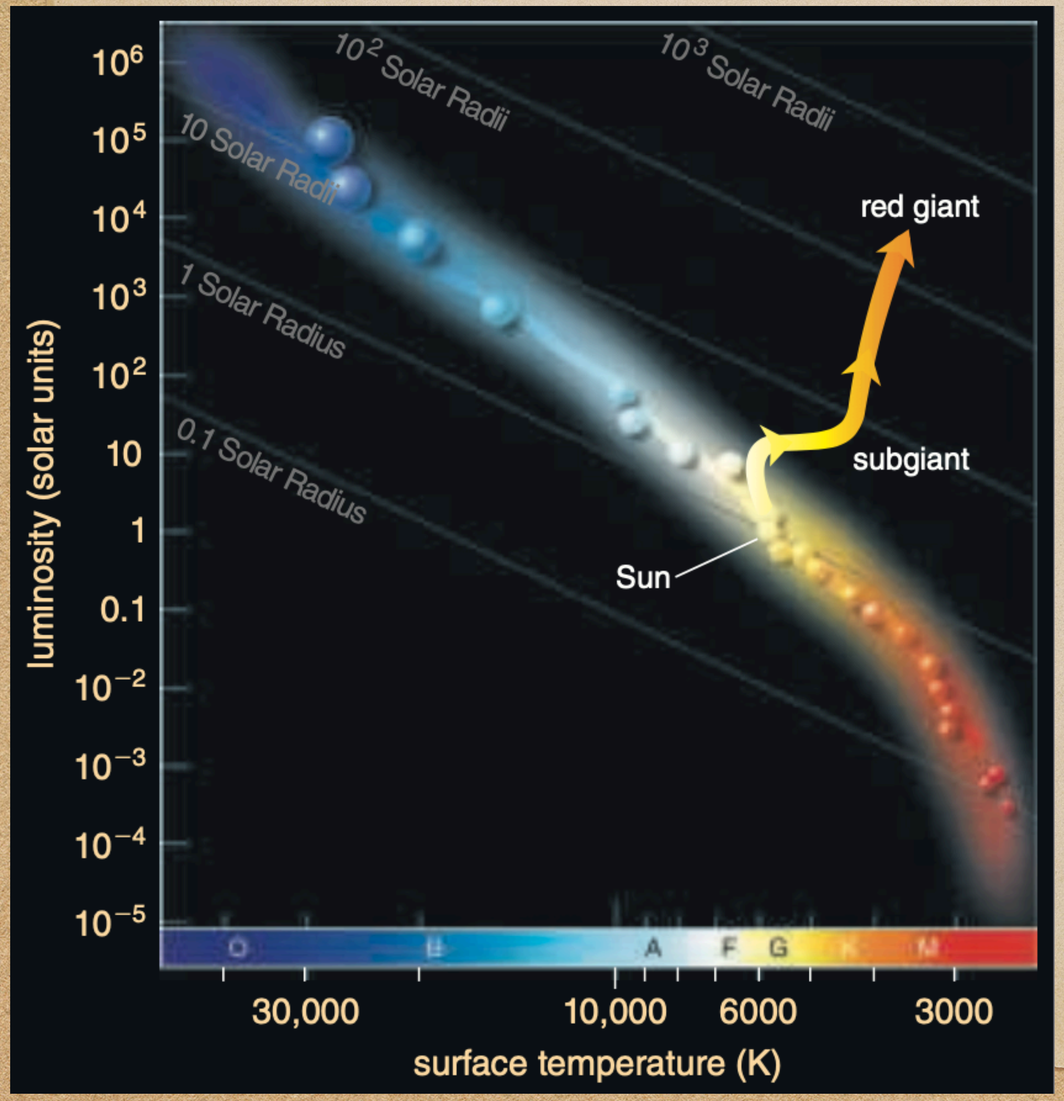
- Now the central temperature is enough to burn He in the nucleus → Helium flash; the star contracts and the brightness decreases
- Then the stars settle on the horizontal branch (HB) to burn He in the center

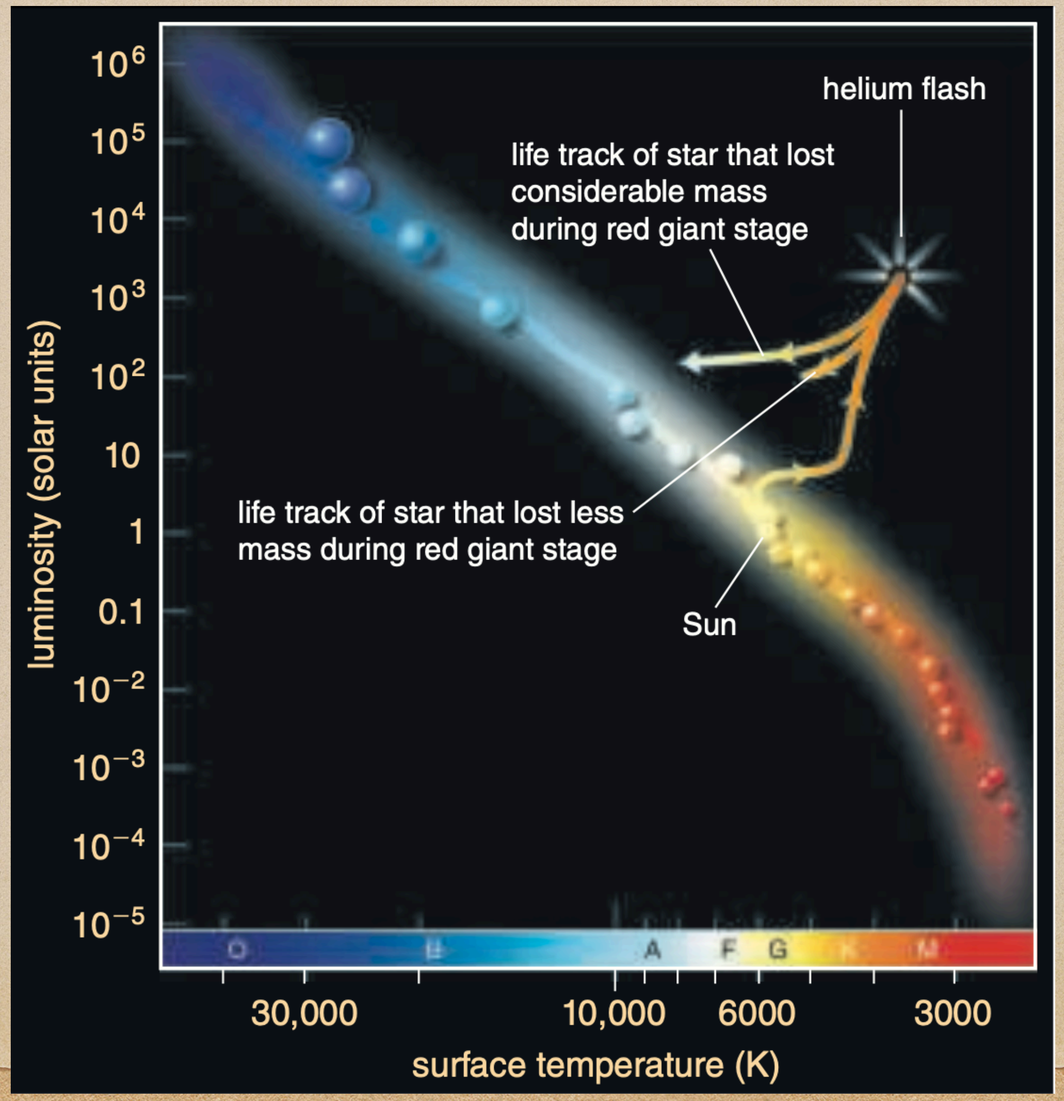
post- Main sequence phases



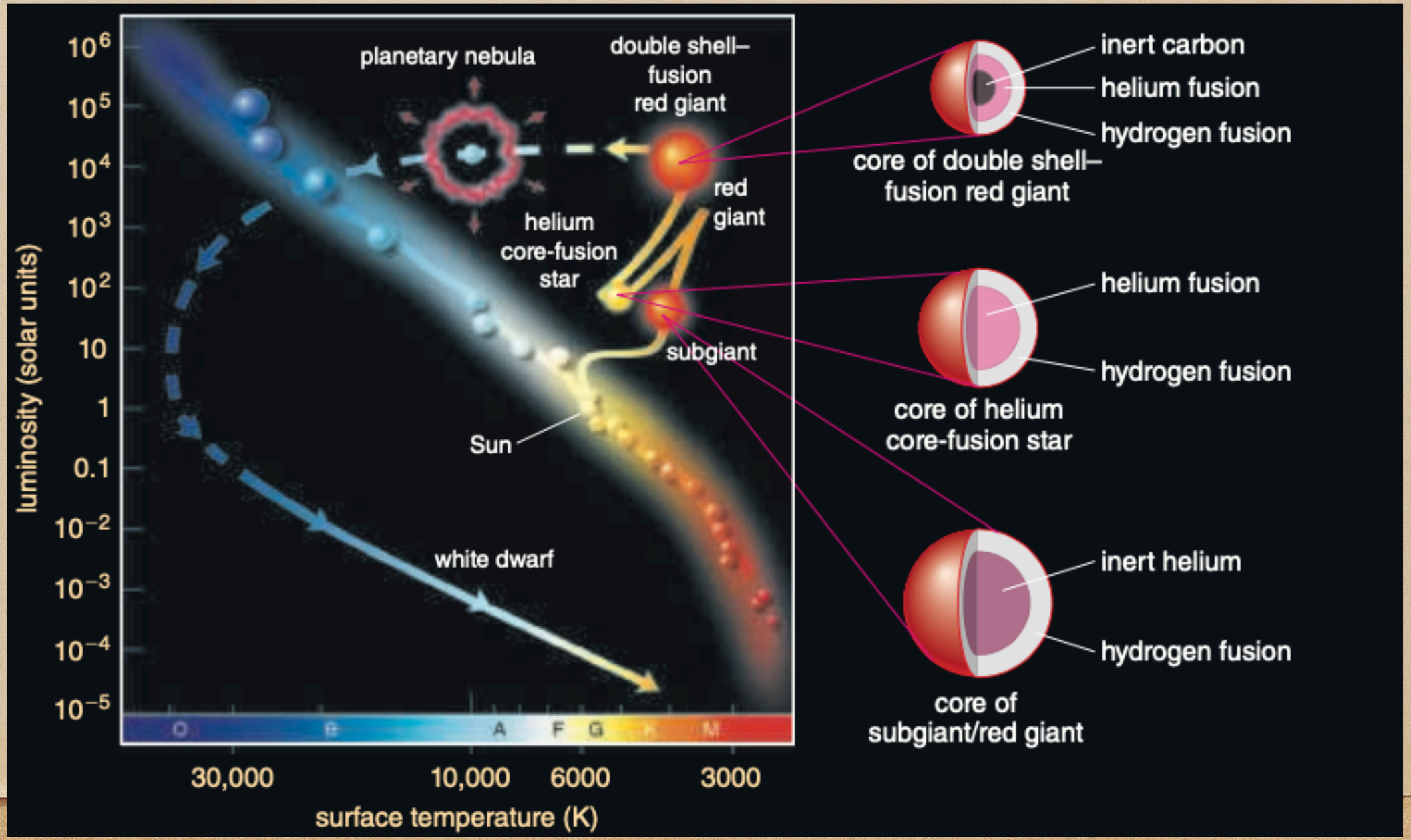
- The phase following that of burning helium in the center depends on the stellar mass, which determines the temperature that can be reached in the center
- The burning of helium in the center has been exhausted, it moves to shell, the star becomes brighter and expands→ AGB: asymptotic giant branch (almost parallel to RGB)
- The star begins to alternate phases of burning hydrogen and helium in the shell→ Thermal Pulsation (TP-AGB)
- This phase ends when the radiation pressure has expelled all the outer layers→ Planetary Nebula
- Stars with M < 8 $M_\odot$ They never get hot enough to burn Carbon; those of smaller mass (e.g. Sun) end up as white dwarfs; the most massive ones explode as supernova
- Stars with M > 8 $M_\odot$ can burn C into Fe

post-MS phase for a 1 $M_\odot$ star (Sun)

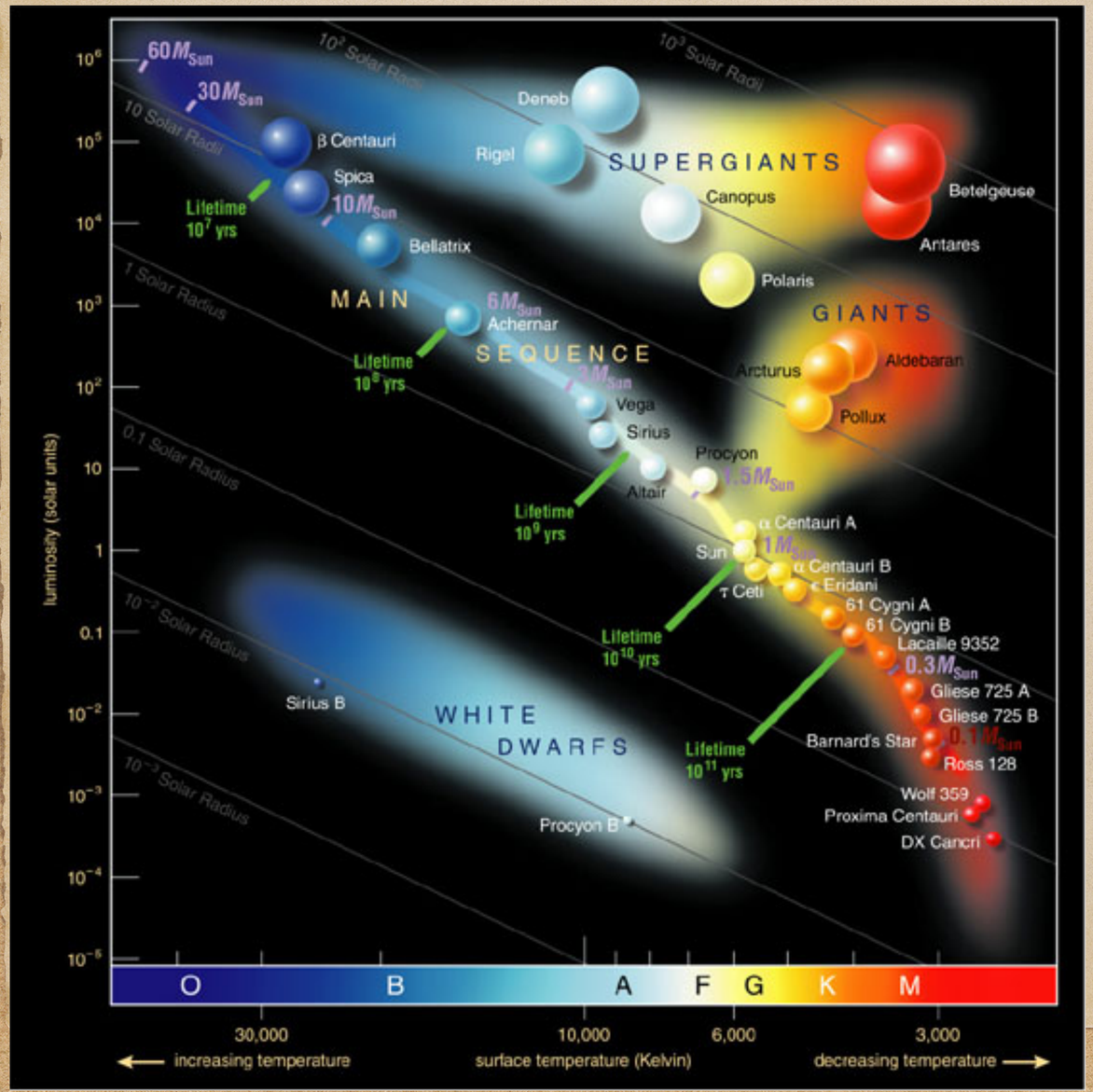




post-MS phase for a star of 1 $M_\odot$ (Sun)

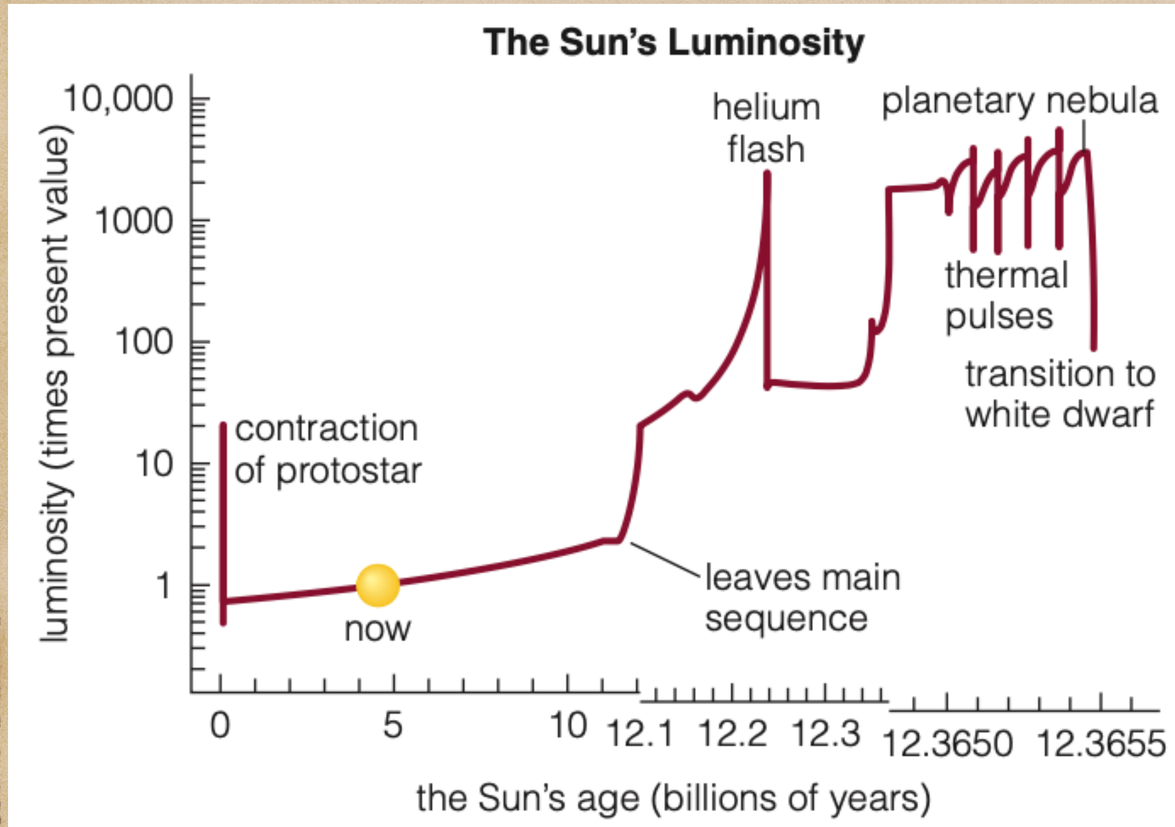


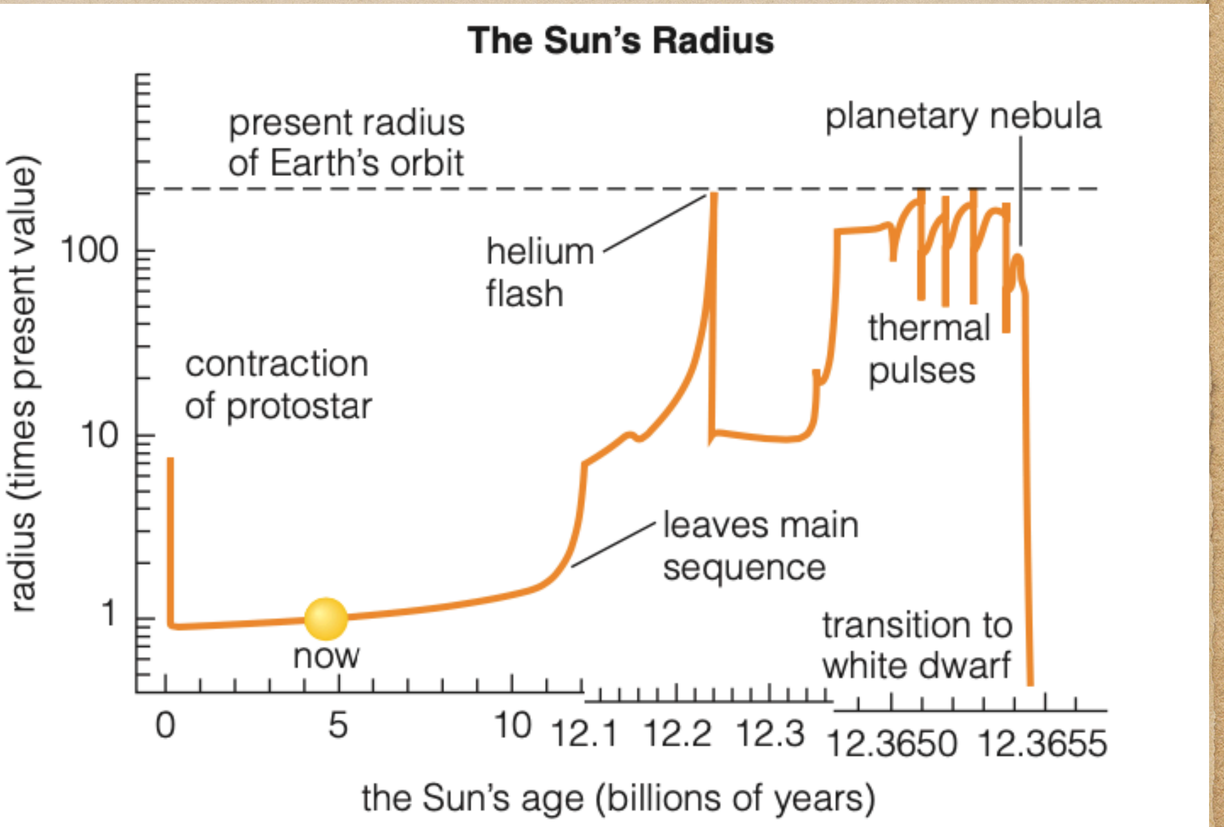
White dwarfs



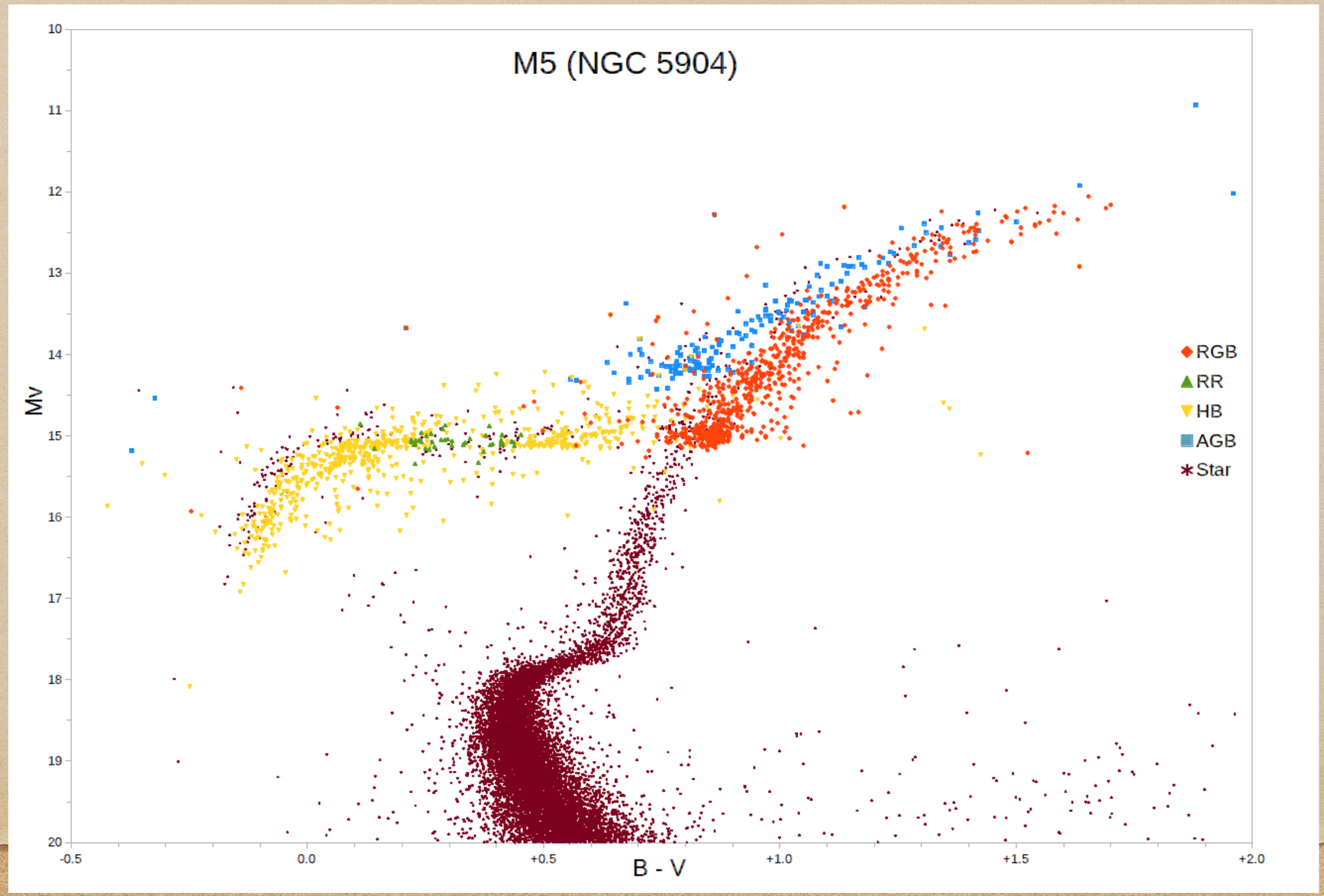
Quiescent nuclei of giant stars (composed mostly of electro-degenerous matter, i.e. plasma of nuclei and unbound electrons) where nuclear reactions have ceased. They are hot because they are exposed stellar nuclei and radiate only their residual heat into space (emission of stored thermal energy). Extremely compact object (mass like the Sun, size like the Earth), e.g. Sirius B.

Evolution of 1 $M_\odot$ star (Sun)

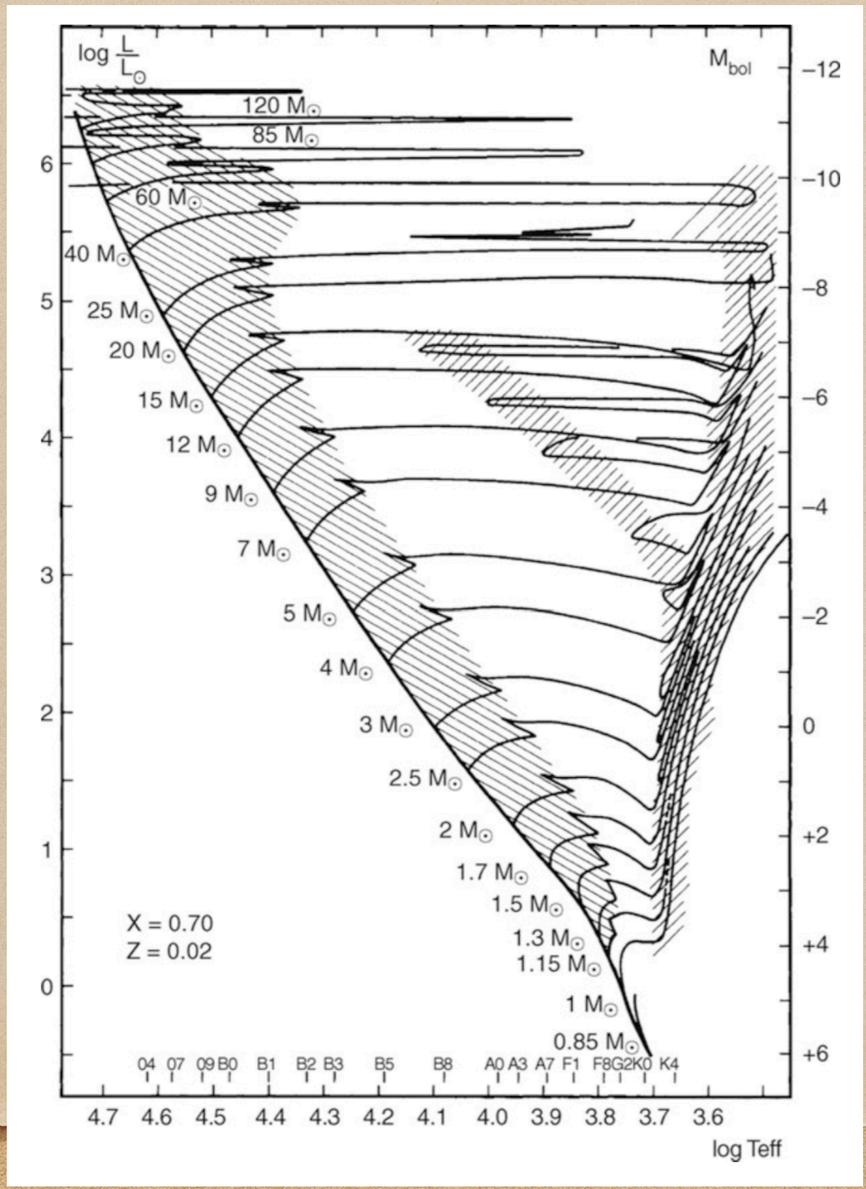




post-Main sequence phases



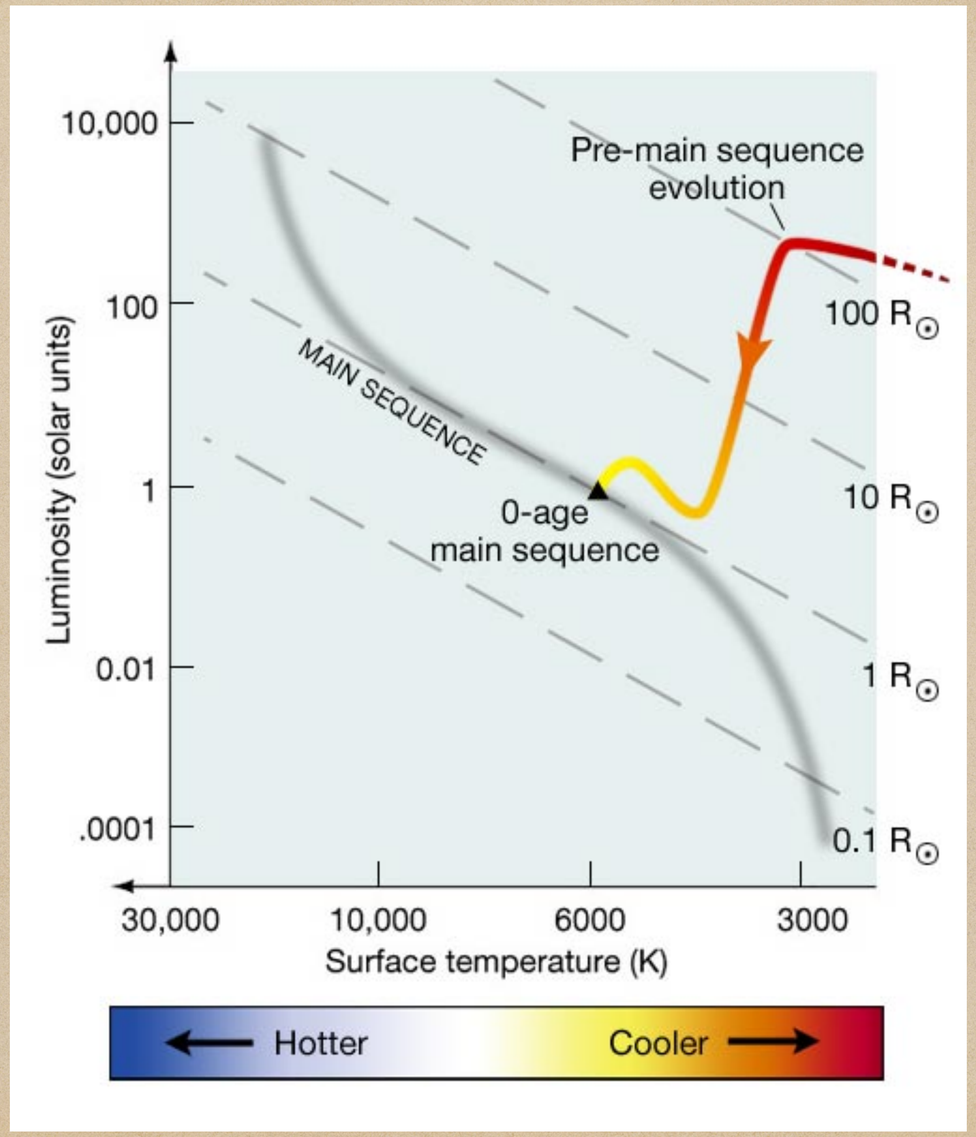
HR diagram as an evolutive diagram

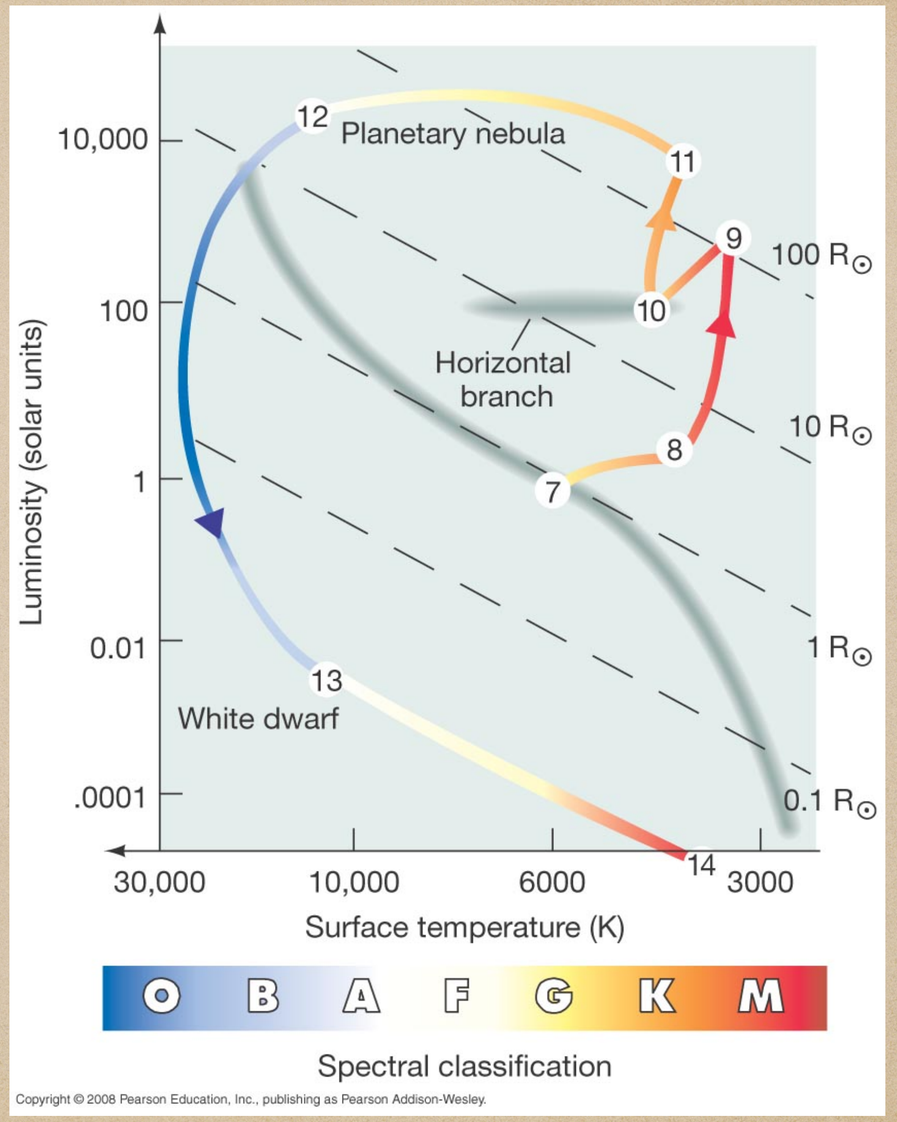


Evolution of stars with different initial masses on the HR diagram

The shaded areas of the diagram are those where evolution is slower, so they are more populated in the HR diagram of a certain population

Sun evolution

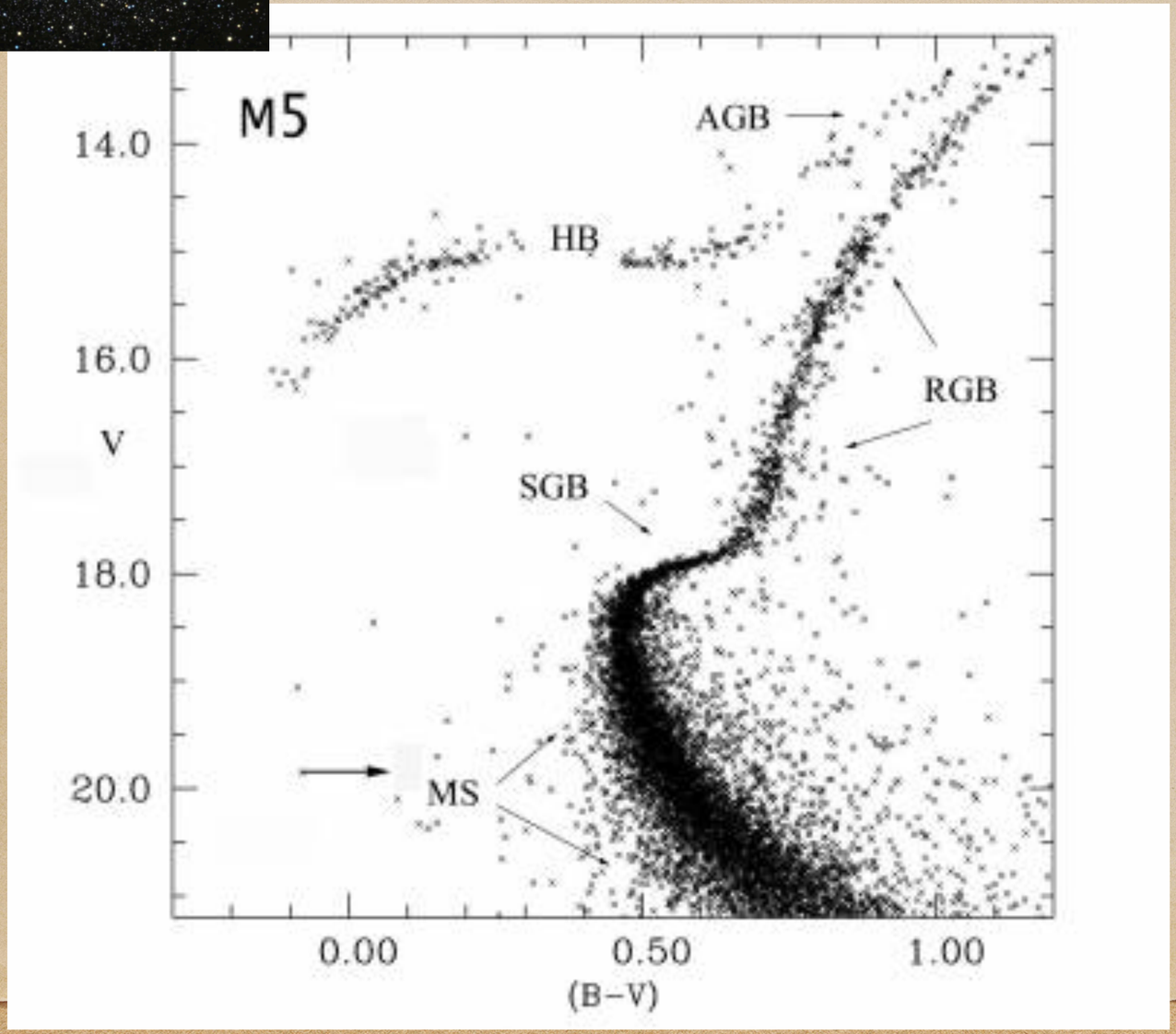




The HR diagram is also an evolutionary diagram!



HR diagram of a globular cluster

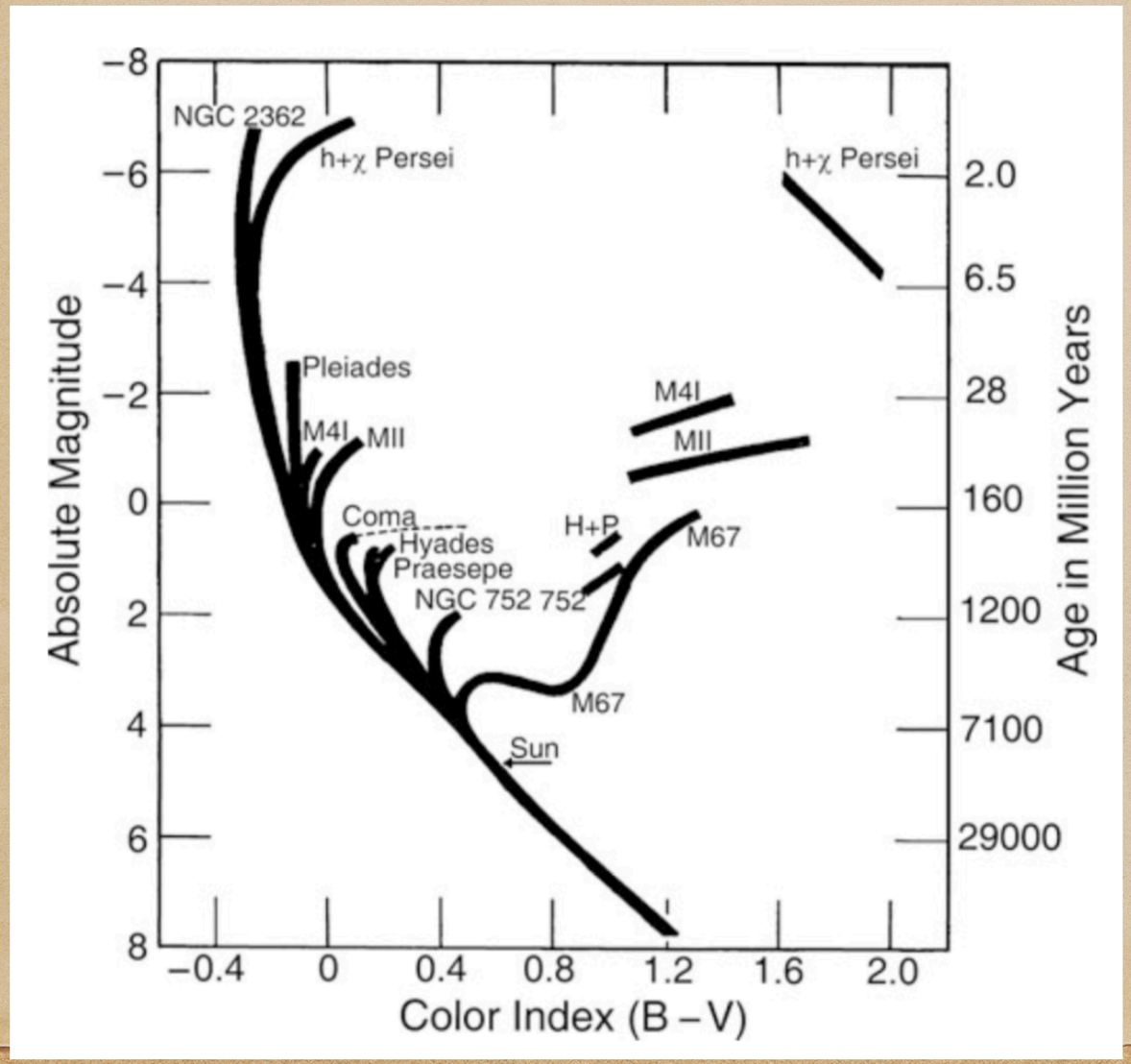


In the first approximation, it is a population of stars all born at the same time

It's equivalent to taking a snapshot: the curve where the stars are located is an ***isochrone***

How to determine the age of a stellar population (for example, a globular cluster)? ***From the MS turn over position***

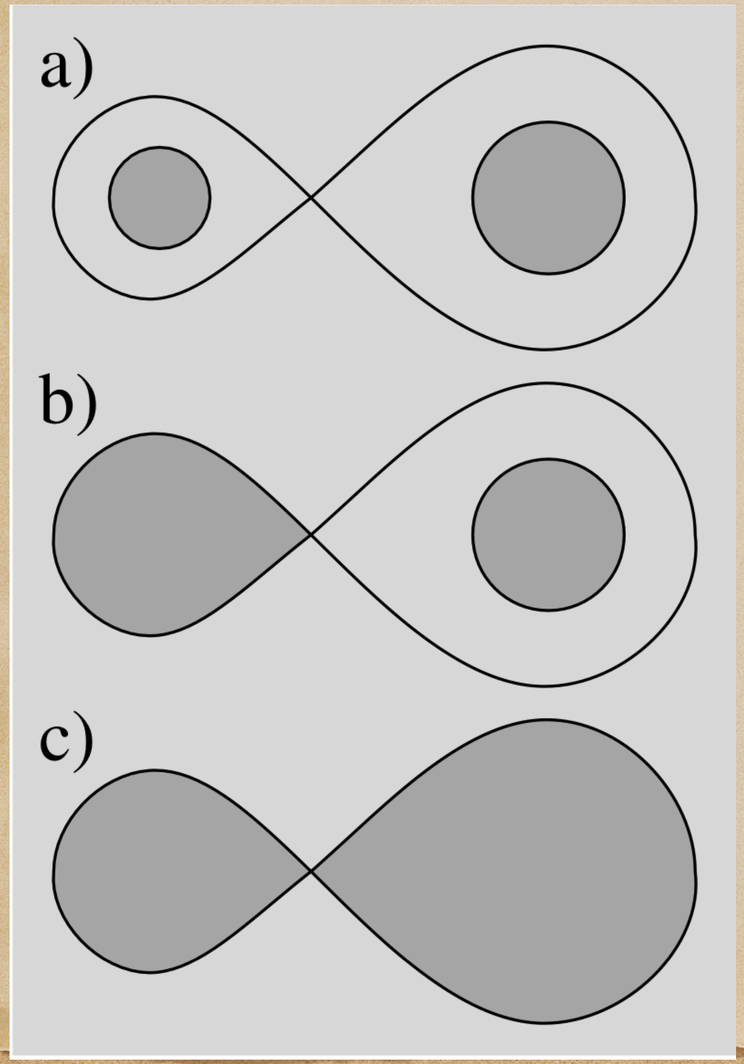
Determination of clusters ages with the HR diagram



The brighter stars have a shorter life on the main sequence than the less bright ones. The *turnoff* from the main sequence therefore corresponds to that stellar mass so that the duration of the main sequence is equal to the age of the cluster.

The age is specified on the right axis as a function of the turnoff position; the Sun will leave the main sequence after about 10 billion years.

Binary systems



Most stars are actually in binary systems

- Roche's lobe: equipotential surface, if the star expands to it, a mass transport on the companion begins
- a) “detached” system ; b) “semi-detached” system; c) “contact” binary
- During MS, the stars are well inside the Roche lobes
- When the most massive evolves out of sequence, it can fill its lobe with Roche and start a mass transfer on the other; at a certain point it will be the other one to be more massive

Binary systems

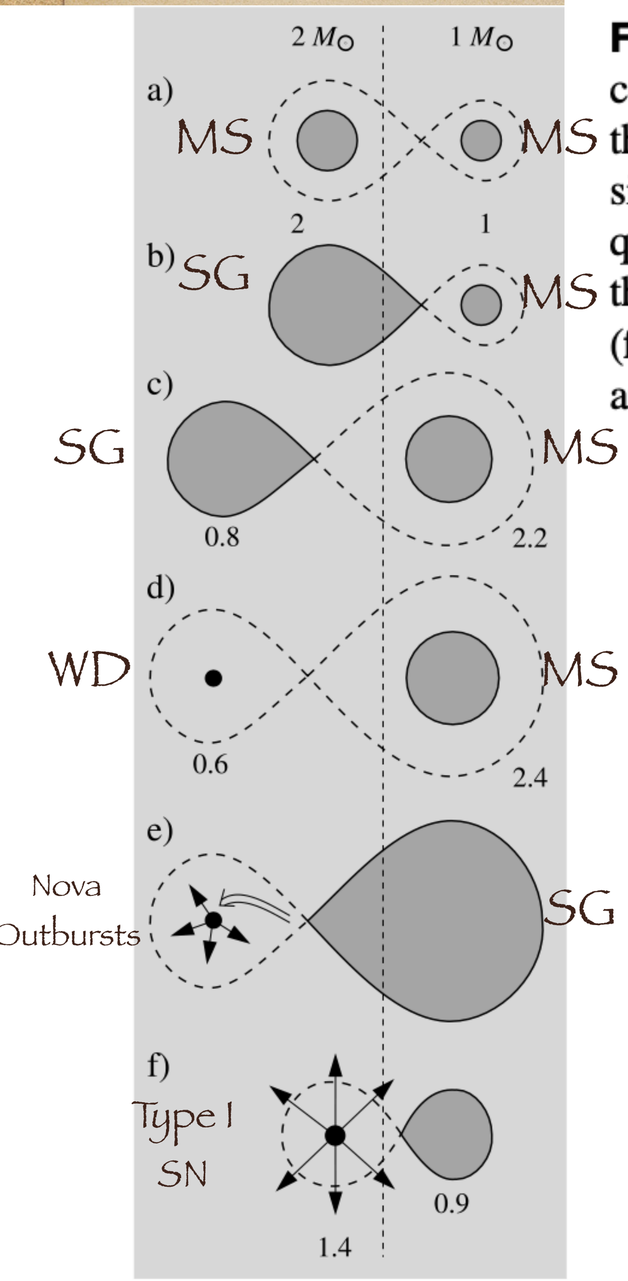


Fig. 12.10 Evolution of a low-mass binary: (**a**) both components on the main sequence; (**b**) mass transfer from the more massive component; (**c**) light subgiant and massive main sequence star; (**d**) white dwarf and main sequence star; (**e**) mass transferred to the white dwarf from the more massive component leads to nova outbursts; (**f**) the white dwarf mass exceeds the Chandrasekhar mass and explodes as a type I supernova

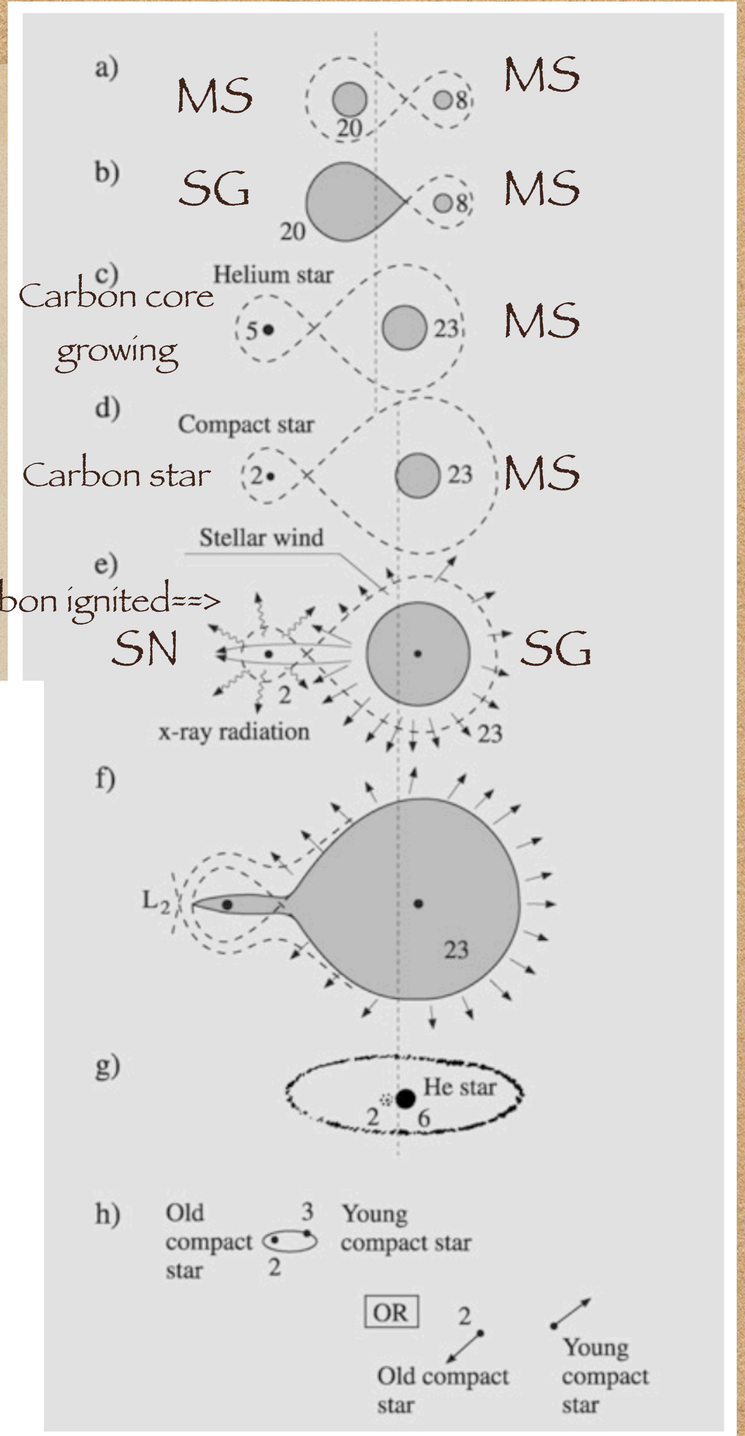


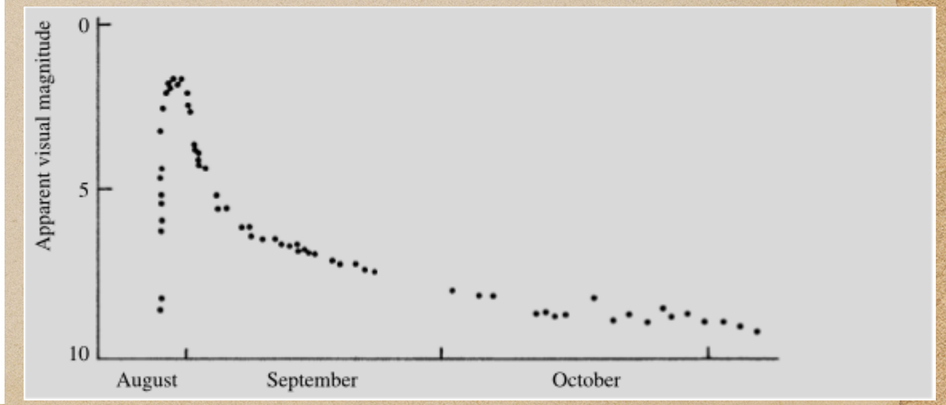
Fig. 12.11 Evolution of a massive binary. It has been assumed that the supernova explosion of a 5 $M_\odot$ helium star leaves a 2 $M_\odot$ compact remnant (neutron star or black hole). (**a**) Main sequence phase; (**b**) beginning of the first mass transfer phase; (**c**) end of the first mass transfer phase; the first Wolf–Rayet phase begins; (**d**) the helium star (Wolf–Rayet star) has exploded as a supernova; (**e**) the 23 $M_\odot$ component becomes a supergiant; the compact component is a strong X-ray source; (**f**) beginning of the second mass transfer phase; the X-ray source is throttled and large-scale mass loss begins; (**g**) second Wolf–Rayet phase; (**h**) the 6 $M_\odot$ helium star has exploded as a supernova; the binary may or may not be disrupted, depending on the remaining mass

Death of the most massive stars

- The most massive stars end their life with an Iron core; iron cannot be combustible by thermonuclear fusion reactions
- This iron-only nucleus has no radiation pressure to support gravity, it collapses into a compact object (neutron star, black hole)
- In the collapse a huge energy is released (100x the energy radiated by the sun in its entire life)
- This energy causes an explosion called **supernova**; so bright that it is visible at cosmological distances
- It is at this stage that elements heavier than iron are formed, by capturing neutrons

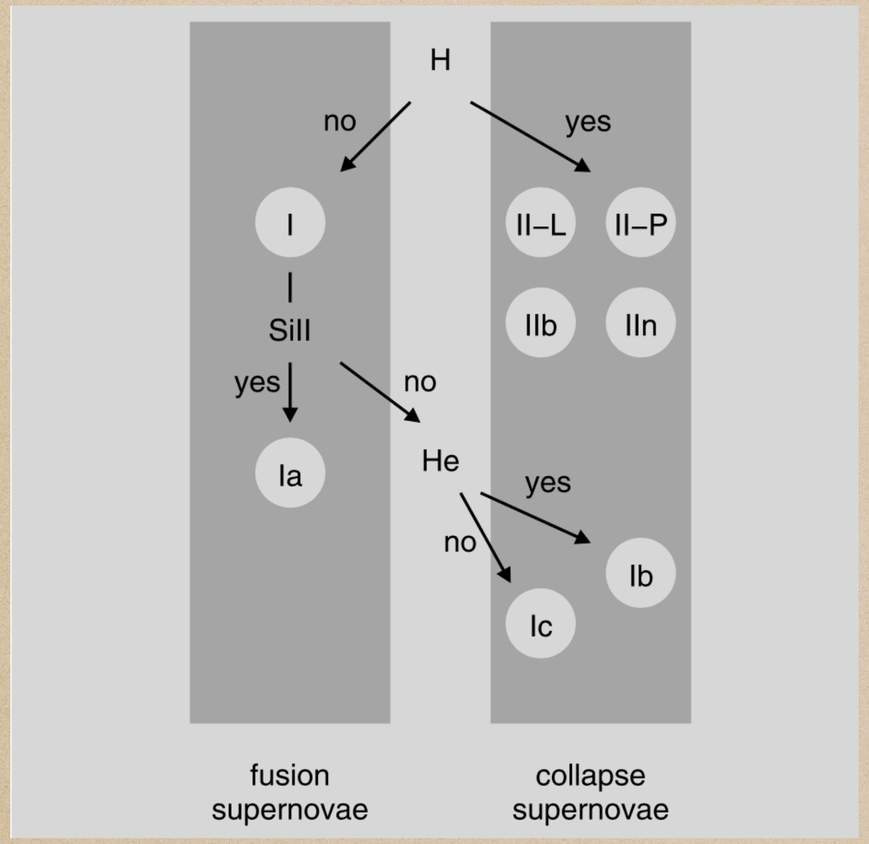


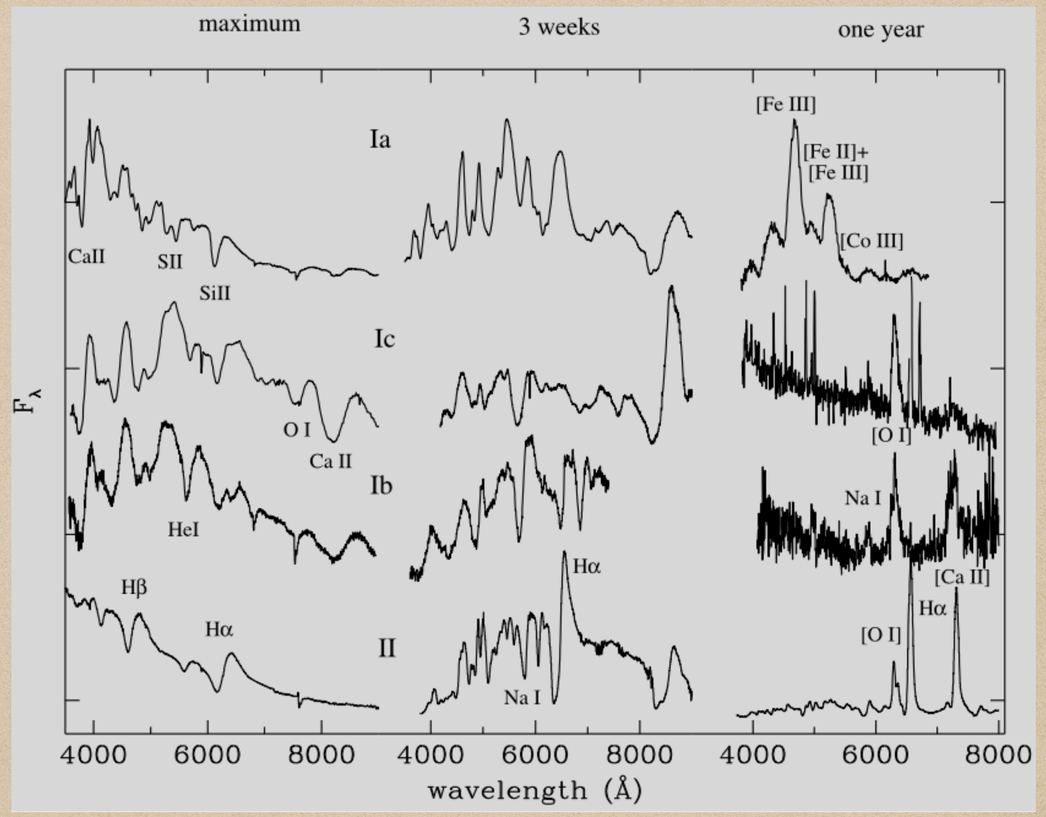




Supernovae

- Original classification based on spectrum (presence or absence of hydrogen)

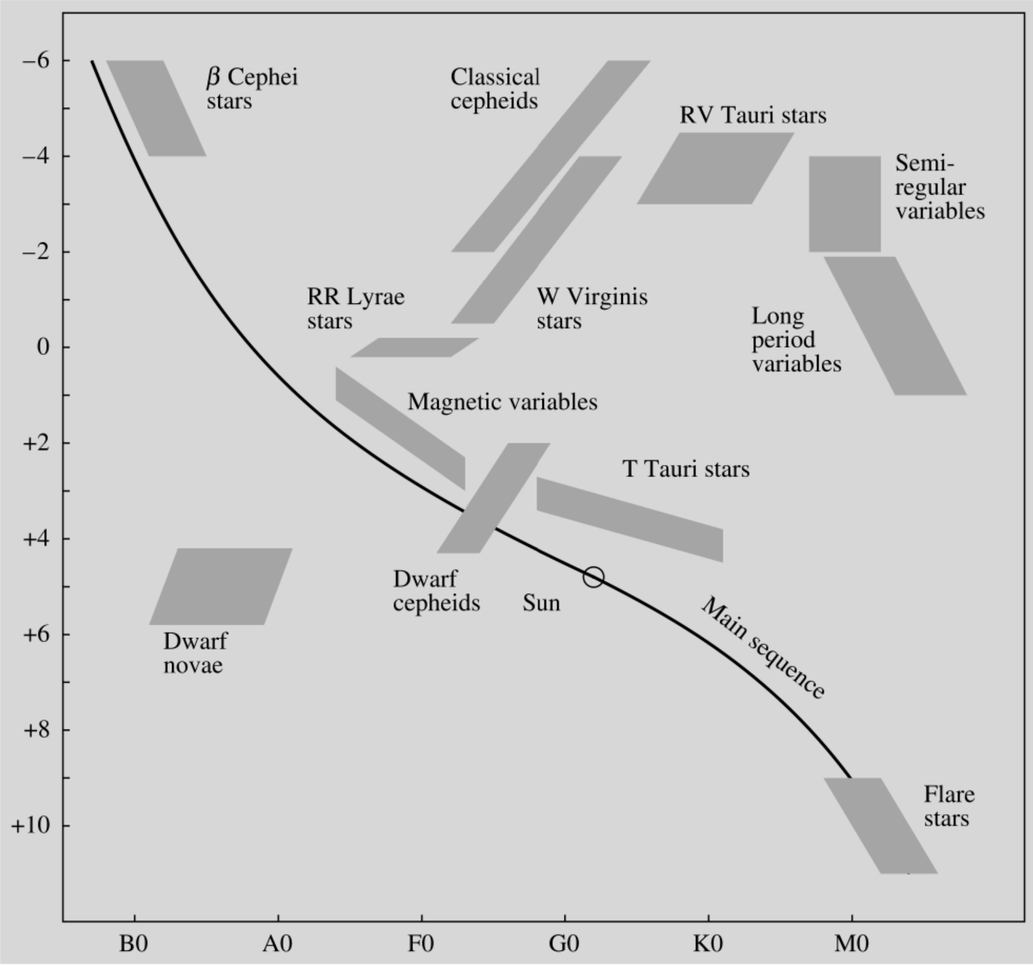


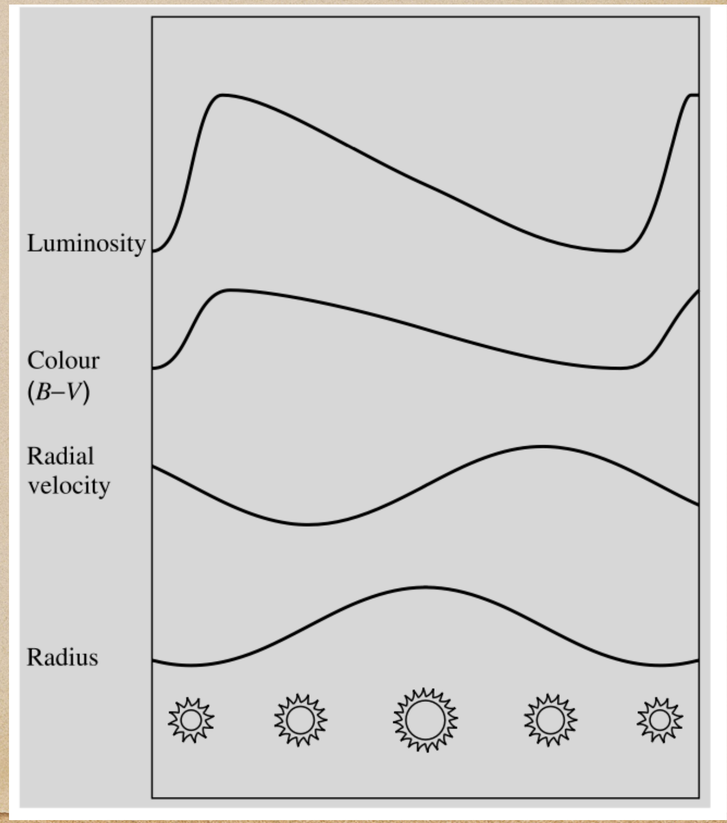


- Type Ia: formed by binary systems of a white dwarf and a companion (in more advanced systems)
- Type II: explosion of massive stars (therefore mainly in young populations)

Variable stars

In various phases of evolution, stars of different masses go through unstable phases in which their brightness varies over time, more or less periodically





Example: Cefeids

Compact stars: white dwarfs

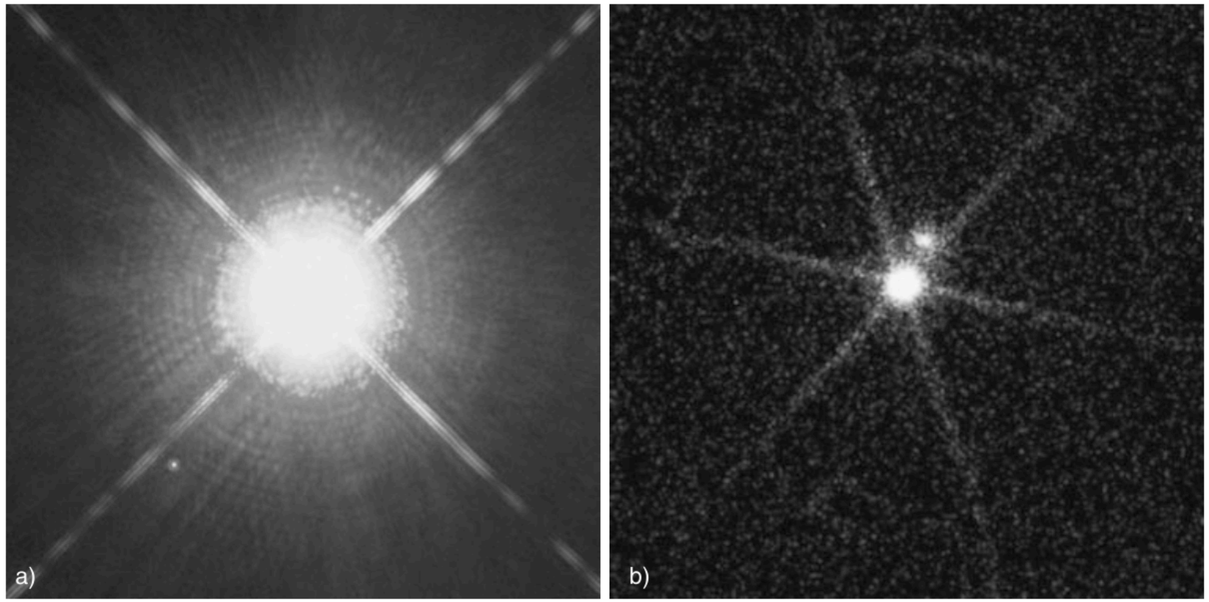
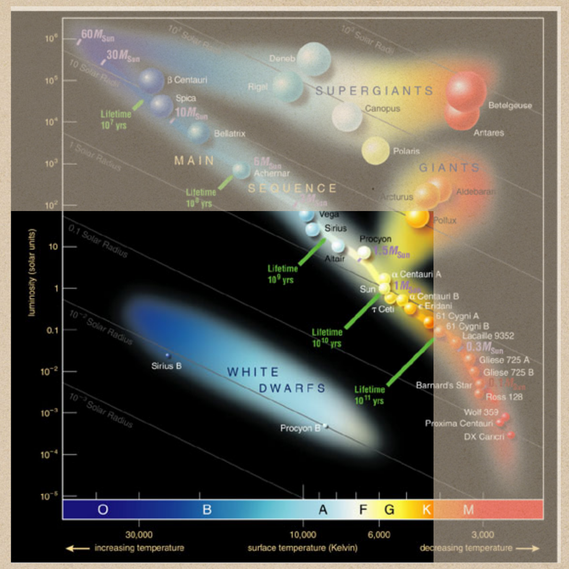


Fig. 15.1 Two views of the best-known white dwarf Sirius B, the small companion to Sirius. *On the left*, a picture in visible light by the Hubble Space Telescope. Sirius B is the tiny white dot on lower left from the overexposed image of Sirius. *On the right*, an X-ray picture of the pair taken by the Chandra X-ray observatory. Sirius B is now the brighter source, and Sirius is weaker, because its surface is much cooler than the surface of the white dwarf. (Photos NASA / HST and Chandra)

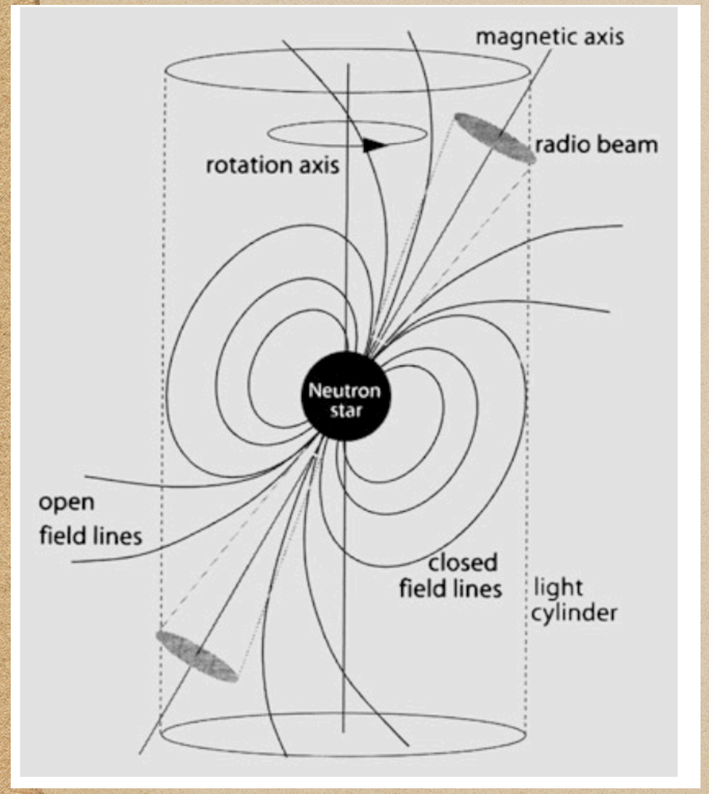


- Stars in which electrons are degenerated
- Chandrasekar mass limit: 1.4 $M_\odot$
- Even if the pressure increases, the temperature remains constant; thermonuclear reactions do not ignite

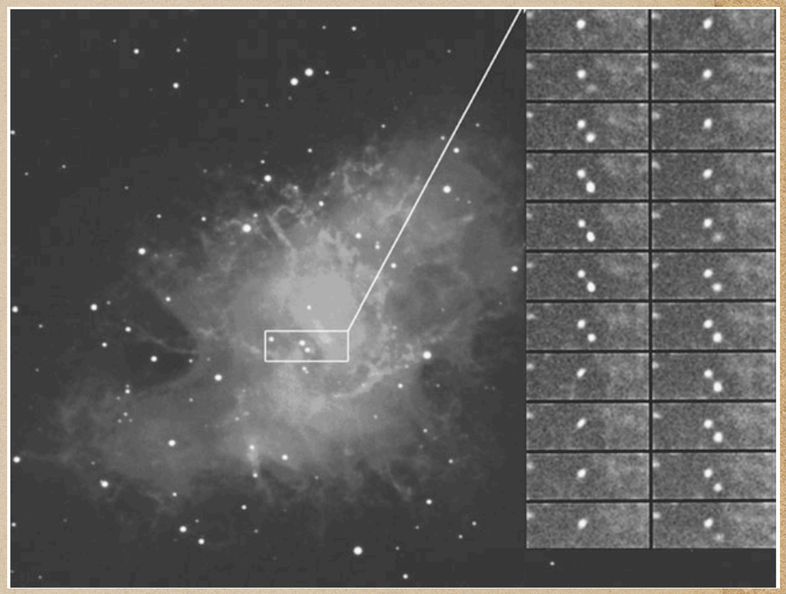
Above 1.4$M_\odot$ ==> gravity overwhelms the pressure and
the star will rapidly contract toward higher densities ==> collapse and birth of a neutron star

Compact stars: neutron stars

- A stellar remnant with a mass greater than that of Chandrasekar: 1.4 $M_\odot$
- The density is so great that protons and electrons merge into neutrons
- Even denser than white dwarfs (radius ~10km, atmosphere of a few cm)
- They typically rotate very fast



- rotating neutron star is surrounded by a strong magnetic field that drags the electrons from the surface and accelerates them at relativistic speeds from the magnetic poles.
- When electrons are accelerated along the magnetic field lines, they radiate
- Because the magnetic axis is misaligned with the axis of rotation, neutron stars move in the sky as in a lighthouse.
- If the Earth is aligned with the beam → *pulsar*



Compact stars: black holes

- A stellar remnant with an even higher mass can become a black hole
- Object so dense that not even the light has a speed greater than that of escape

$$v_e = \sqrt{\frac{2GM}{R}}.$$

Escape velocity from a body of mass M and radius R

If $R<R_s$ then $v_e>c$

$$R_S = 2GM/c^2.$$

R_s Schwarzschild radius

- For the Sun R_s~3km
- But the Sun can't become a black hole; R_s minimal black hole ~10 km
- Black hole properties → general relativity theory

