Radiative processes & magnitudes

Topic 3

Material from
Chap. 4&5 "Fundamental Astronomy" edition, by H. Karttunen, H. Oja, M. Poutanen, K. J. Donner

Electromagnetic radiation

Until a few years ago, most of the information used by astronomy traveled on electromagnetic waves:

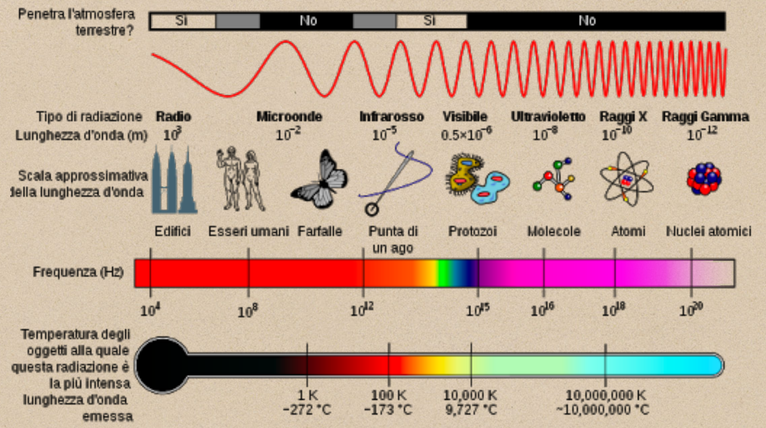


















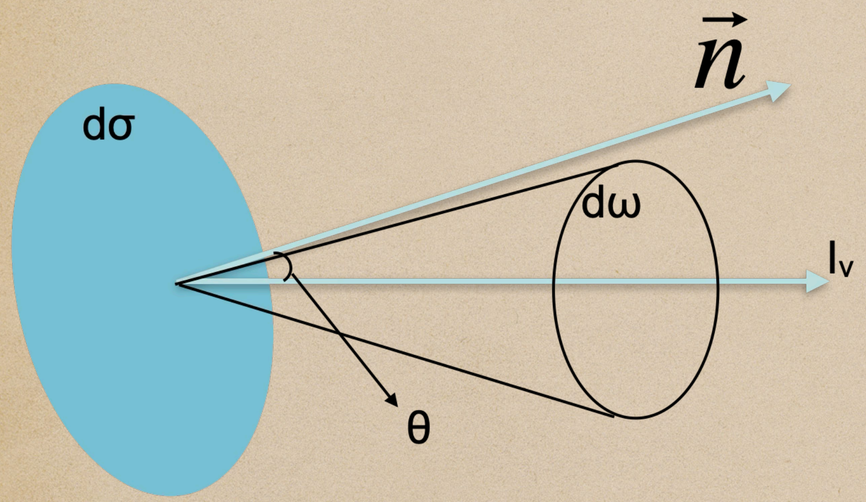


(In recent years, multi-messenger astronomy has developed: gravitational waves and particles are also used)

Radiation field parameters

We neglect the possibility of a refractive medium; electromagnetic radiation travels in a vacuum, therefore:

$$\lambda = \frac{c}{\nu} \quad , \quad \mathrm{d}\lambda = -\frac{c}{\nu^2}\mathrm{d}\nu = -\frac{\lambda^2}{c}\mathrm{d}\nu$$



The specific energy emitted by a source in the unit of time in the frequency range dv, by an infinitesimal surface dσ within the solid angle dω in a direction that forms an angle θ with the normal *n* is given by

$$dE_\nu = I_\nu \cos\theta d\sigma d\nu dt d\omega$$

$\cos\theta d\sigma$= projection of $d\sigma$ as seen from the direction θ

Radiation field parameters

We define the ***specific intensity,*** that is measured in ***erg cm^{-2} s^{-1}ster^{-1} Hz^{-1}***

$$I_\nu = \frac{dE_\nu}{\cos\theta d\sigma d\nu dt d\omega}$$

It defines the ***surface brightness*** of the object

The ***total intensity*** is defined from the specific one by integrating on all frequencies

$$I = \int_0^\infty I_\nu \, d\nu$$

Intensity is an unmeasurable amount that is preserved, if there is no absorption or emission of photons between the source and the observer

$$\frac{dI_\nu}{ds} = 0$$

The *surface brightness* of the object does not depend on the distance

Radiation field parameters

The ***specific flux density*** out of a surface dσ is obtained by integrating the specific intensity on all directions:

$$F_\nu = \int_\Omega I_\nu(\varphi,\theta)\cos\theta d\omega = 2\pi \int_0^{\pi/2} I_\nu(\varphi,\theta)\cos\theta\sin\theta d\theta$$

$$d\omega = \sin\theta\, d\theta\, d\phi$$

Solid angle in spherical coordinates

And it is measured in ***erg cm^{-2} s^{-1} Hz^{-1} (o Jansky)*** [1Jy=10^{-23} ***erg cm^{-2} s^{-1} Hz^{-1}***]

The ***total flux density*** is obtained by integrating the specific one on all frequencies:

$$F = \int_0^\infty F_\nu\, d\nu$$

It is measured in ***erg cm^{-2} s^{-1}***

The flux density is a measurable amount

Radiation field parameters

The ***specific luminosity*** is obtained by integrating the specific flux density over the entire emitting surface:

$$L_\nu = \int_S F_\nu \, dS = 4\pi R^2 F_\nu$$

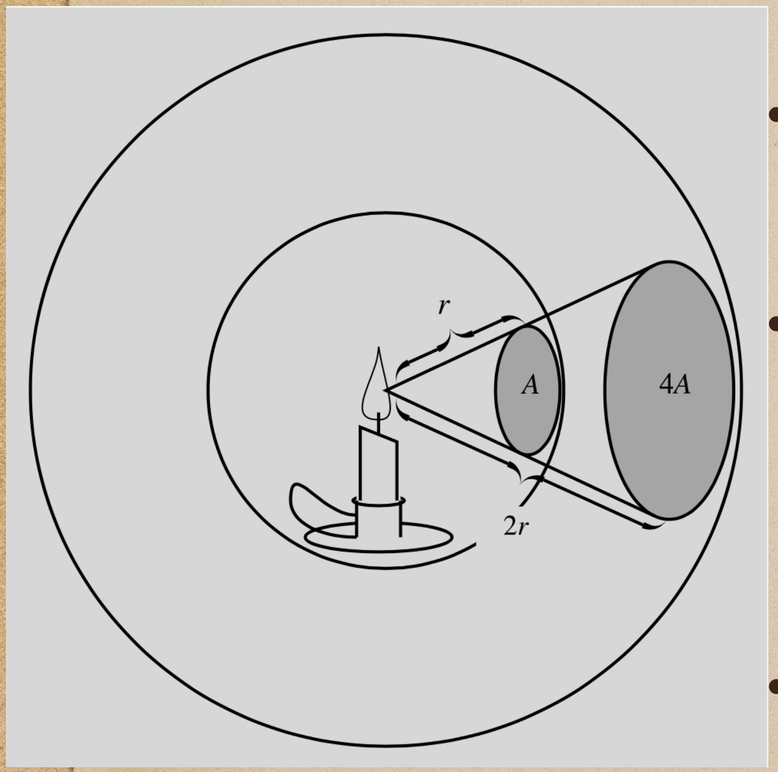
(Assuming it's a sphere); it is measured in ***erg s^{-1} Hz^{-1}***

The ***total luminosity*** is obtained by integrating the specific one on all frequencies:

$$L = \int_0^\infty L_\nu \, d\nu = 4\pi R^2 F$$

It is measured in ***erg s^{-1}***;
It's also called power

Reverse distance square law



Suppose we have a star that radiates in an isotropic way (*Iv* does not depend on the direction)

- The radiation radiated at a distance *r* will be distributed over a sphere of radius *r* and surface *4πr2*
- If the flux passing through this surface is *F*, we will have that the total luminosity is:

$$L = 4\pi r^2 F$$

- If we are outside the source, for energy conservation, L=cost (does not depend on the distance)

$$\Rightarrow F = \frac{L}{4\pi r^2}$$

Radiation field parameters

If there is absorption/emission of photons along the line of sight:

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + j_\nu$$

Radiative transport equation

Absorption

Emission

Absorption coefficient k_ν and emission coefficient j_ν

The absorption and emission coefficients take into account both the actual absorption and emission processes and the scattering of radiation. In fact, scattering can be considered as an absorption that is immediately followed by the emission of a photon.

Radiation field parameters

The optical depth along the line of sight is defined as

$$\tau_\nu(s) = \int_{s_0}^{s} ds' \kappa_\nu(s')$$

Combining this with the equation of radiative transport

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} = -I_\nu + S_\nu$$

“Source function”

and by integrating:

$$I_\nu(\tau_\nu) = I_\nu(0)\exp(-\tau_\nu) + \int_0^{\tau_\nu} d\tau'_\nu \exp(\tau'_\nu - \tau_\nu) S_\nu(\tau'_\nu)$$

Radiation field parameters

$$I_\nu(\tau_\nu) = I_\nu(0)\exp(-\tau_\nu) + \int_0^{\tau_\nu} d\tau'_\nu \exp(\tau'_\nu - \tau_\nu) S_\nu(\tau'_\nu)$$

This equation has a simple interpretation. If $I_\nu(0)$ is the incident intensity, it will be reduced by absorption to a value $I_\nu(0)\exp(-\tau_\nu)$ after an optical depth of τ_ν …

…. while the second term takes into account the radiation added along the line of sight

Black body

For a body in thermal equilibrium S_ν only depends on the temperature:

$$S_\nu = B_\nu(T)$$

Since S_ν doesn't depend on the optical path:

$$I_\nu(\tau_\nu) = I_\nu(0)\exp(-\tau_\nu) + B_\nu(T)\int_0^{\tau_\nu} d\tau'_\nu \exp(\tau'_\nu - \tau_\nu) =$$

$$= I_\nu(0)\exp(-\tau_\nu) + B_\nu(T)\left[1 - \exp(-\tau_\nu)\right]$$

If the optical depth is large enough

$$\exp(-\tau_\nu) \to 0 \quad \Rightarrow \quad \boxed{I_\nu(\tau_\nu) = B_\nu(T)}$$

Black Body

If we imagine a cavity maintained at constant temperature T, from opaque walls, the radiation will be continuously absorbed and re-emitted, therefore the optical path will be infinite

Black body

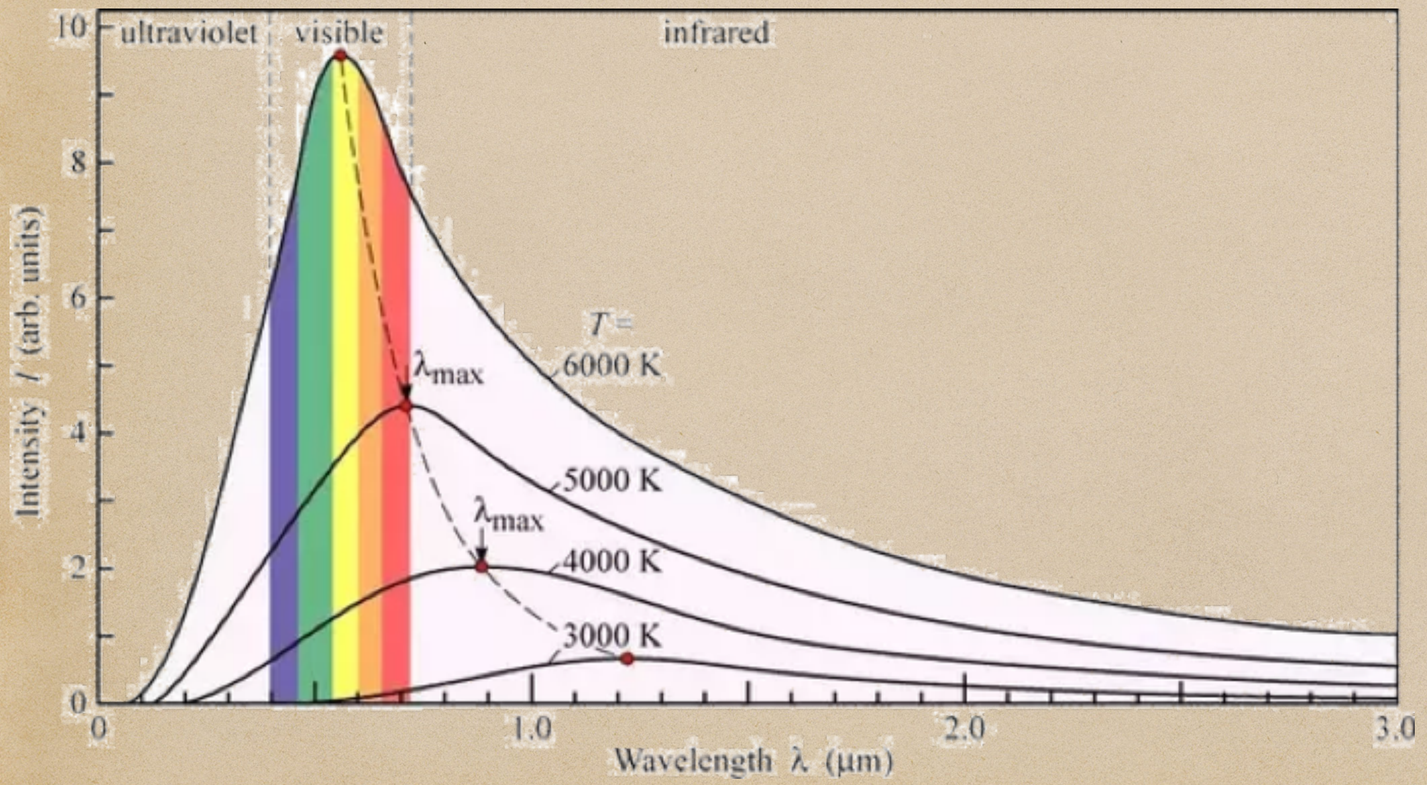
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/k\lambda T}-1} \quad , \quad B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

Planck's law, determined in 1900

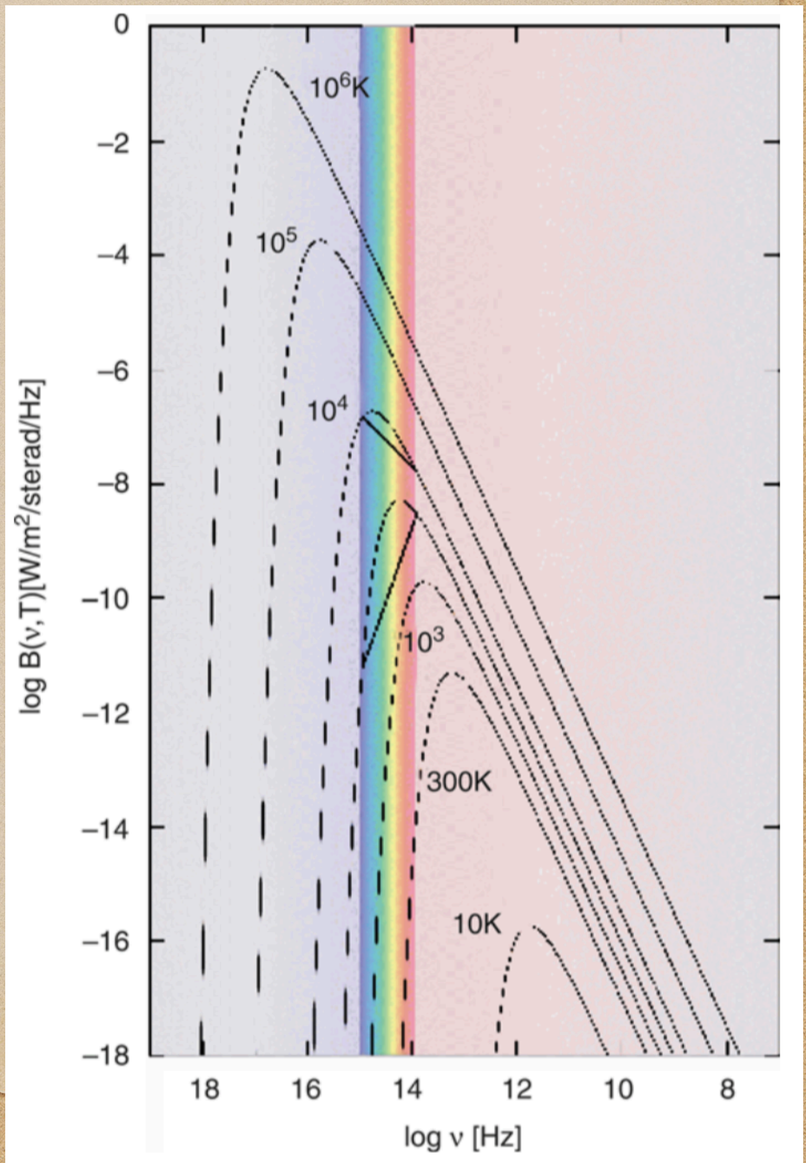
Where *B* is the spectral irradiance, *h* is the Planck's constant (h = 6.57×10^{-27}erg s^{-1}), and *k* is Boltzmann's constant (*k* = 1.38×10^{-16}erg/K). B_λ (or B_ν) Then expresses the radiance per unit surface per unit of time per unit of band in the unitary solid angle (i.e. per steradian). In the cgs system it is therefore in erg·cm^{-2}·s^{-1}·sr^{-1}·$Å^{-1}$, o in erg·cm^{-2}·s^{-1}·sr^{-1}·Hz^{-1}.

Black body

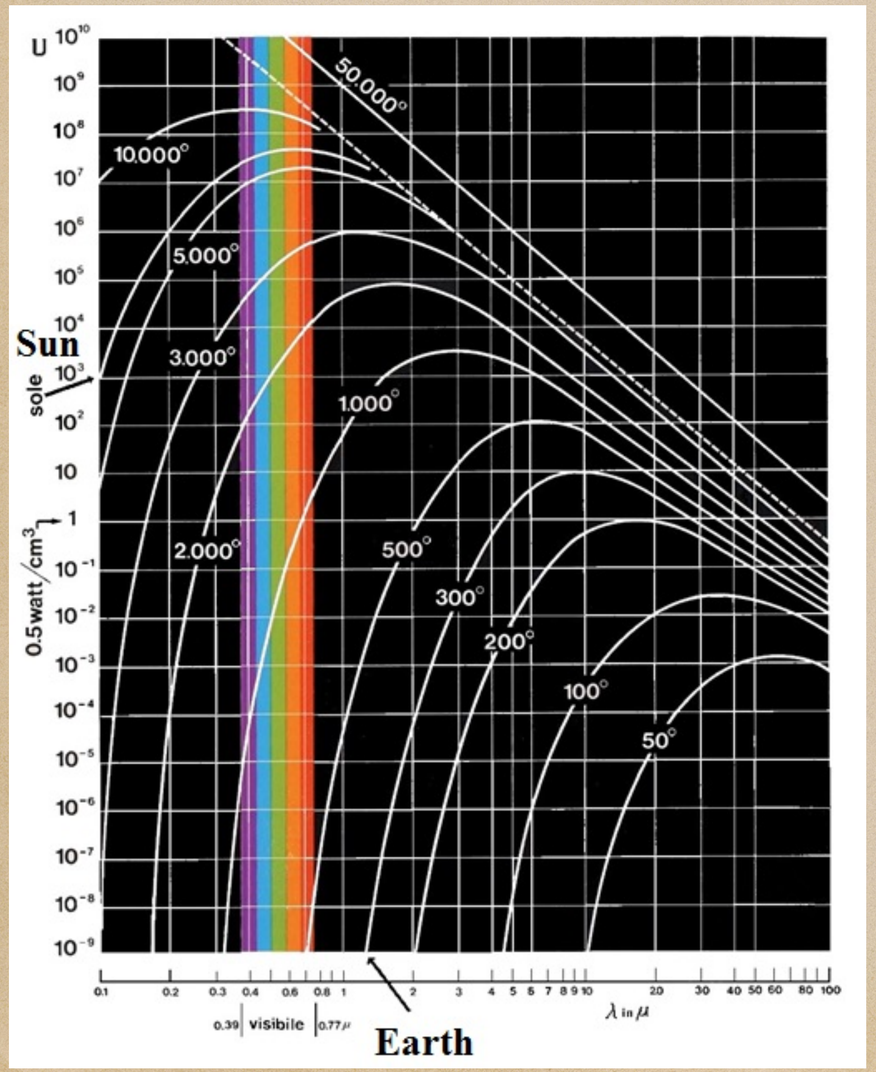
Planck's law vs λ



Planck's law vs ν

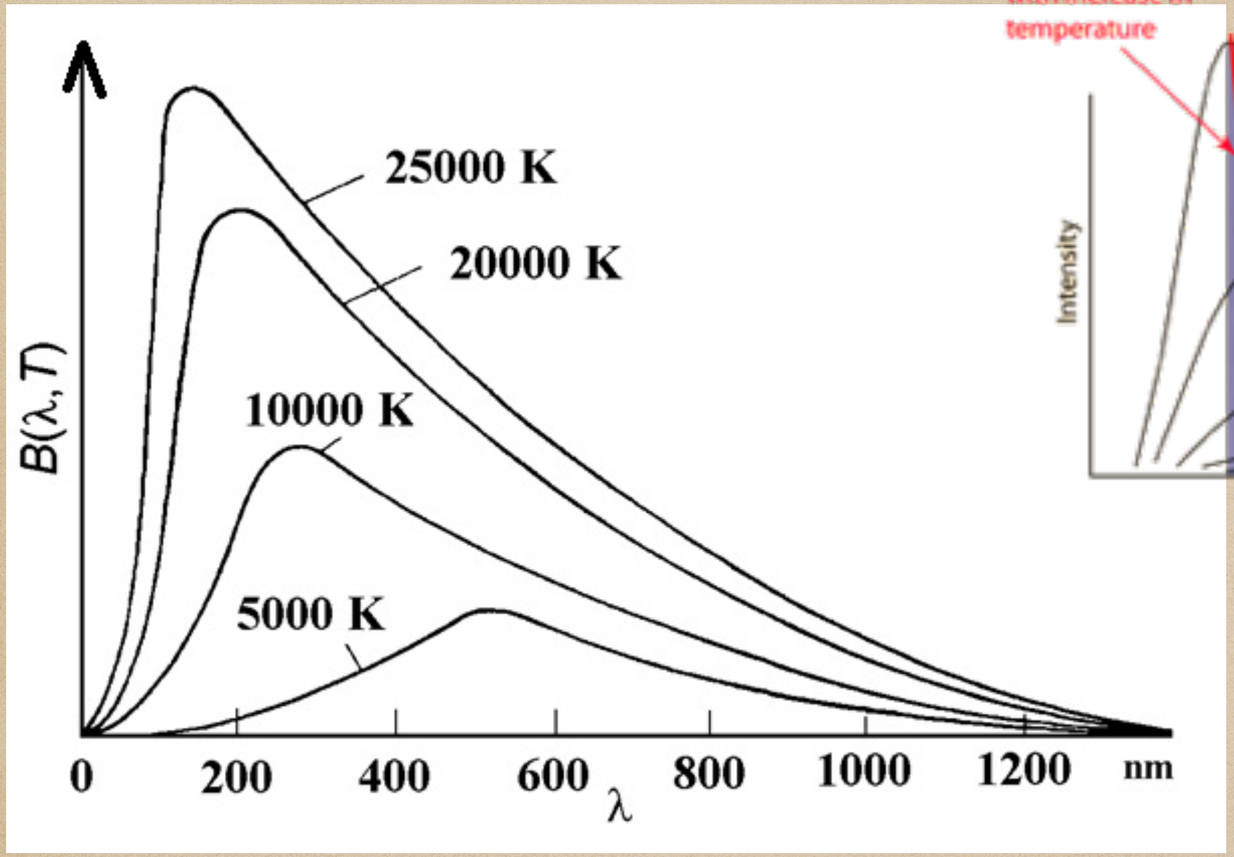


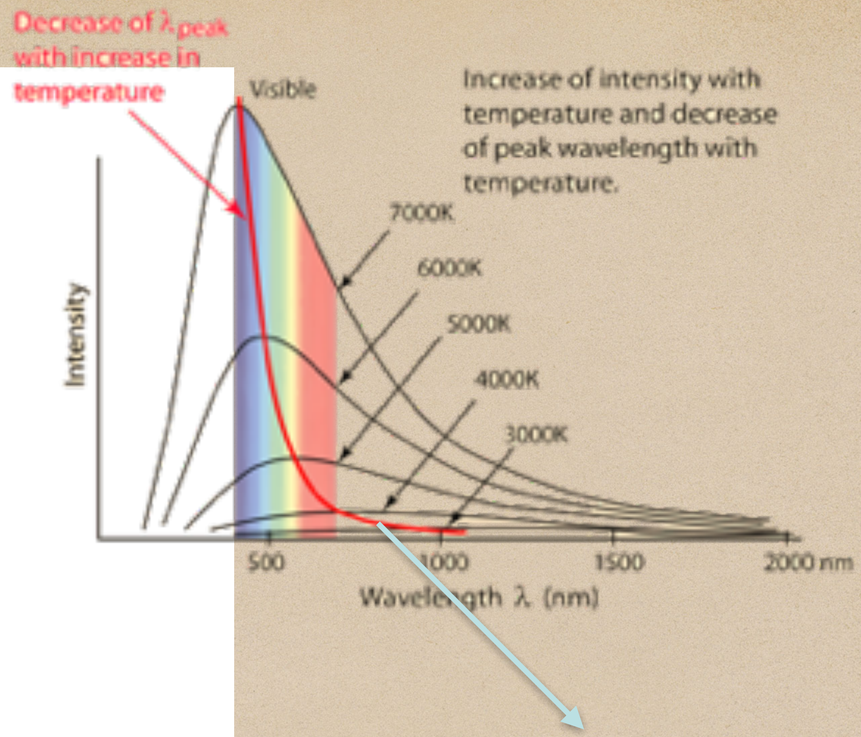
Black body



Planck's law on a bi-logarithmic scale

Black body





If T = 10^4 K then λ_{max} = 2900 A
(In the ultraviolet, invisible from the Earth due to the atmospheric cut)

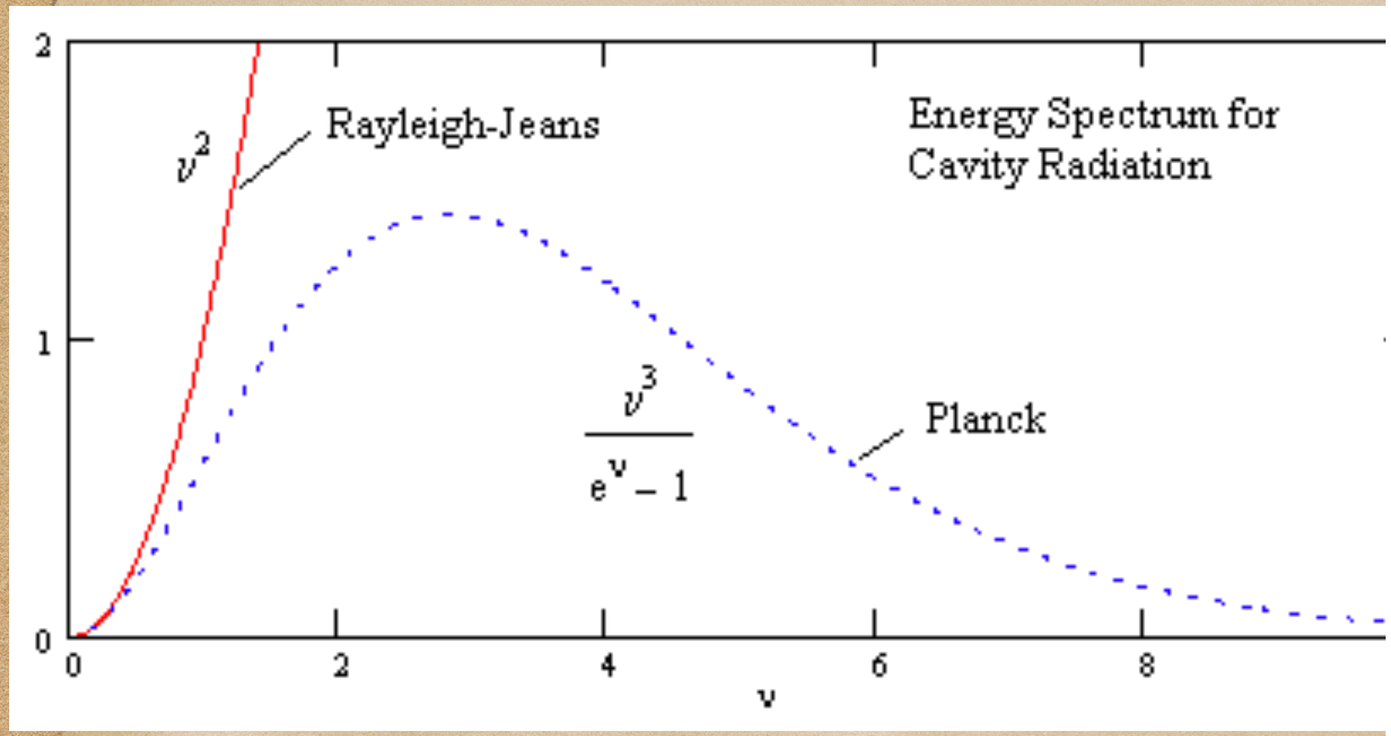
On a linear scale it is clear how the peak moves as a function of temperature T (Wien's law):

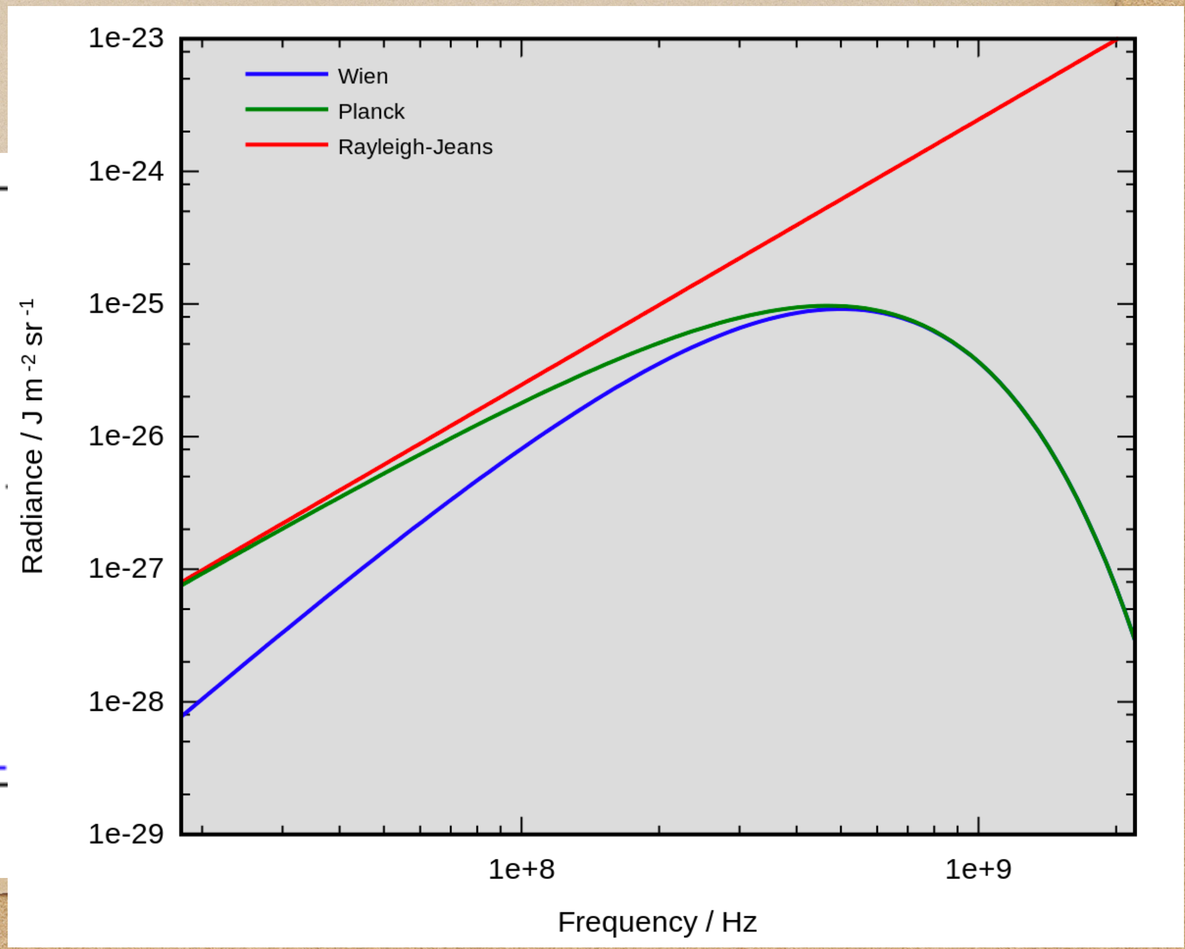
λ_{max} T=const

Black body: Rayleigh-Jeans regime

When $h\nu \ll kT$

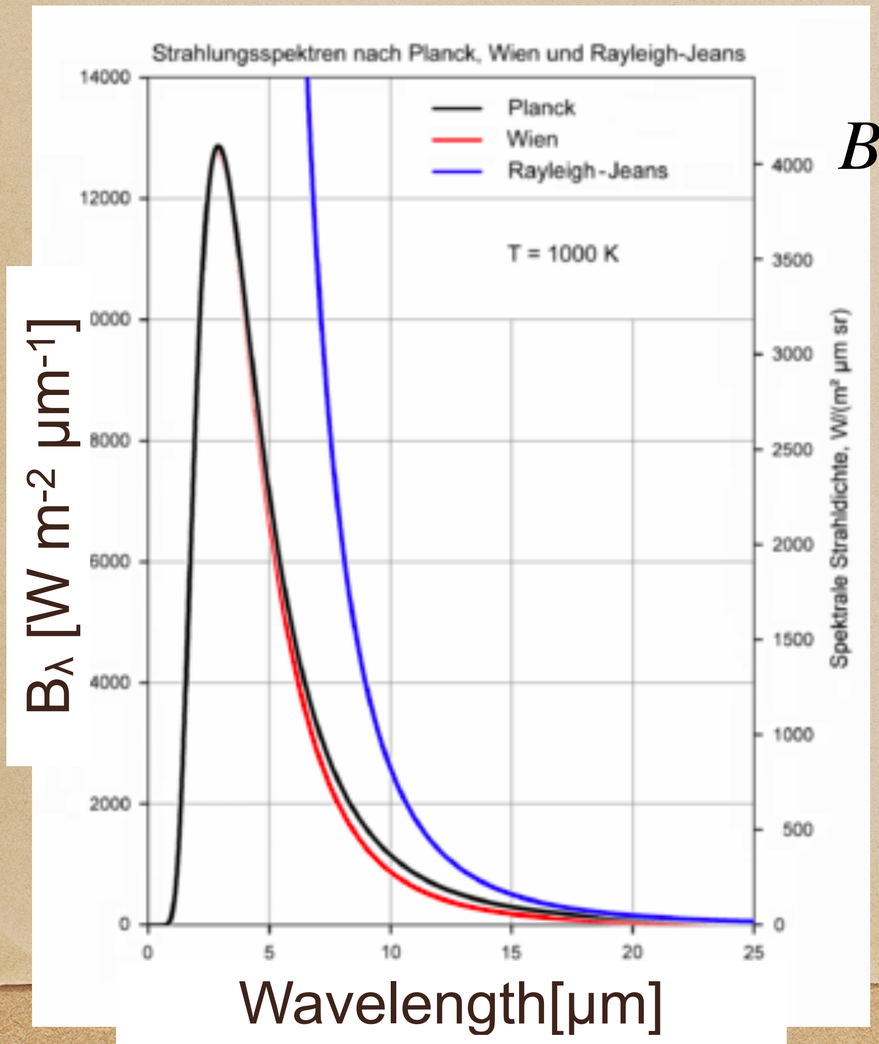
$$B_\nu(T) \approx B_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$



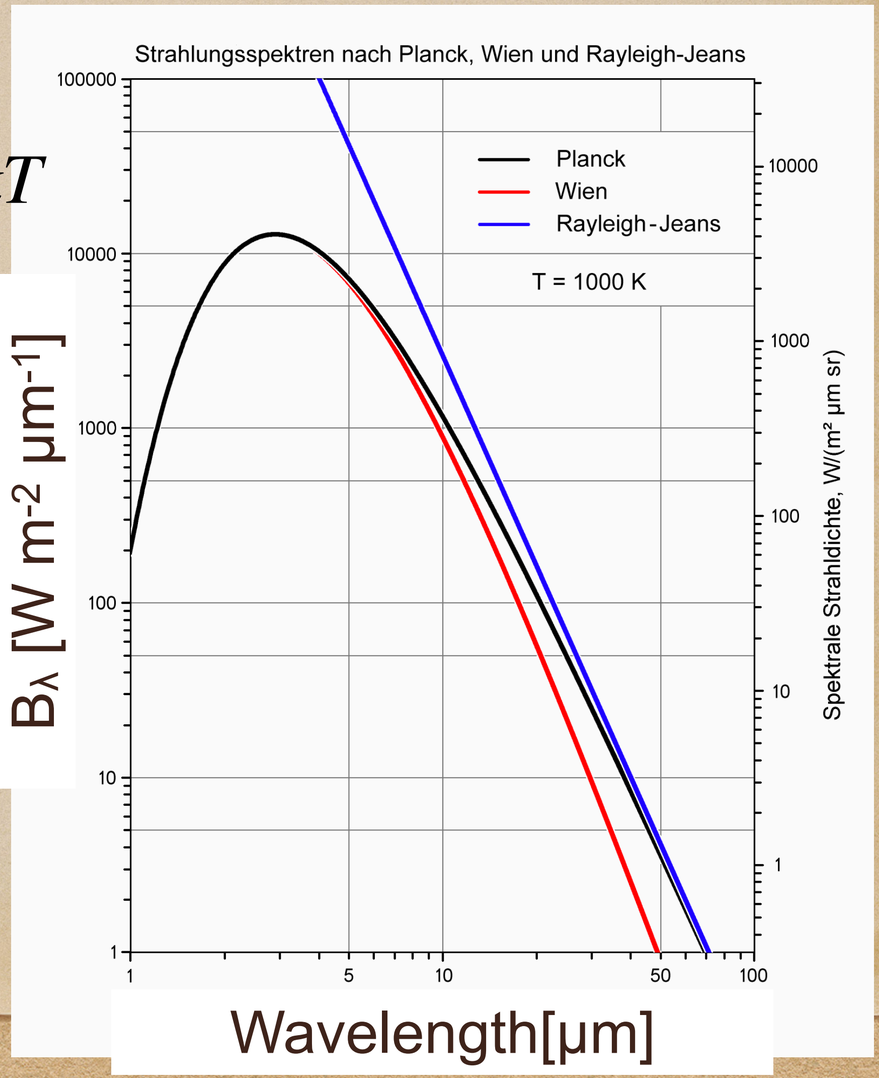


Black body: Rayleigh-Jeans regime

Rayleigh-Jeans vs λ



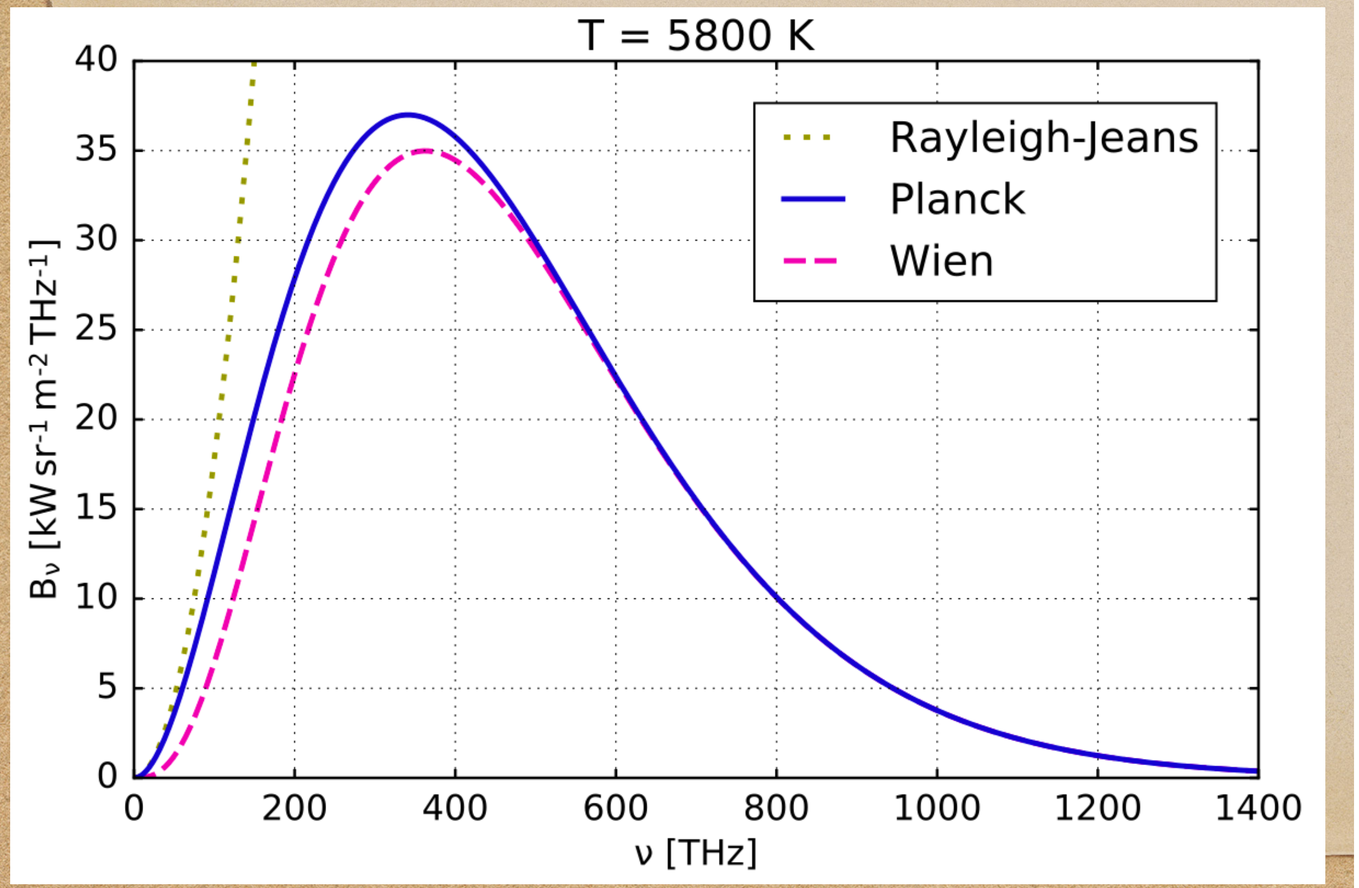
$$B_\lambda(T) \approx B_\lambda^{RJ}(T) = \frac{2c}{\lambda^4} kT$$

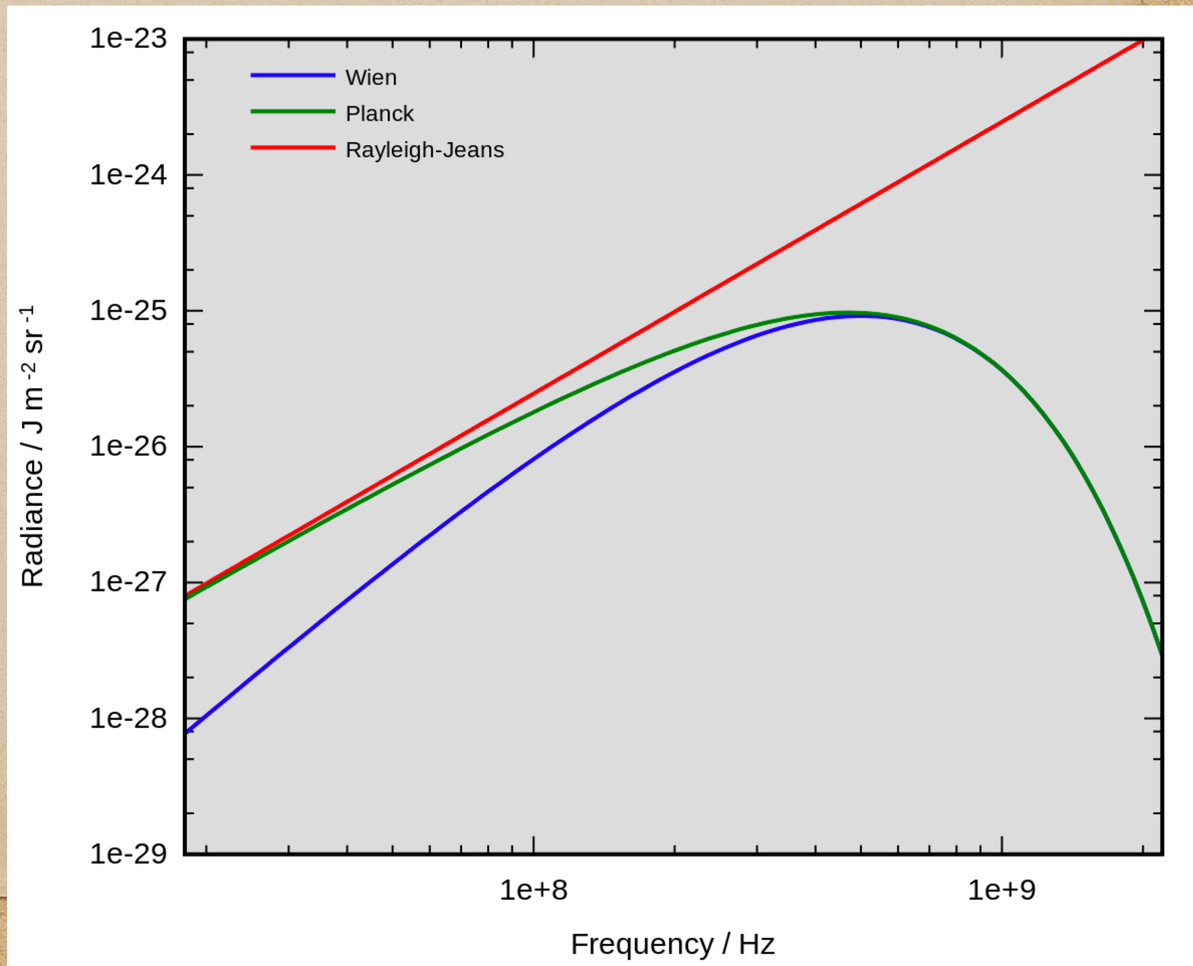


Black body: Wien regime

When $h\nu \gg kT$

$$B_\nu(T) \approx B_\nu^W(T) = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$

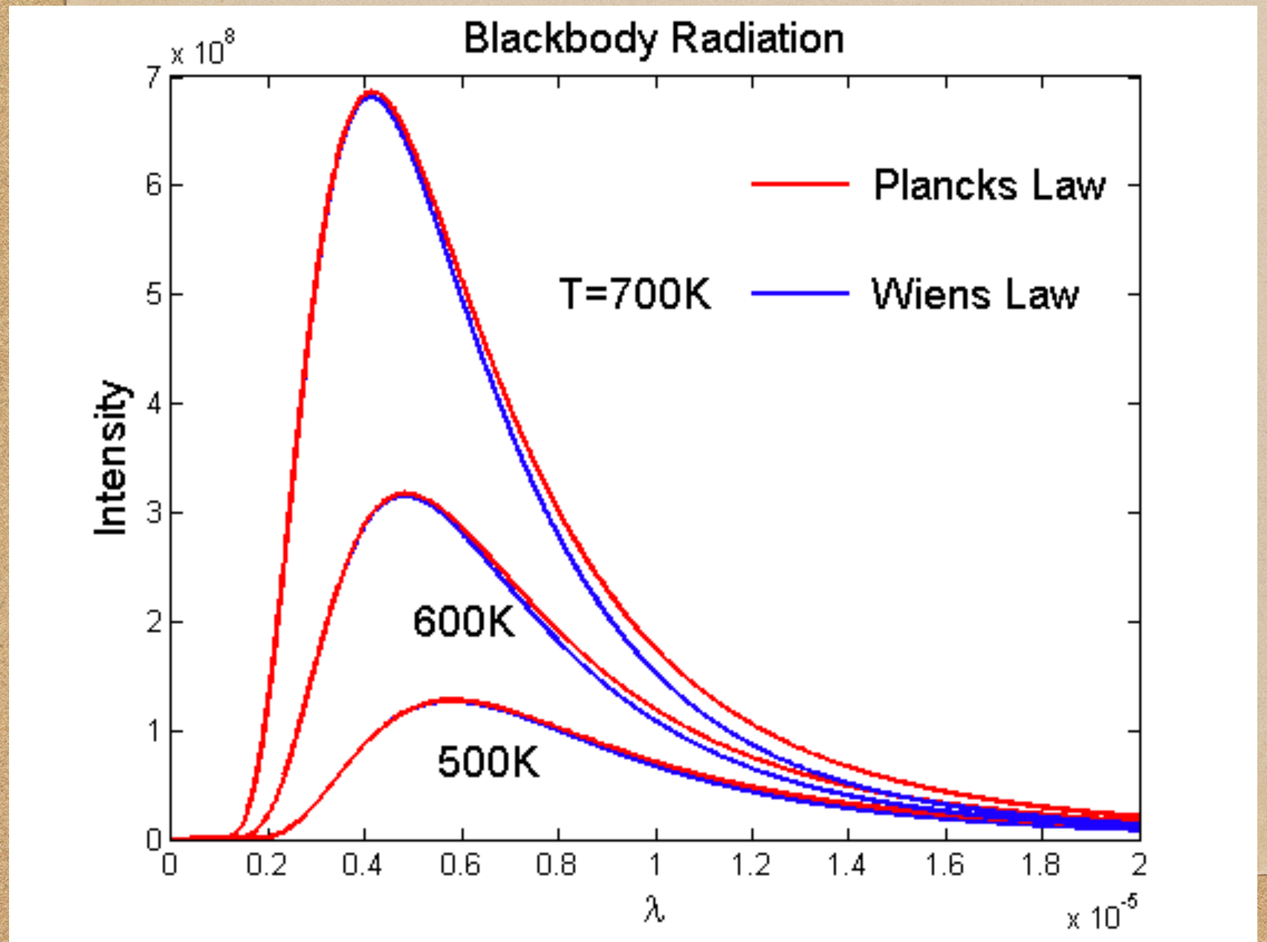


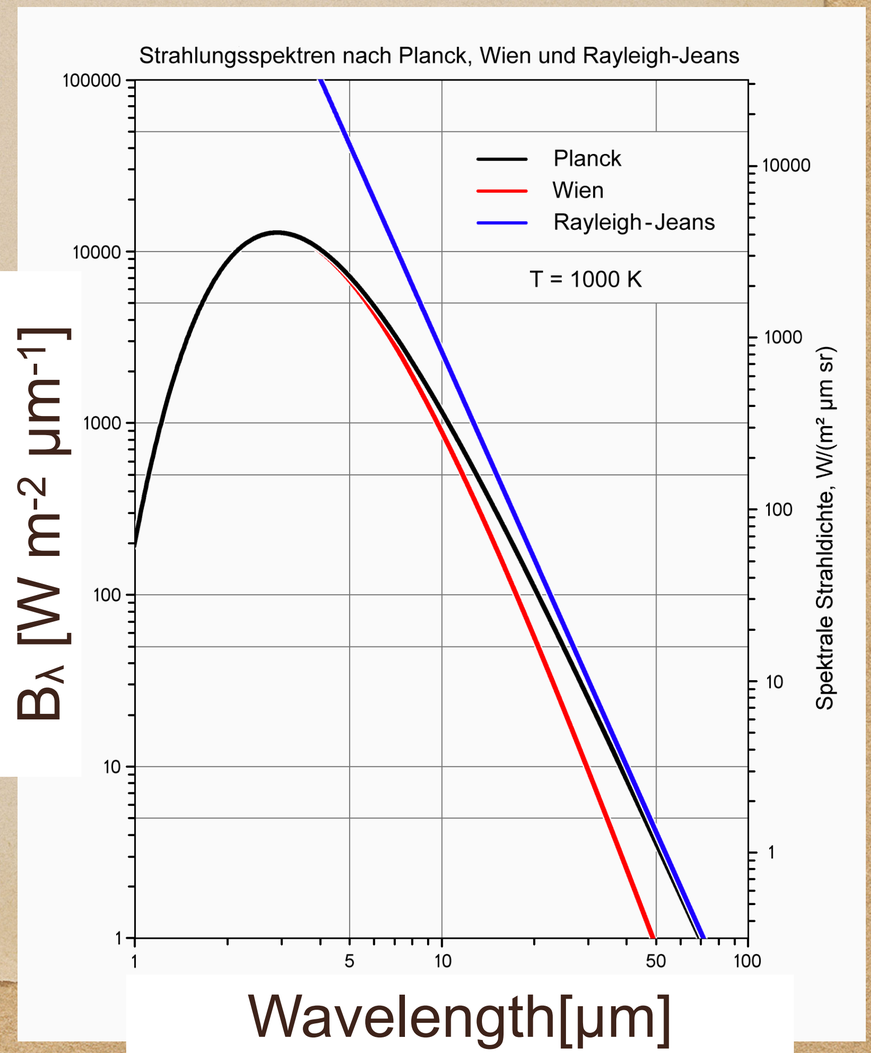


Black body: Wien regime

Wien regime vs wavelength

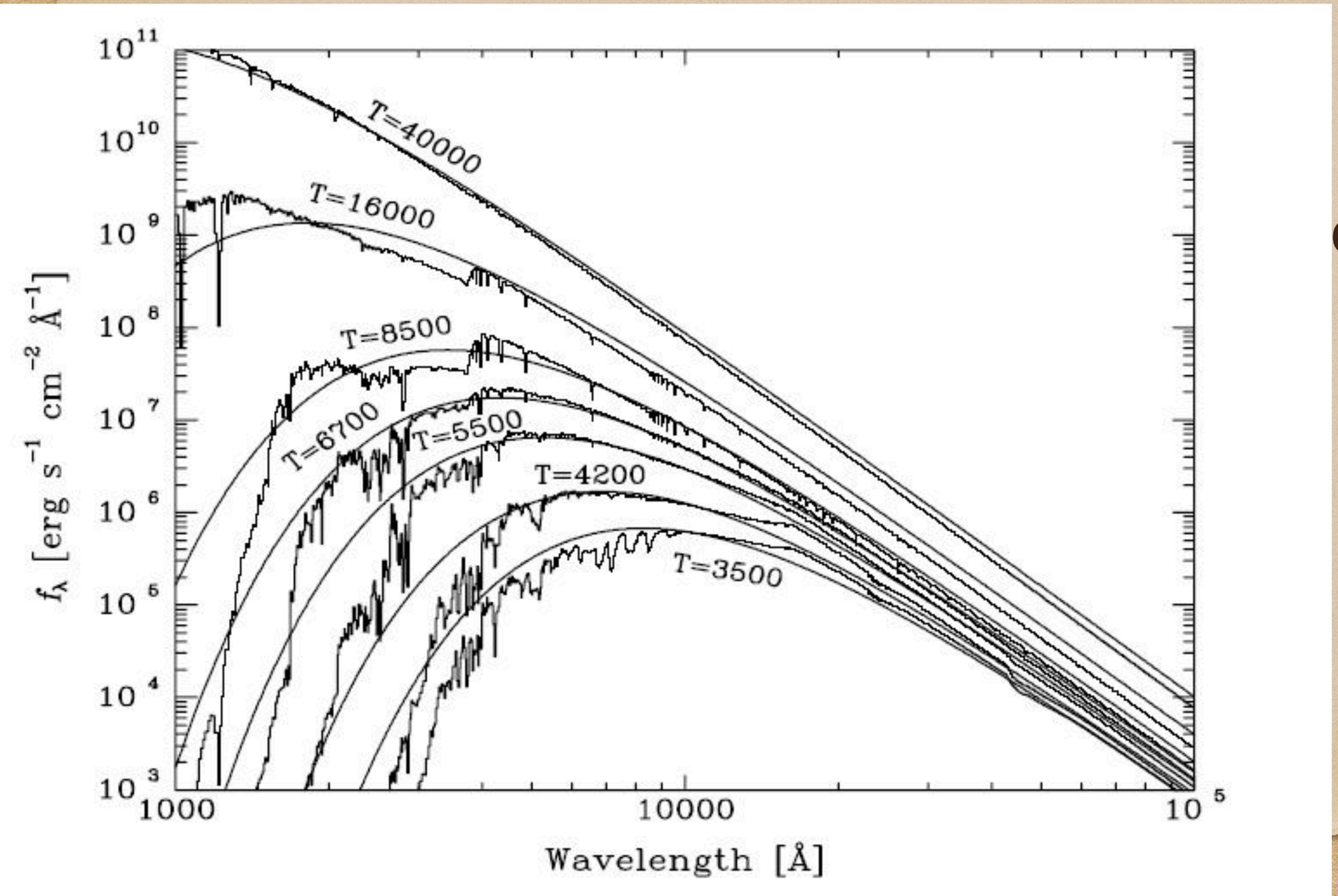
$$B_\lambda(T) \approx B_\lambda^W(T) = \frac{2hc^2}{\lambda^5} e^{-hc/\lambda kT}$$





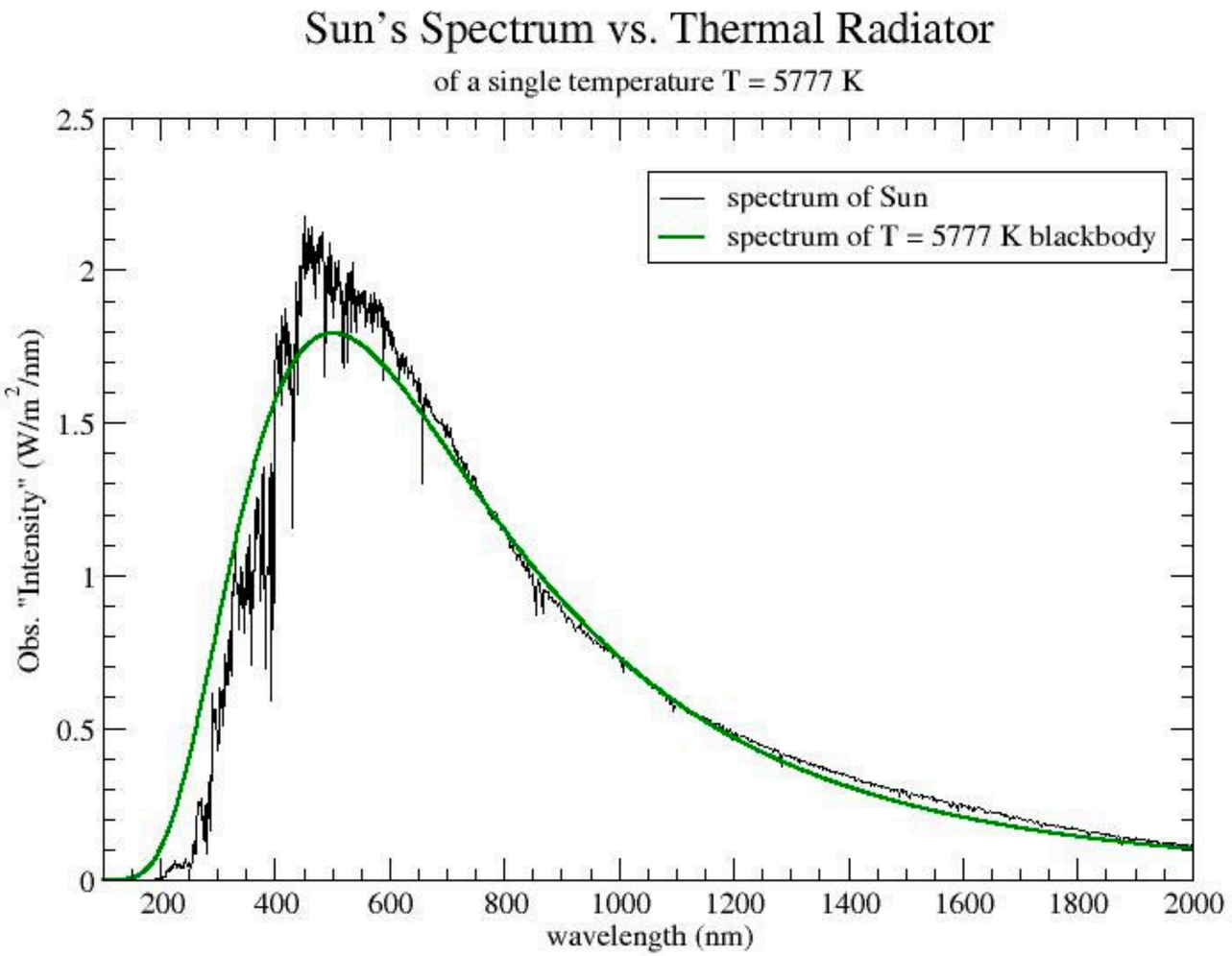
Stars as black bodies

In the first approximation, the stars are black bodies



Or rather, their spectral energy distribution is well represented by B_ν (o B_λ)

Stars as black bodies



In a solar spectrum at ***Low resolution*** the lines are practically absent, and the continuum is well represented by a **black body at 5800 K**.

There are significant differences, especially in UV, but here they are neglected.

Stefan-Boltzmann's law

The total energy emitted by a black body obviously depends only on the temperature T, and is obtained by integrating $B_\nu(T)$ over all the ν (or $B_\lambda(T)$ over all the λ):

$$B(T) = \int_0^\infty B_\nu(T)\,d\nu = A\,T^4$$

The flux F(T) that is emitted by a black body per unit of area (also called emittance) is given by

$$F = \int_0^\infty F_\nu\,d\nu = \pi \int_0^\infty B_\nu(T)\,d\nu = \pi B(T) = \sigma T^4$$

Where $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1} is the Stefan-Boltzmann constant

Stefan-Boltzmann's law

$$\sigma\ (= \pi A)$$

Stefan-Boltzmann's law

The total energy emitted by a black body obviously depends only on the temperature T, and is obtained by integrating $B_\nu(T)$ over all the ν (or $B_\lambda(T)$ over all the λ):

$$B(T) = \int_0^\infty B_\nu(T)\,d\nu = \frac{2h}{c^2}\int_0^\infty \frac{\nu^3\,d\nu}{e^{h\nu/(kT)}-1}.$$

We now change the integration variable to $x = h\nu/(kT)$, whence $d\nu = (kT/h)\,dx$:

$$B(T) = \frac{2h}{c^2}\frac{k^4}{h^4}T^4\int_0^\infty \frac{x^3\,dx}{e^x-1}.$$

The definite integral in this expression is just a real number, independent of the temperature. Thus we find that

$$B(T) = AT^4, \qquad (5.16)$$

where the constant A has the value

$$A = \frac{2k^4}{c^2h^3}\frac{\pi^4}{15}. \qquad (5.17)$$

$$B(T) = \int_0^\infty B_\nu(T)\,d\nu = A\,T^4$$

emitted by a black body per unit of
alled emittance) is given by

$$= \pi\int_0^\infty B_\nu(T)\,d\nu = \pi B(T) = \sigma T^4$$

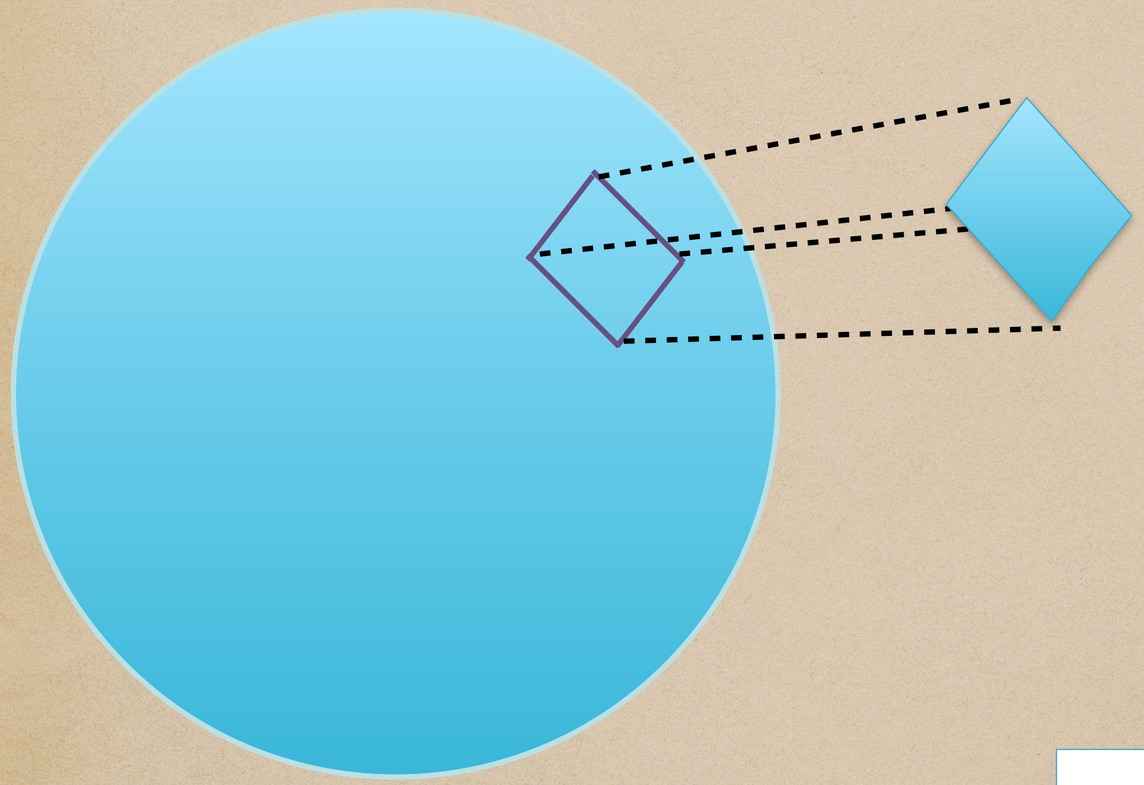
$^{-5}$ erg cm^{-2} K^{-4} s^{-1} is the Stefan-Boltzmann constant

efan-Boltzmann's law

$$\sigma\ (=\pi A)$$

Stefan-Boltzmann's law

Suppose a star radiates like a black body:



An infinitesimal area of the surface will radiate σT^4

Sphere area $A=4\pi R^2$

So the whole sphere will radiate:

$$L = 4\pi R^2 F = 4\pi R^2 \sigma T^4$$

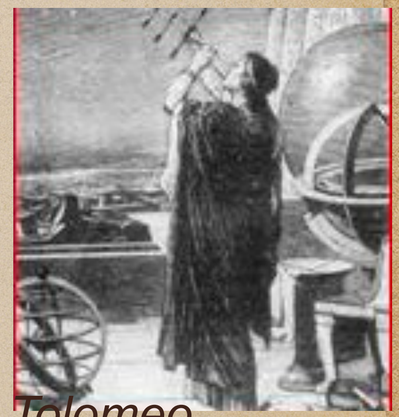
If I know the brightness and temperature of a star, I can determine its radius (or from radius and T I can determine L, …)

The astronomical magnitude

- The apparent brightness of celestial bodies can be measured with relative simplicity even with the naked eye. Already Ipparco, and then Ptolemy, had divided the stars visible to the naked eye into ***six classes of descending brightness (or more precisely of magnitudes)***, Indicated by the Greek letters α, β, γ, δ, ε, ζ, followed by the Latin name of the constellation in which they are, for example α Cyg or β Lyr.
- The ideal criterion is that by moving from one class to another, the eye perceives the same difference in visual stimulation...
- …But the eye has a logarithmic response: the same difference in stimulus between stars of brightness 1:10 or 10:100
- …And can estimate this difference well only on small angular distances, and for objects with similar color, so the classification is not consistent on the entire celestial sphere.
- Even today, the problem is not easy to solve, and you have to use a lot of caution to transfer a sequence from one area of the sky to another very far away from it.



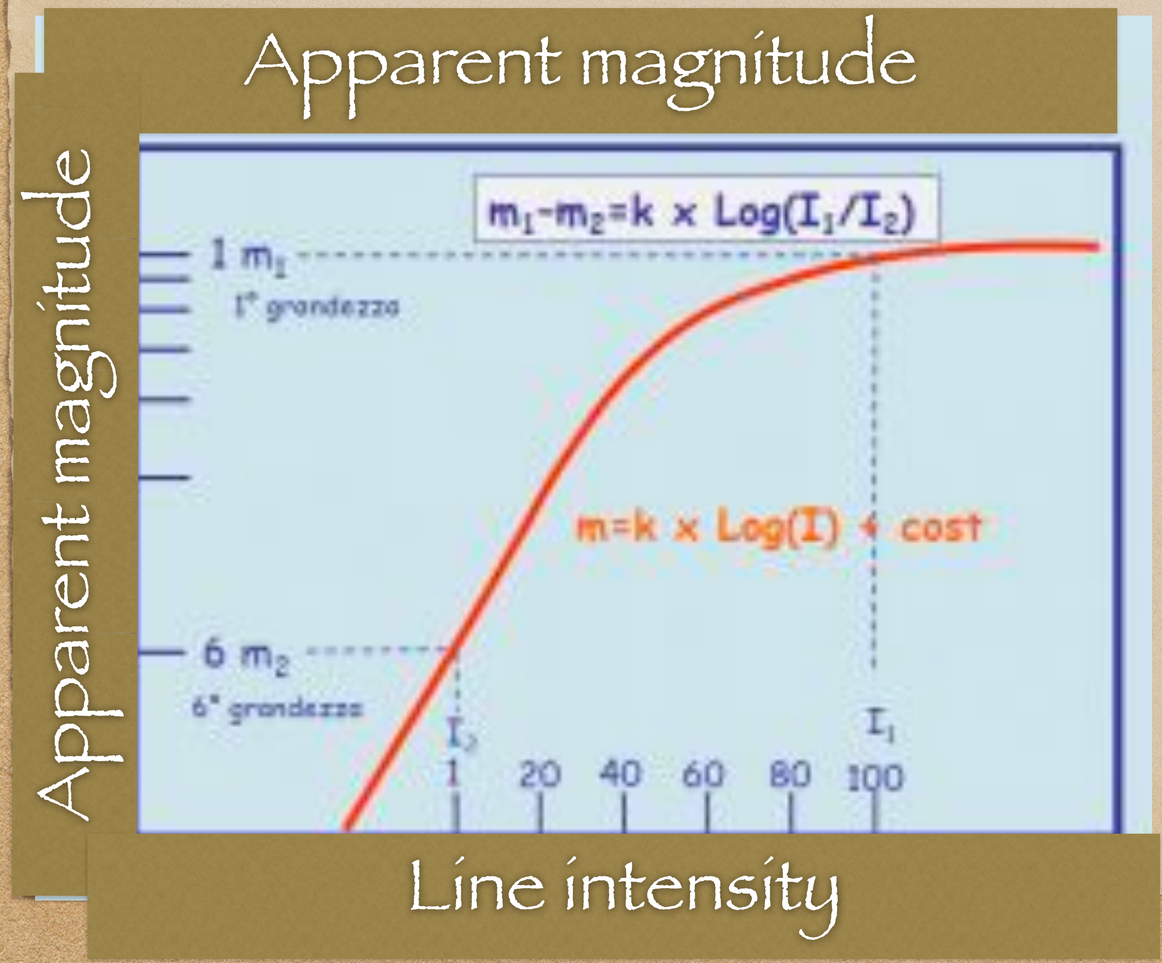
Ipparco



Tolomeo

The astronomical magnitude

Pogson, 1856: *Light intensity vs apparent magnitude*



Given then two stars whose luminous fluxes observed at a generic wavelength λ are, respectively, $S_1(\lambda)$ and $S_2(\lambda)$, the difference in apparent magnitude to that λ is defined as the quantity:

$$\Delta m(\lambda) = m_1(\lambda) - m_2(\lambda) = -2.5 \log_{10} \frac{S_1(\lambda)}{S_2(\lambda)}$$

Pogson's law

Apparent Magnitude

Apparent magnitude is the brightness of a celestial object as observed from Earth.



It is a logarithmic scale where bright objects have low and sometimes negative magnitudes, while dim objects have higher positive magnitudes.

A difference of 1 magnitude corresponds to a brightness ratio of ~2.512.

The astronomical magnitude

$$\Delta m(\lambda) = m_1(\lambda) - m_2(\lambda) = -2.5 \log_{10} \frac{S_1(\lambda)}{S_2(\lambda)}$$

Keep in mind the **negative sign**: The larger the magnitude, the smaller the intensity; a brighter star has a smaller magnitude

Otherwise:

$$\frac{S_1}{S_2} = 10^{-0.4(m_1 - m_2)}$$

$$m = m_0 - 2.5 \log_{10} S$$

$$\log_{10} S = \frac{\log_e S}{\log_e 10} = \frac{\log_e S}{2.302...}$$

$$\Rightarrow dm = -1.08 \frac{ds}{S}$$

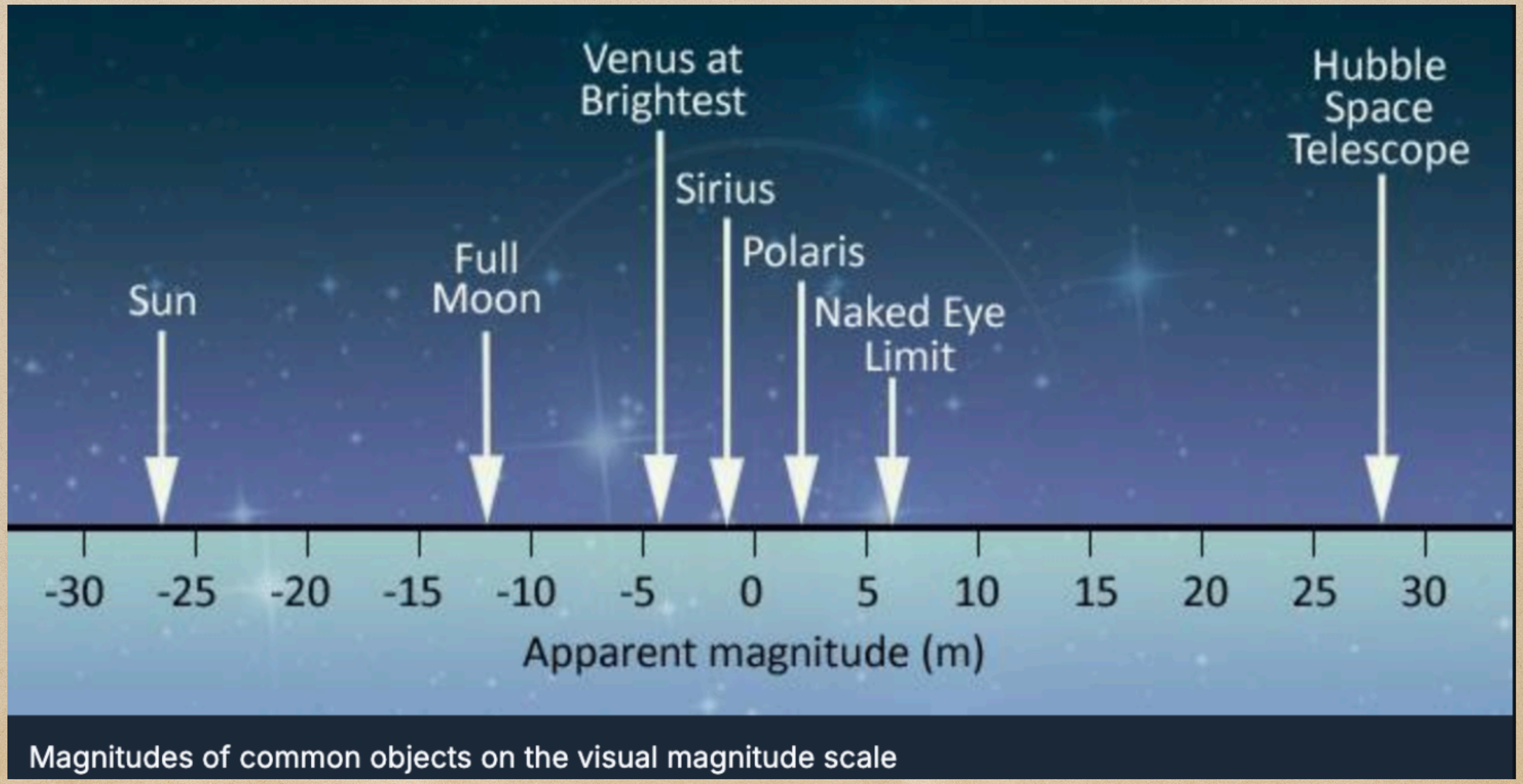
The astronomical magnitude

With reference to a standard object (Vega or Polar Star):

$$m(\lambda) = m_0(\lambda) - 2.5 \log_{10} S(\lambda)$$

With m_0 appropriate constant that depends on λ and that ***takes care of the a-dimensionality of the logarithmic argument.***

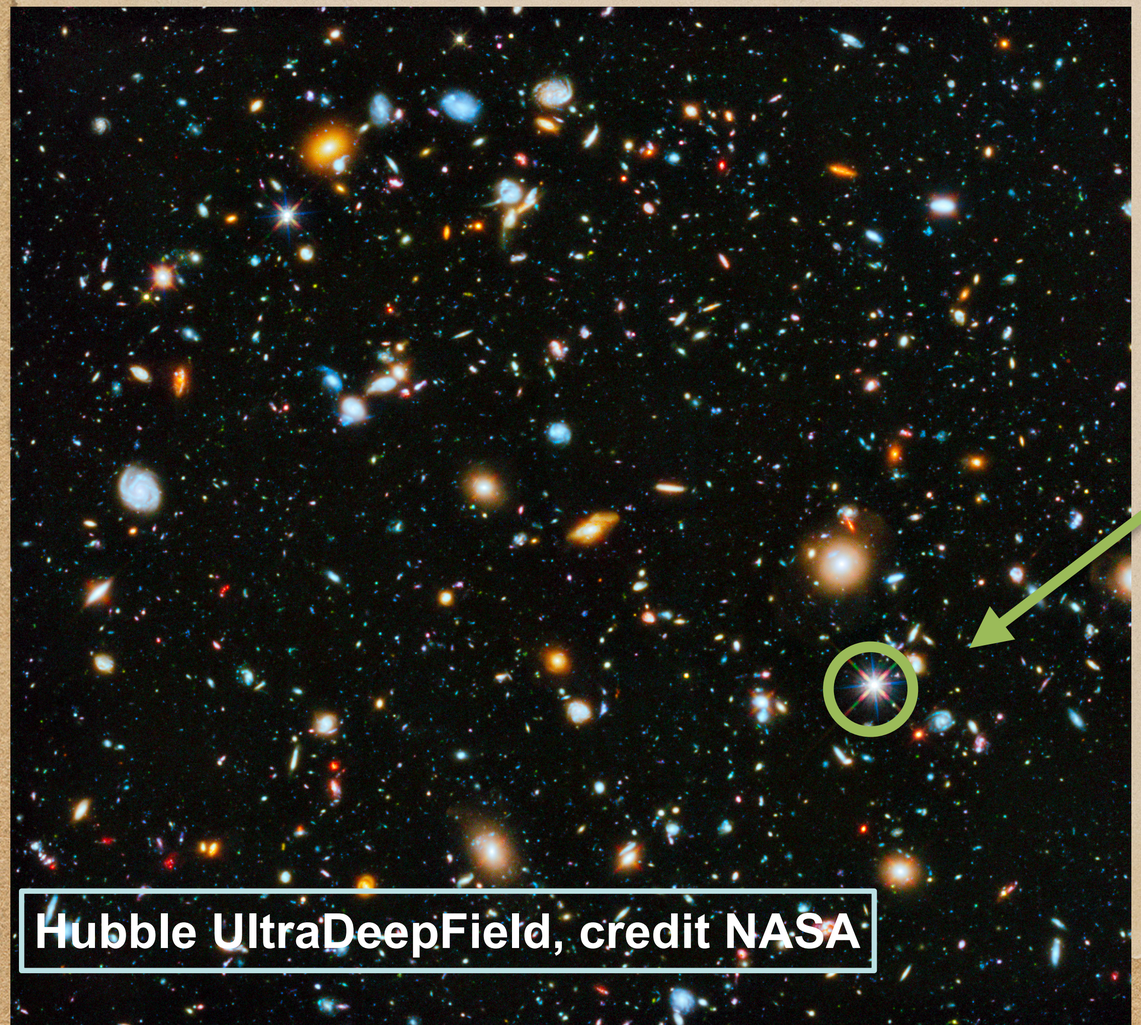
In the visible, the polar star has m = + 2.1, the brightest stars are Sirius, Canopus, Vega, whose magnitudes are between -1.5 and 0; The weakest stars visible to the naked eye are around + 6. Venus can reach -4.5. The Sun has m = -26.8, the full Moon m = -12.5. However, the Sun, the Moon, Venus are extended bodies, not point sources. We could therefore define a mag for the unit area (or 'surface brightness', expressed e.g. In mag for square arcseconds, or for steradiants), dividing the flux by the area underlying the object.



Magnitudes of common objects on the visual magnitude scale

The astronomical magnitude

In fact, in astronomy absolute fluxes are almost never measured, but only flux ratios (or differences in magnitude)



Hubble UltraDeepField, credit NASA

Standard star, whose magnitude I know (at that λ)

The magnitudes of the celestial bodies in the image are calculated based on the flux ratios with respect to the standard star



Same job in the JWST era!

SMACS 0723

The astronomical magnitude

Let's now give a more precise operational definition of apparent magnitude, taking into account the ***spectral energy distribution F(λ)*** of the star (i.e. the flux per unit receiving area), the ***reflectivity of the R optics***, the ***quantum efficiency of the CCD QE, the filter transmission curve*** and any S filter:

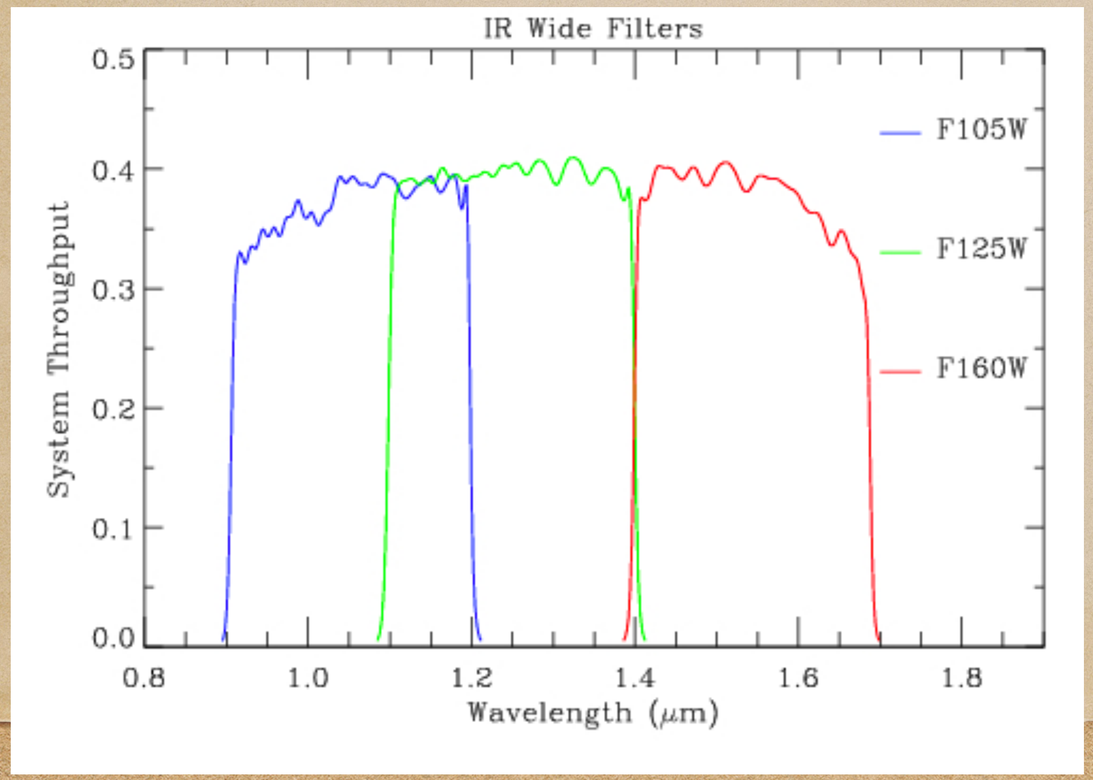
$$m(\lambda_c) = m_0(\lambda_c) - 2.5\log_{10}\left(\frac{\int_{\lambda_1}^{\lambda_2} F(\lambda)\times R^n(\lambda)\times QE(\lambda)\times S(\lambda)d\lambda}{\int_{\lambda_1}^{\lambda_2} R^n(\lambda)\times QE(\lambda)\times S(\lambda)d\lambda}\right)$$

Where A is the telescope area, n the number of reflective mirrors and λc an appropriate wavelength within the range (λ_1, λ_2), generally approximately coincides with the central wavelength, but whose exact value depends on the spectral characteristics of the star (or even its temperature).

The astronomical magnitude

$$m(\lambda_c) = m_0(\lambda_c) - 2.5\log_{10}\left(\frac{\int_{\lambda_1}^{\lambda_2} F(\lambda)\times T(\lambda)d\lambda}{\int_{\lambda_1}^{\lambda_2} T(\lambda)d\lambda}\right)$$

To simplify, the filter responses usually incorporates: S(λ), the reflexivity of optics $R^n(\lambda)$ and the quantum efficiency of the detector QE(λ) into the ***photometric system response T(λ) or system throughput***

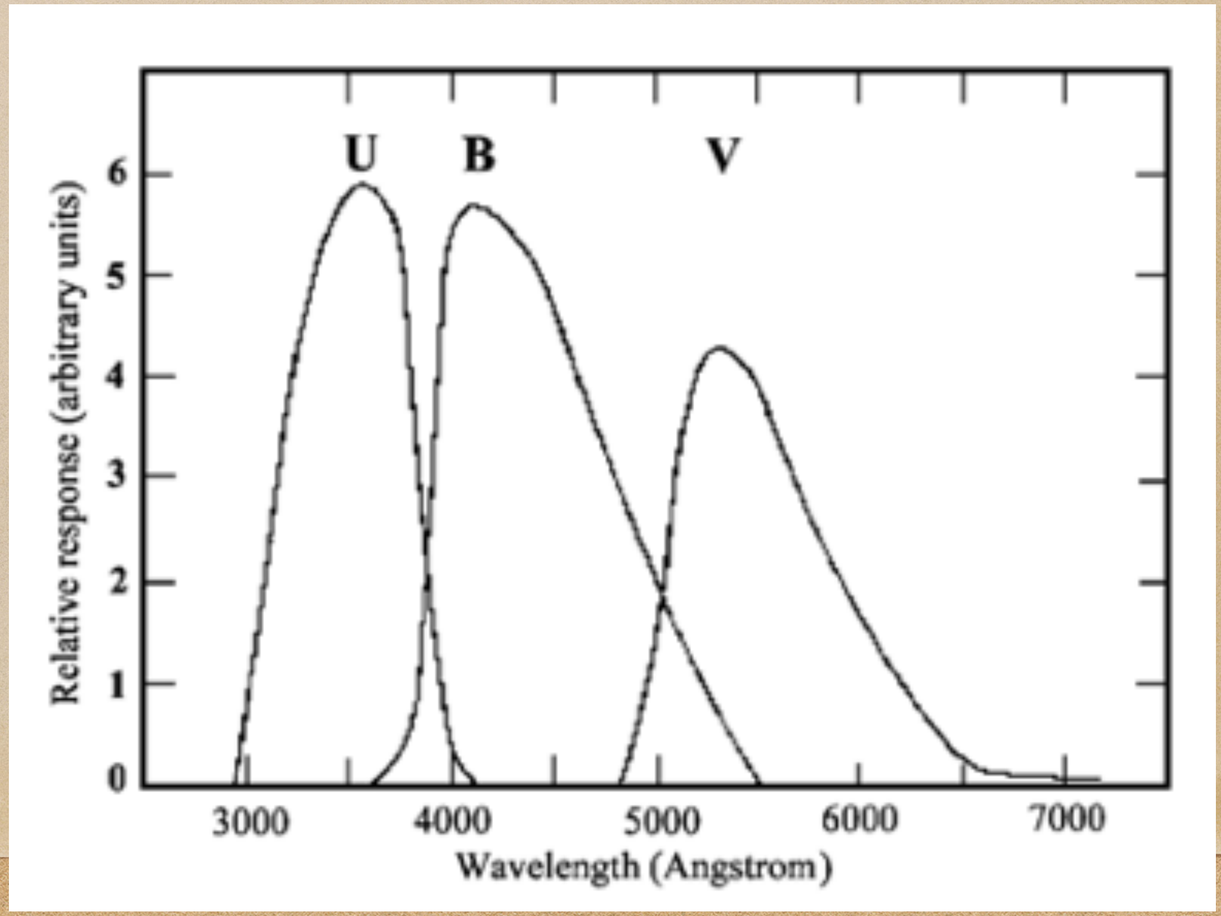


HST/WFC3 filters

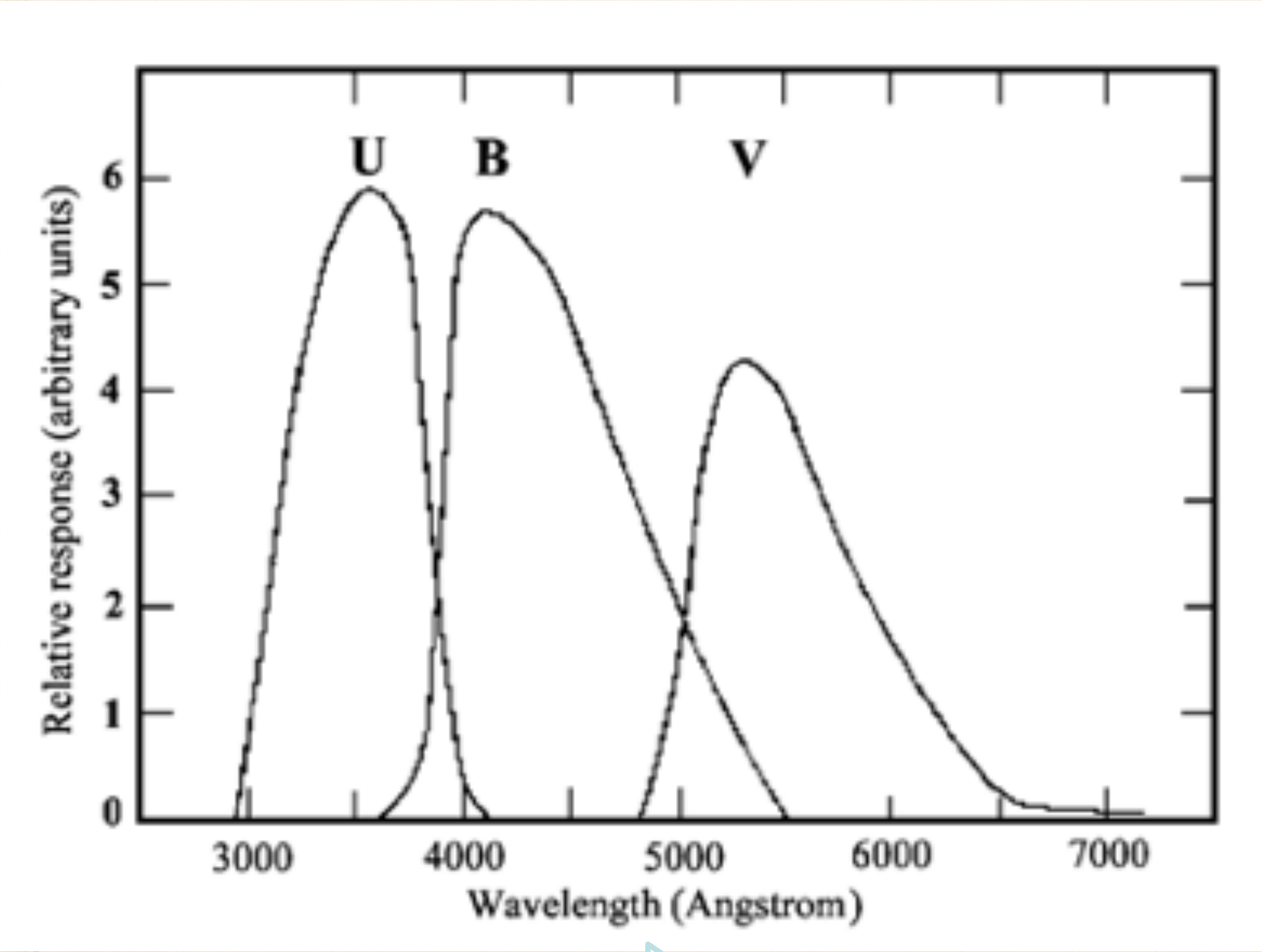
UBV photometric system (Johnson)

In history, various photometric systems have been developed, suitable for visual observations, photo detectors or solid-state detectors (such as the CCD) and with appropriate filters to limit bandwidth.

Among the most used is Johnson's UBV (broadband) introduced in the 1950s:



Vega magnitudes



The filters are selected so that the average wavelengths of the response functions are 364 nm for U, 442 nm for B, 540 nm for V. The zero point of B-V and U-B was defined as to be about zero for A0 stars in the main sequence not affected by interstellar reddening.

Vega magnitude system

Vega magnitudes

$$m(\lambda_c) = m_0(\lambda_c) - 2.5\log_{10}\left(\frac{\int_{\lambda_1}^{\lambda_2} F(\lambda)\times R^n(\lambda)\times QE(\lambda)\times T(\lambda)d\lambda}{\int_{\lambda_1}^{\lambda_2} R^n(\lambda)\times QE(\lambda)\times T(\lambda)d\lambda}\right)$$

$m_0(\lambda_c)$ It is determined based on the energy distribution of Vega, a star type A0; by definition these stars have equal magnitudes in all bands U=B=V

Vega magnitudes

AB Magnitudes

Unlike the Vega system, magnitudes AB are defined without using a spectral distribution of energy, but the reference flux is assumed to be the same in all bands.

$S_V^{ref} = S_V^{AB} = 2.89 \times 10^{-21}$ erg s^{-1} cm^{-2} Hz^{-1}

This value was chosen so that A0 stars like Vega have the same magnitude in Johnson's V-band and in the AB system

$m^{AB}_V = m_V$

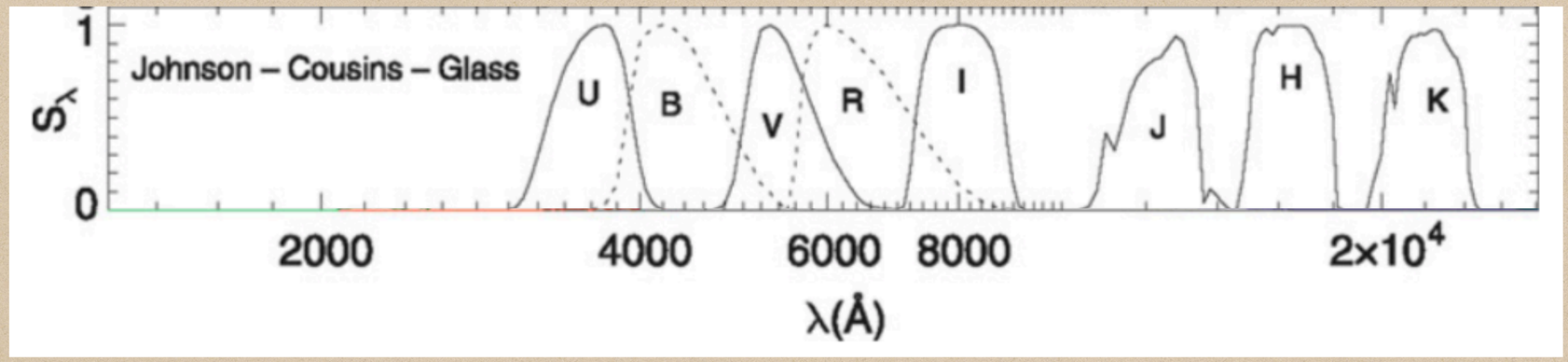
$$m_{\mathrm{AB}\to\mathrm{Vega}} := m_X^{\mathrm{AB}} - m_X^{\mathrm{Vega}}$$

$$= -2.5 \log \left(\frac{\int \mathrm{d}\nu \; T_X(\nu) \, S_\nu^{\mathrm{AB}}}{\int \mathrm{d}\nu \; T_X(\nu) \, S_\nu^{\mathrm{Vega}}} \right)$$

AB Magnitudes

Extension to NIR

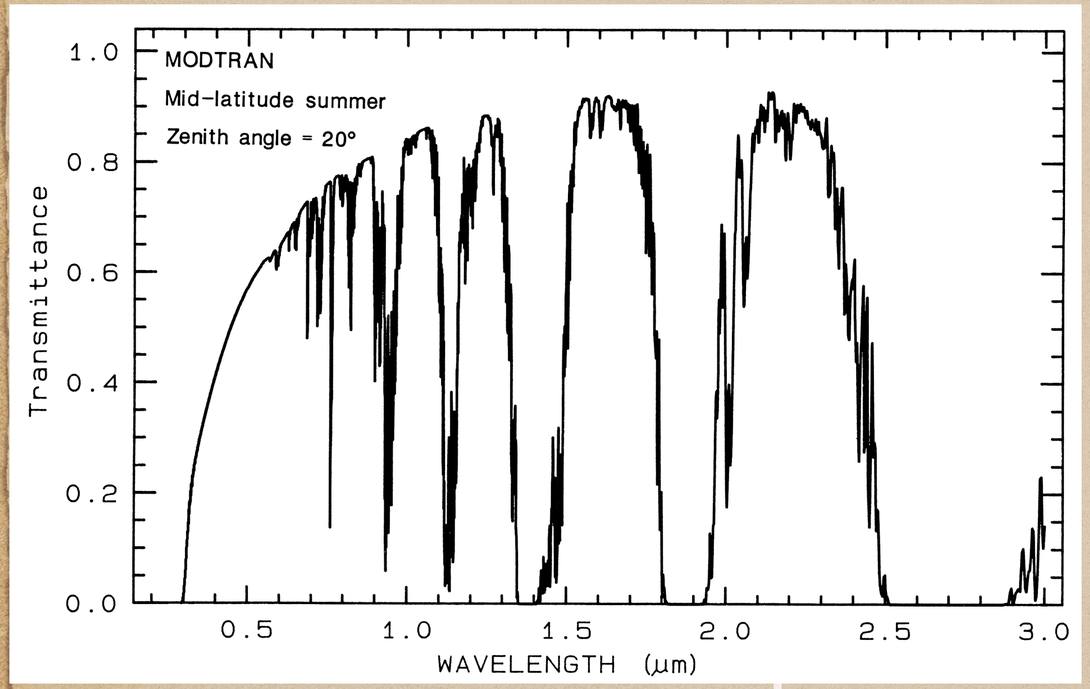
Extension to the near infrared, addition of new bands

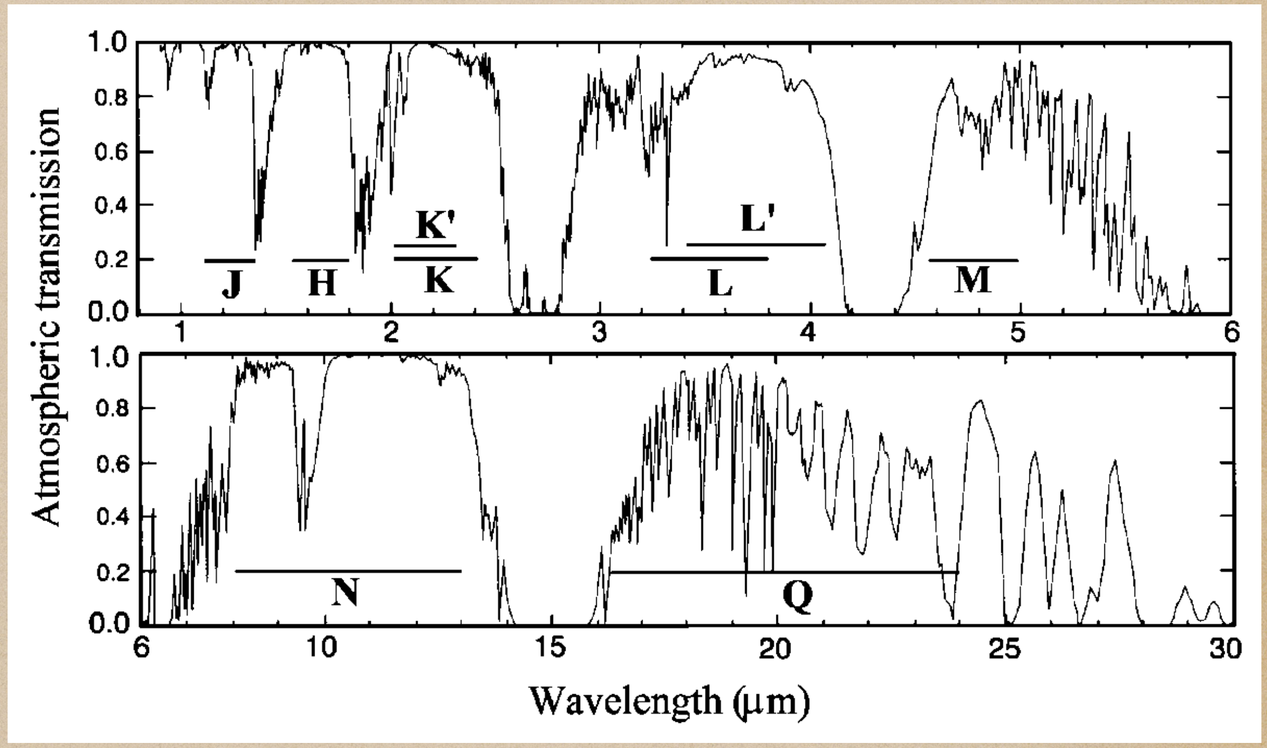


Johnson	U	B	V	R	I	J	H	K
λ_{eff}(nm)	367	436	545	638	797	1220	1630	2190
$\Delta\lambda$(nm)	66	94	85	160	149	213	307	39

λ_{eff} It is the wavelength that best represents the observation (and in reality it depends on the energy spectral distribution of the source)

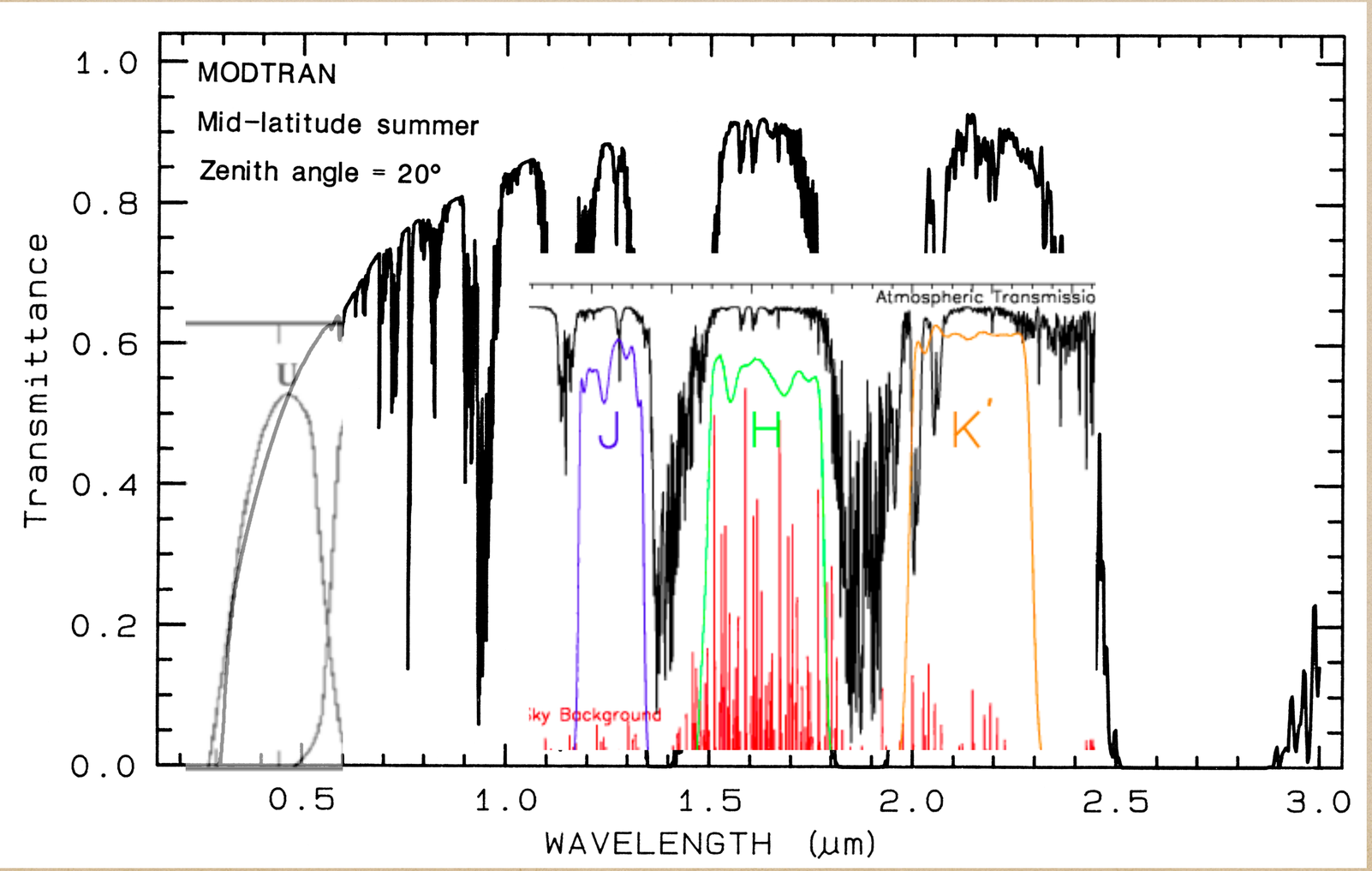
Atmospheric transmission





The Earth's atmosphere is not transparent at all wavelengths: there are deep absorption bands; near-infrared bands are built in high transparency intervals

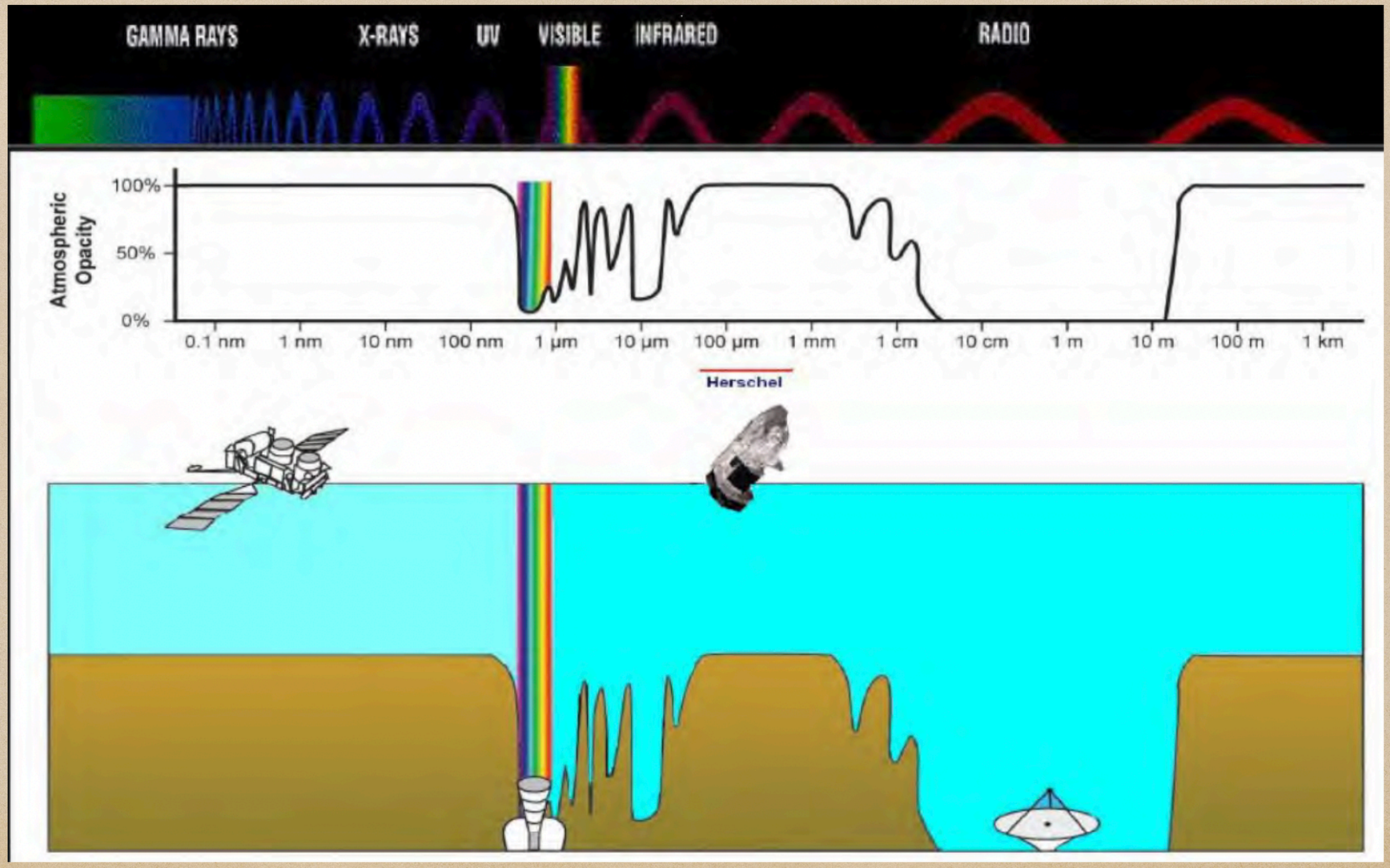
Atmospheric transmission



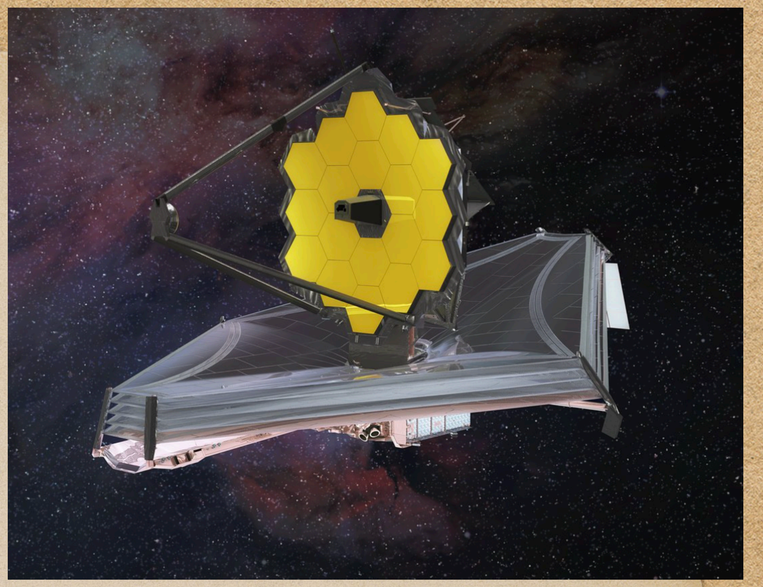
The blue cutoff of the U filter is given purely by atmospheric transmission

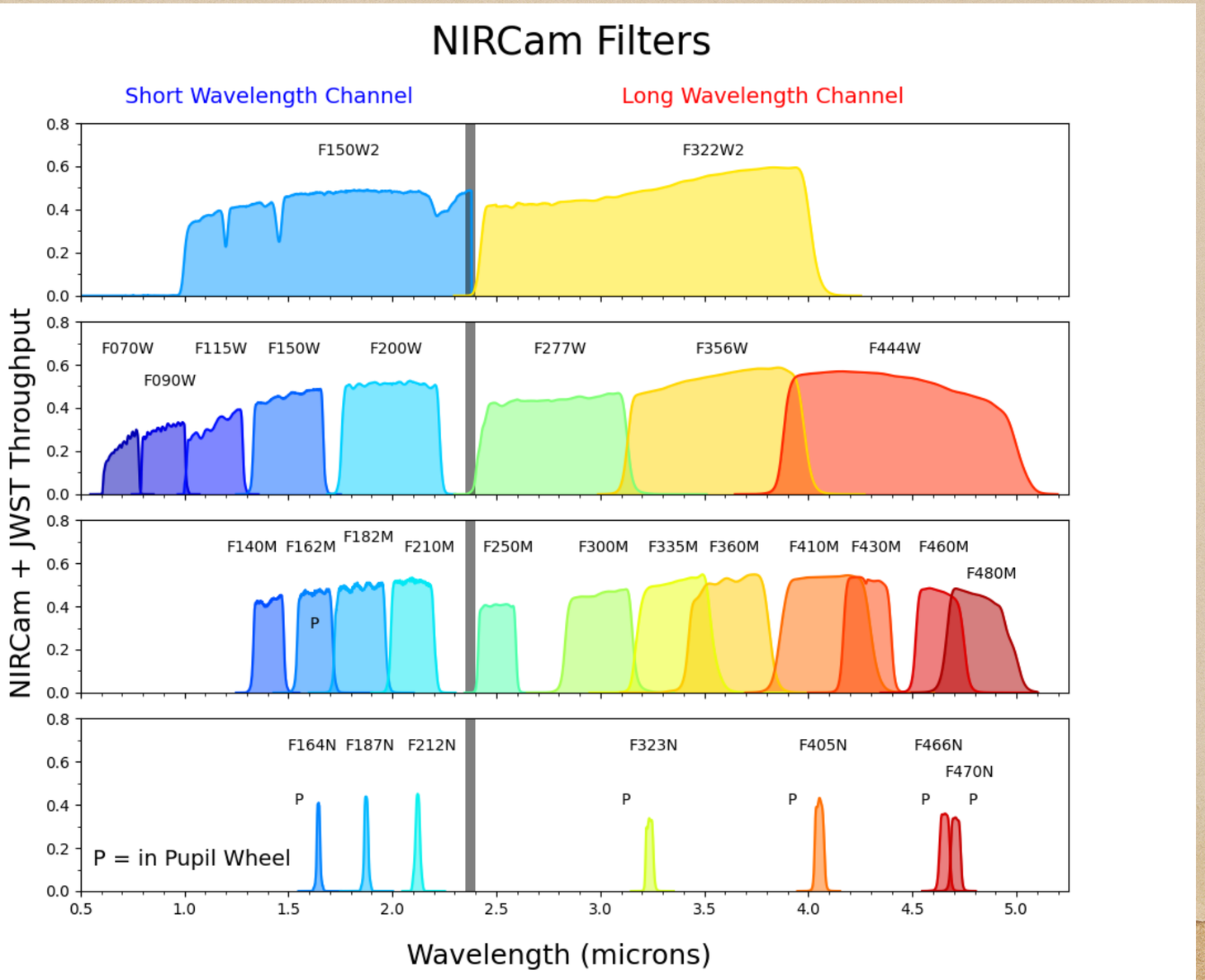
NIR filters are built in "windows" of high atmospheric transmission

Helps to establish the shape of photometric filters

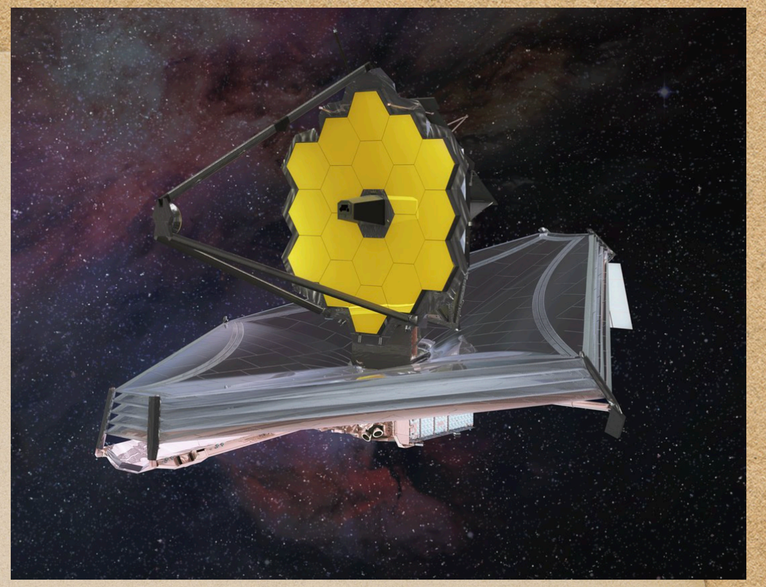

GAMMA RAYS
X-RAYS
UV
VISIBLE
INFRARED
RADIO
Atmospheric Opacity
100%
50%
0%
0.1 nm
1 nm
10 nm
100 nm
1 μm
10 μm
100 μm
1 mm
1 cm
10 cm
1 m
10 m
100 m
1 km
Herschel

The JWST infrared space frontier

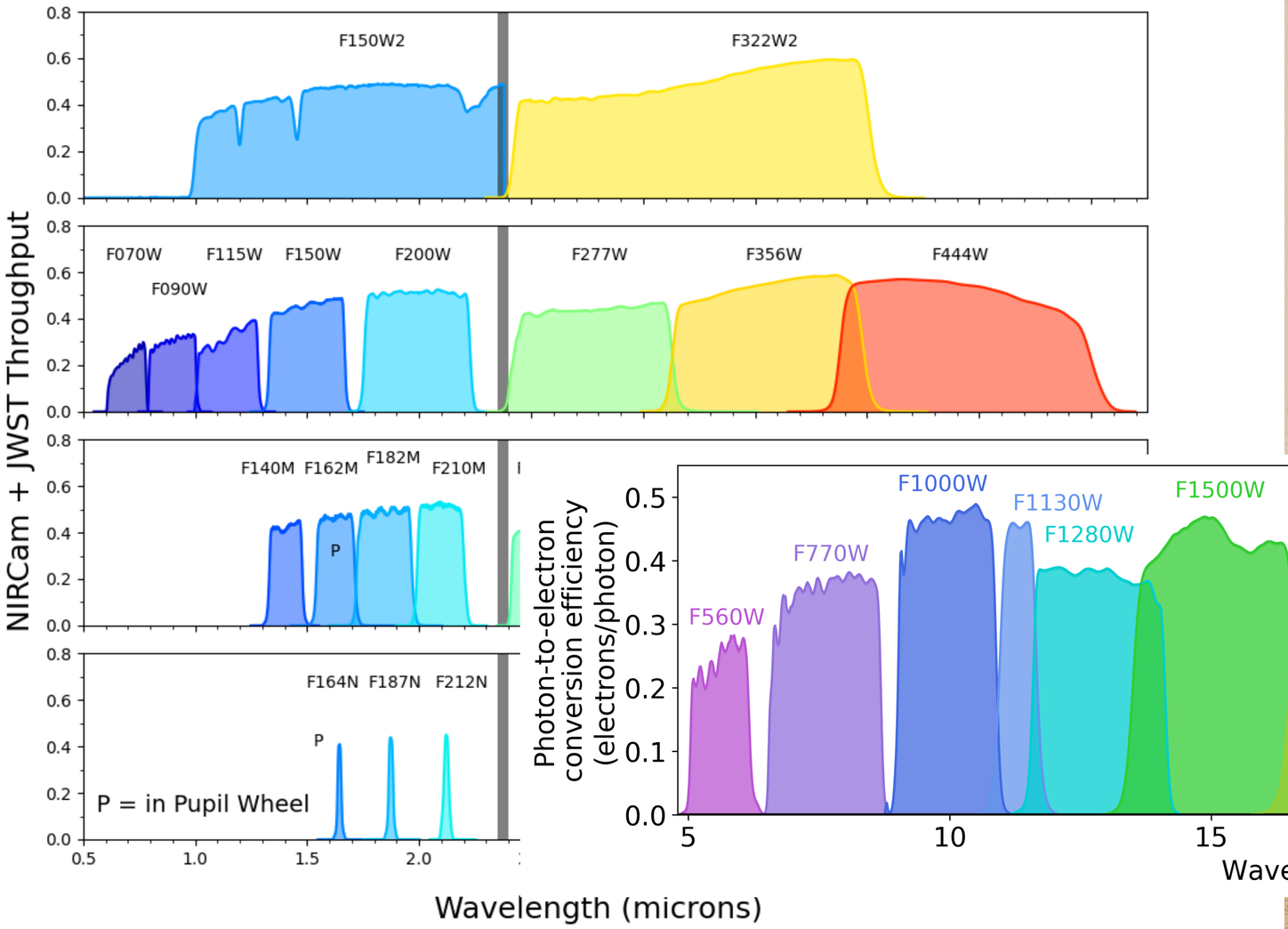




The JWST infrared space frontier



NIRCam Filters



MIRI filters

The astronomical magnitude

The calibration of the UBV system into energy units

U = 0 corrisponds to S(3700 A) = 4.4×10^{-9} erg×cm^{-2}×s^{-1}A^{-1}

B = 0 corrisponds to S(4450 A) = 7.2×10^{-9} erg×cm^{-2}×s^{-1}A^{-1}

V = 0 corrisponds to S(5556 A) = 3.44×10^{-9} erg×cm^{-2}×s^{-1}A^{-1}

(Note: 10^{-9} erg×cm^{-2}×s^{-1}A^{-1} = 10^{-8} W×m^{-2}×m^{-1})

(1 erg = 10^{-7} J)

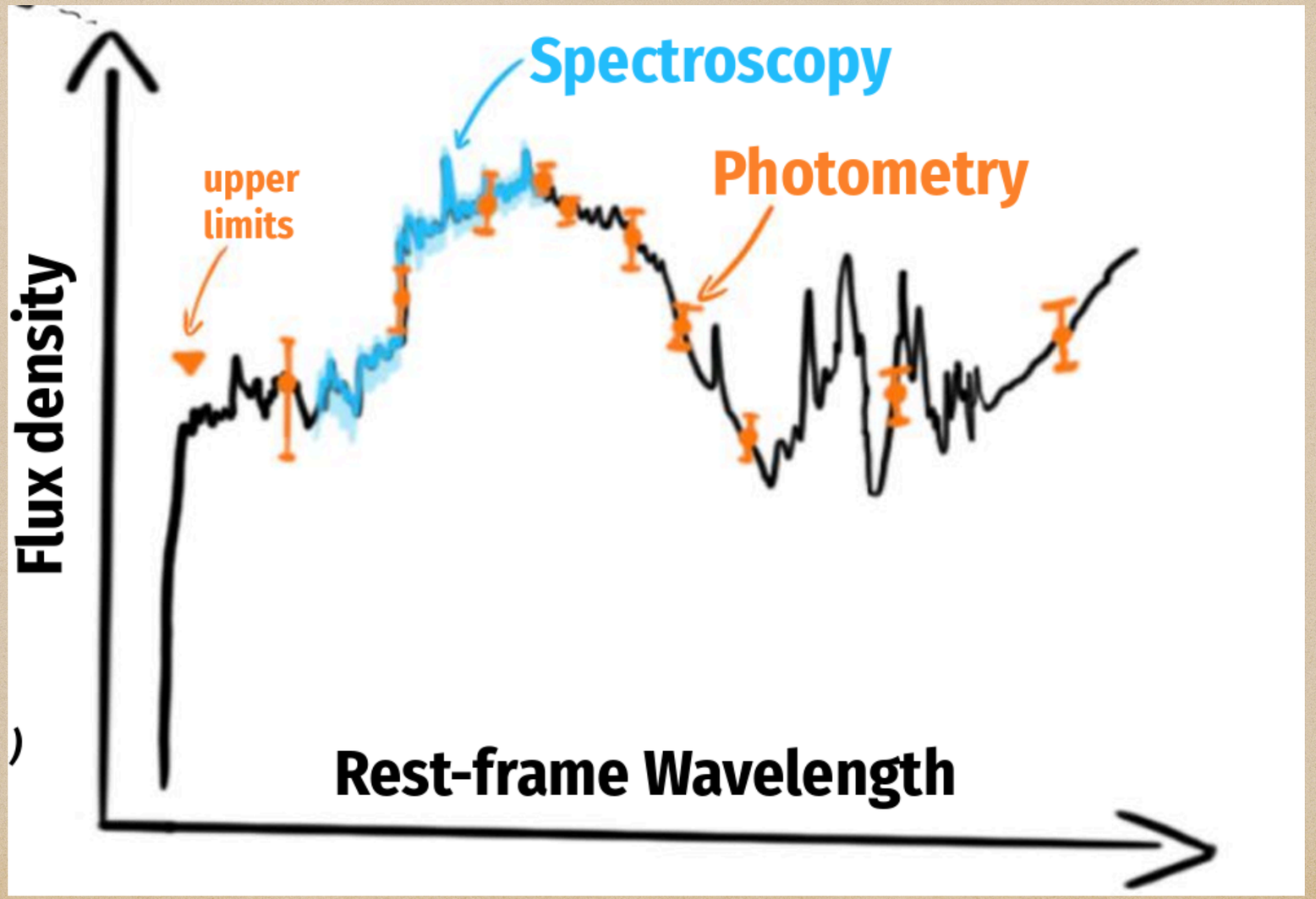
The astronomical magnitude

For a generic star of magnitudes U, B e V:

$S(\lambda_U) = 10^{-0.4(U+20.90)}$,
$S(\lambda_B) = 10^{-0.4(B+20.36)}$,
$S(\lambda_V) = 10^{-0.4(V+21.08)}$

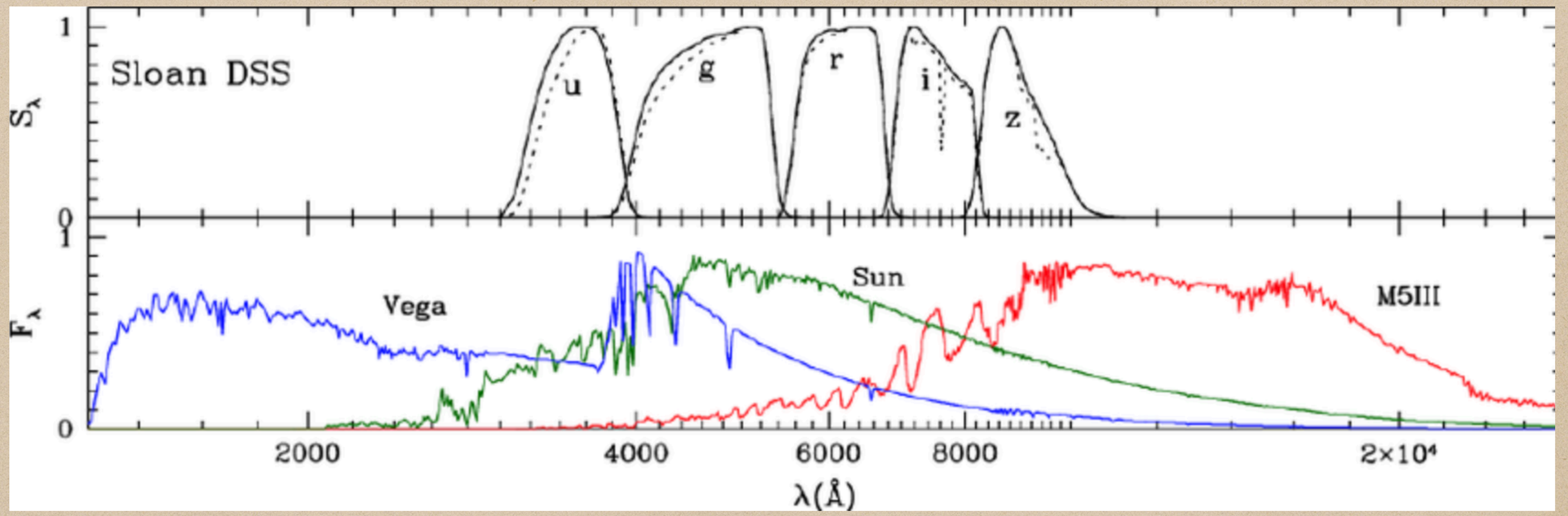
I

Concepts/Examples of broad band photometry (vs spectroscopy)



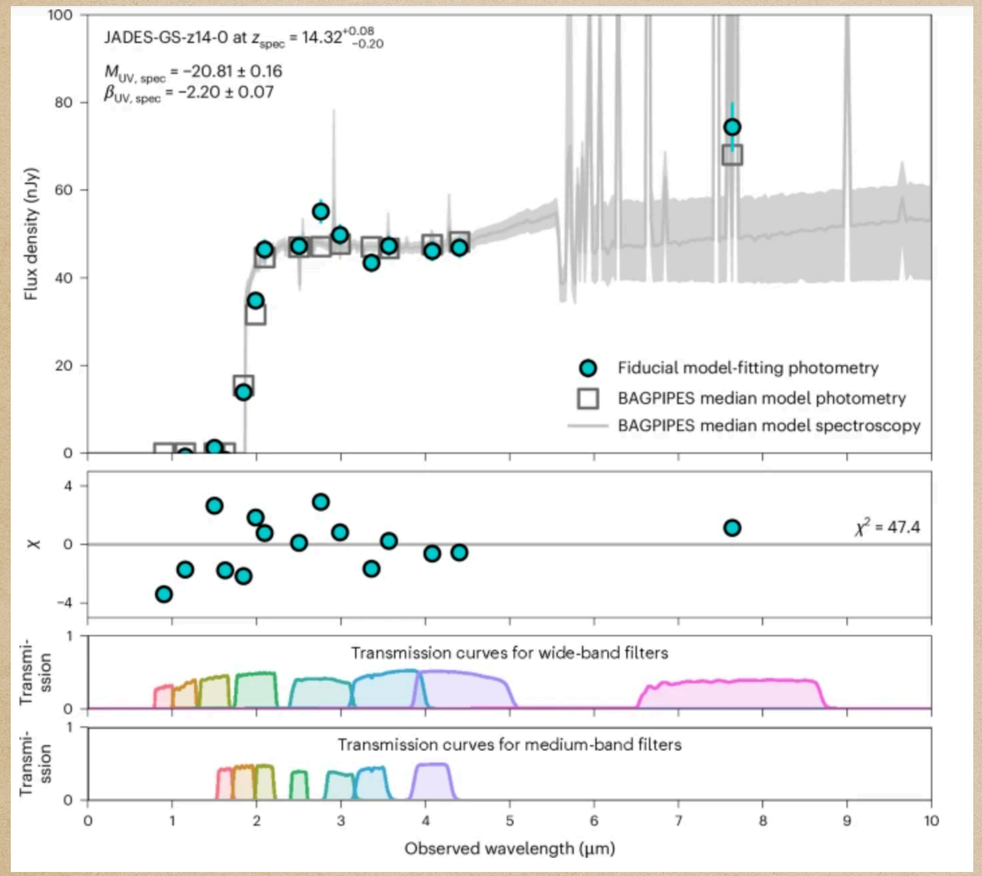
Iyer+25

UV-optical photometric system: Different stellar spectra



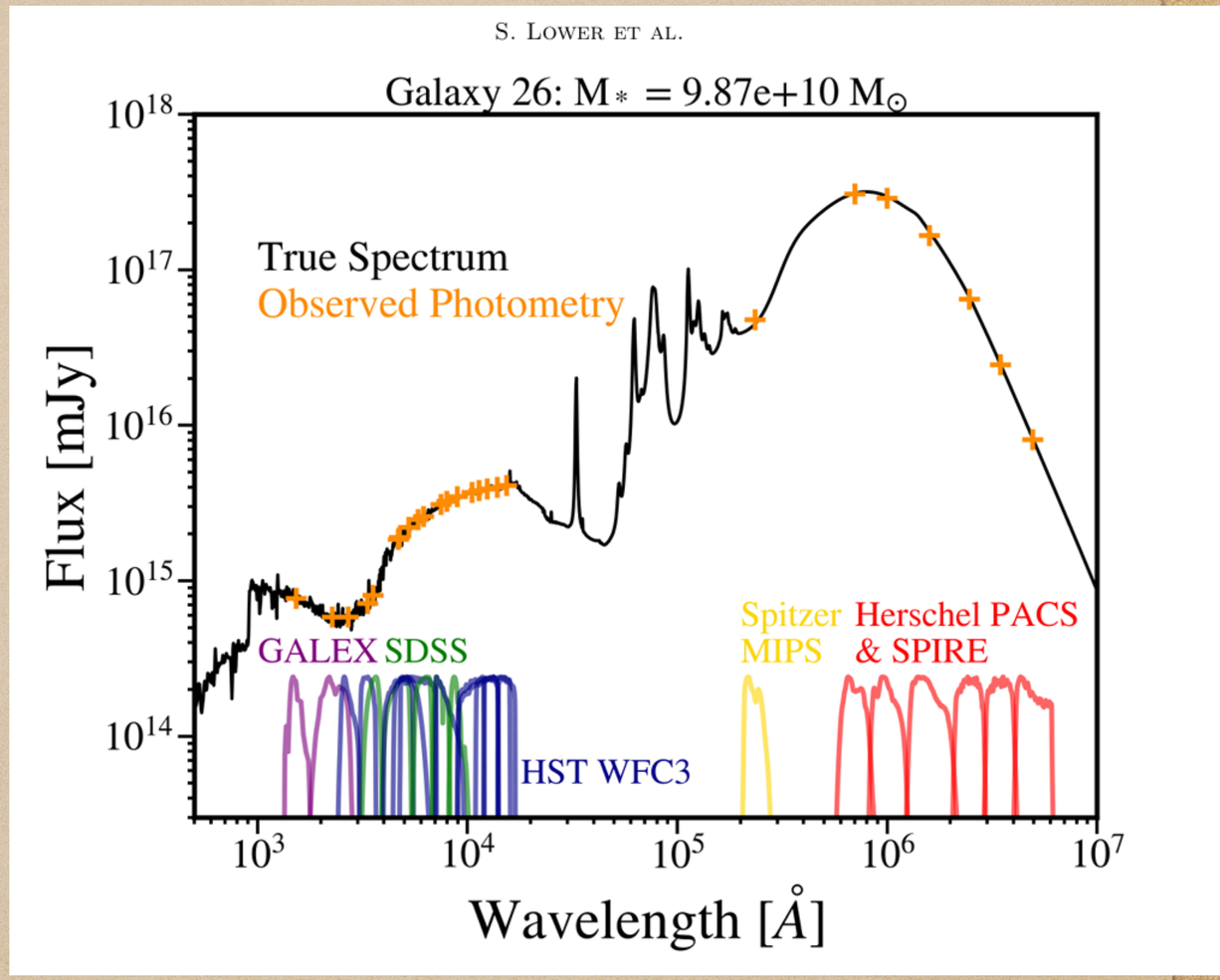
Hodapp & Chambers (2006)

UV-Optical spectrum of a distant galaxy



Helton+25

Panchromatic (UV-to-submillimeter) photometry of a dusty galaxy



Lower+20

Color index or color

Consider a star that is observed at 2 wavelengths λ_1, λ_2, with $\lambda_1 < \lambda_2$. We define the *color index (or color)* of this star the parameter:

$$c_{\lambda_1\lambda_2} = m_{\lambda_1} - m_{\lambda_2} = c_{0\lambda_1\lambda_2} - 2.5\log_{10}\frac{S(\lambda_1)}{S(\lambda_2)}$$

(with c_0 appropriate constant that depends on the 2 wavelengths)

The color does not depend on the ray of the star. If the space is transparent, the color is also independent of the distance of the star.

If the star radiates like a black body, you would also have:

$$c_{\lambda_1\lambda_2} = m_{\lambda_1} - m_{\lambda_2} = c_{0\lambda_1\lambda_2} - 2.5\log_{10}\frac{B(\lambda_1)}{B(\lambda_2)}$$

Color index or color

By international convention, the arbitrary constants $c_{0\lambda 1\lambda 2}$ were chosen so that the color indexes are = 0, whatever the pair of wavelengths λ_1, λ_2 , for a set of stars with spectrophotometric characteristics similar to those of Vega.

These stars are also indicated by the spectral type A0-V (A zero fifth), as we will see later. For them we therefore have:

U - B = B - V = V - R = R - I = … = 0

Evidently, color index = 0

does not mean equal flux in the two bands.

Absolute magnitude

We define absolute magnitude M the magnitude m that the object would have if it were positioned 10 pc from us. If the space is perfectly transparent, the observed luminous flux S decreases with the inverse of the square of the distance from the source, therefore, if ***R*** is the radius of the object and ***d*** its distance:

$$S(\lambda) = \frac{L(\lambda)}{4\pi d^2} \qquad (1)$$

where $L(\lambda)$ is the intrinsic brightness of the source

We write (1) for 2 distances:

$$S_1(\lambda) = \frac{L(\lambda)}{4\pi {d_1}^2} \qquad S_2(\lambda) = \frac{L(\lambda)}{4\pi {d_2}^2}$$

Absolute magnitude

It follows from Pogson's law that:

$$m_1(\lambda) = m_{0\lambda} - 2.5\log\left[F_1(\lambda)\right] = m_{0\lambda} - 2.5\log\left[\frac{L(\lambda)}{4\pi d_1^2}\right] \quad (2)$$

And:

$$m_2(\lambda) = m_{0\lambda} - 2.5\log\left[F_1(\lambda)\right] = m_{0\lambda} - 2.5\log\left[\frac{L(\lambda)}{4\pi d_2^2}\right] \quad (3)$$

By subtracting 3 from 2:

$$m_1(\lambda) - m_2(\lambda) = -2.5\log\left[\frac{L(\lambda)}{4\pi d_1^2}\right] + 2.5\log\left[\frac{L(\lambda)}{4\pi d_2^2}\right] = -2.5\log\left(\frac{d_2^2}{d_1^2}\right) \quad (4)$$

Absolute magnitude

Now in (4) we define *m1=M*, magnitude at *d1=10 p*c and *m2=m* apparent magnitude at distance *d*:

$$M(\lambda) - m(\lambda) = -2.5\log\left(\frac{d^2}{10^2}\right) = -5\log(d) + 2.5\log(100) = -5\log(d) + 5$$

And in the end it follows that:

$$M(\lambda) = m(\lambda) - 5\log(d) + 5$$

If I know *M* I know *d*, and vice versa

Distance module

$$DM_\lambda = m_\lambda - M_\lambda = 5 \log d - 5$$

is called the ***distance module*** of that star.

Dependence on λ (which at first glance should not be there) was explicitly held to warn that ***if space is not transparent*** the magnitude observed depends on λ, and so will the distance module.

So the distance modules obtained with different photometric systems may differ from each other, due to the imperfect knowledge of interstellar absorption.

Bolometric magnitude

The total luminosity L of a source is the integral of the specific luminosity L_v over all frequencies (or L_λ over all wavelengths). Consequently, the total flux S of a source is the specific flux S_v integrated over all frequencies v. The apparent bolometric magnitude m_{bol} is defined as:

$$m_{bol} = const - 2.5 \log S$$

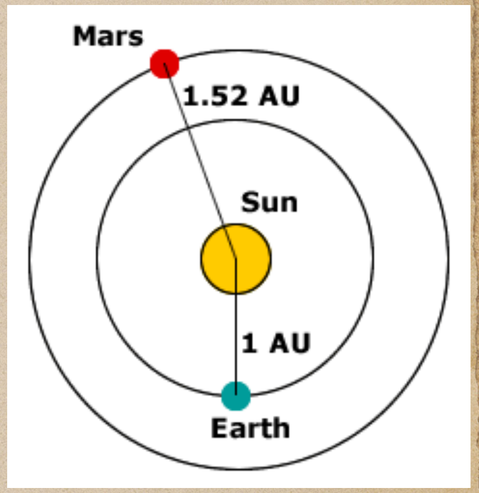
The absolute bolometric magnitude M_{bol} will therefore be:

$$M_{bol} = const - 2.5 \log L \qquad (5)$$

Bolometric magnitude

And the constant can be fixed using the parameters of the Sun:

$$m_{\odot bol} = -26.83 \qquad DM(1AU) = -31.47$$



(1AU= distance Sun-Earth, 1/206265 pc or 1.5×10^8 km)

It follows that the absolute bolometric magnitude of the Sun is:

$$M_{\odot bol} = m_{\odot bol} - DM(1AU) = -26.3 + 31.47 = 4.74$$

1 parsec=206265 AU

Bolometric magnitude

It follows that:

$$M_{\odot bol} = -2.5 \log L_{\odot} + const = 4.74$$

$$const = 4.74 + 2.5 \log L_{\odot}$$

$L_{\odot}$ bolometric luminosity of the Sun

And finally the equation (5) can be written:

$$M_{bol} = 4.74 - 2.5 \log \frac{L}{L_{\odot}} \qquad (6)$$

And the bolometric luminosity of the Sun will be:

$$L_{\odot} = 3.85 \times 10^{33} erg \cdot s^{-1}$$

The difference in bolometric absolute magnitude between the generic star and the Sun is (writing (6) for a generic star and for the Sun, then subtracting):

$$M_{bol} = 4.74 - 2.5 \log \frac{L}{L_\odot} \qquad M_{\odot bol} = 4.74$$

$$M_{bol} - M_\odot = -2.5 \log \frac{L}{L_\odot} = -2.5 \log \frac{R^2 T^4}{R_\odot^2 T_\odot^4}$$

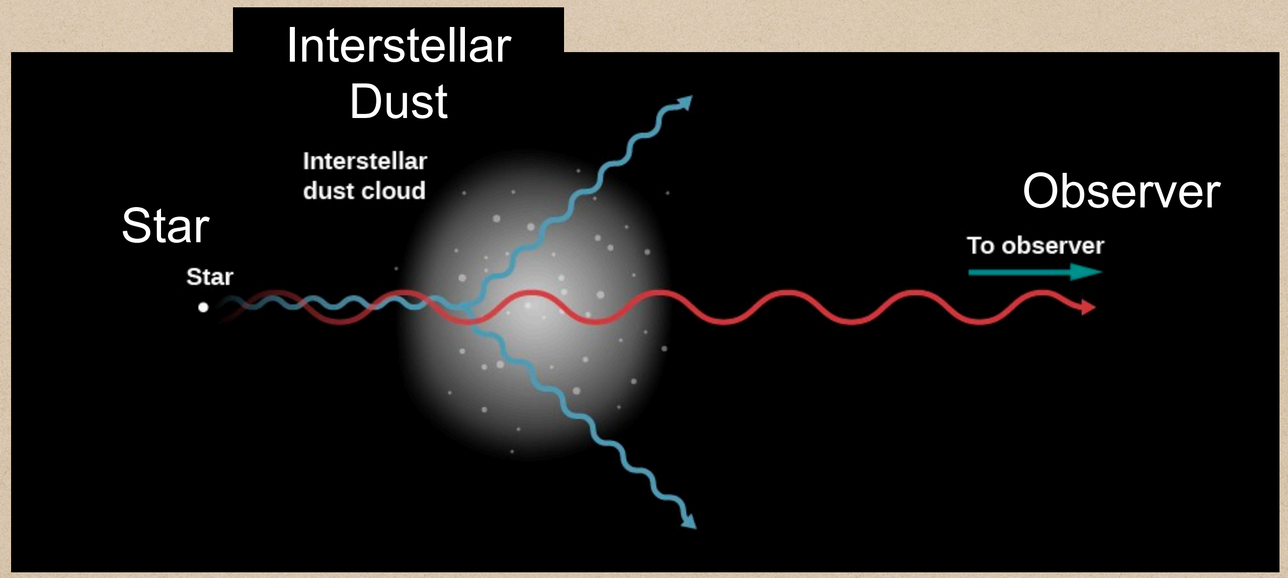
Thus making the dependence on the radius and temperature

Interstellar absorption

The space between the stars is not empty, but filled with ***gas and dust***

The dust is produced in the atmospheres of giant and super giant stars (therefore in the last stages of life of very massive stars)

These are conglomerates of silicates or carbonates, often covered with water/ ice, a few microns in diameter.

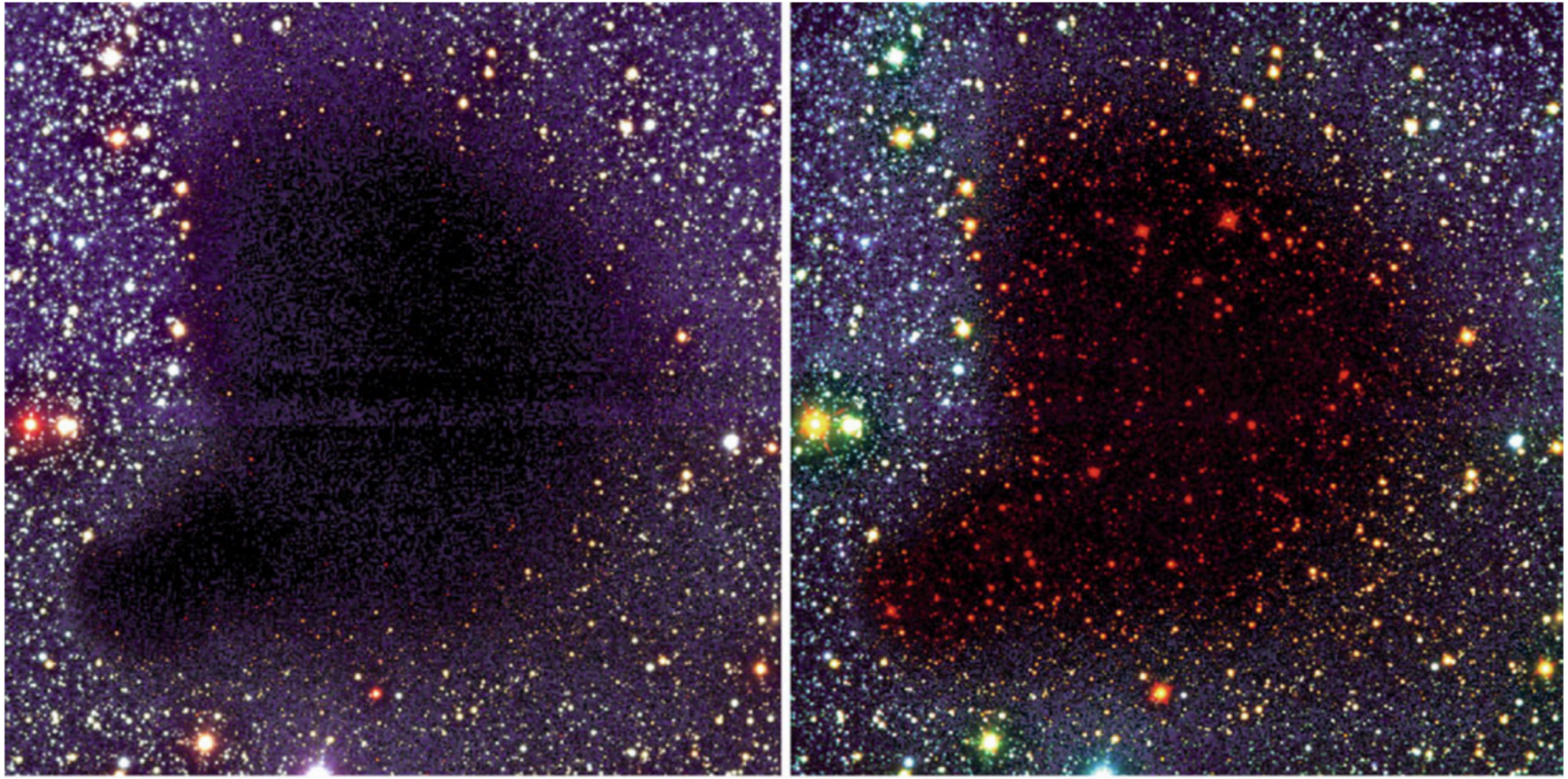


Dust is very efficient in absorbing/scattering blue light; the transmitted light is ***attenuated and reddened***

Interstellar absorption

Attenuation

Reddening



B, V, I

Barnard cloud 68

B, I, K

Interstellar absorption

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu$$

Radiative transport equation in the presence of absorption/scattering only

I can rewrite it like this:

$$\mathrm{d}\ln I_\nu = \mathrm{d}I_\nu / I_\nu = -\kappa_\nu \,\mathrm{d}s$$

and integrate:

Optical depth

$$\ln I_\nu(s) - \ln I_\nu(0) = -\int_0^s \mathrm{d}s'\, \kappa_\nu(s') \equiv -\tau_\nu(s)$$

$$I_\nu(s) = I_\nu(0)\, \mathrm{e}^{-\tau_\nu(s)}$$

The specific intensity is reduced by a factor $\tau_\nu(s)$

Interstellar absorption

Similarly, the flux will be reduced, at the distance s, in the same way:

$$S_\nu = S_\nu(0)\, e^{-\tau_\nu(s)}$$

Where $S_\nu(0)$ is the flux without absorption. Moving to magnitudes:

$$\frac{S_\nu}{S_{\nu,0}} = 10^{-0.4(m-m_0)} = e^{-\tau_\nu} = 10^{-\log(e)\tau_\nu}$$

$$A_\nu := m - m_0 = -2.5\, \log(S_\nu/S_{\nu,0})$$

$$= 2.5\, \log(e)\, \tau_\nu = 1.086\tau_\nu\ .$$

Where m is the observed magnitude, while m_0 is the one you would have without absorption. A_ν is called ***Extinction coefficient***, Which is nothing more than a measurement of ***the optical depth at that frequency***

Interstellar absorption

Since the coefficient κ_V depends on the frequency (or λ), ***the absorption*** also implies a change of ***color. Color excess is defined as follows:***

$$E(X-Y) := A_X - A_Y = (X - X_0) - (Y - Y_0)$$
$$= (X-Y) - (X-Y)_0.$$

The color excess describes the change in the color index (X-Y), measured in two filters X and Y. Note that:

$$E(X-Y) = A_X - A_Y = A_X \left(1 - \frac{A_Y}{A_X}\right) \equiv A_X \, R_X^{-1}$$

The color excess is thus proportional to the extinction coefficient.

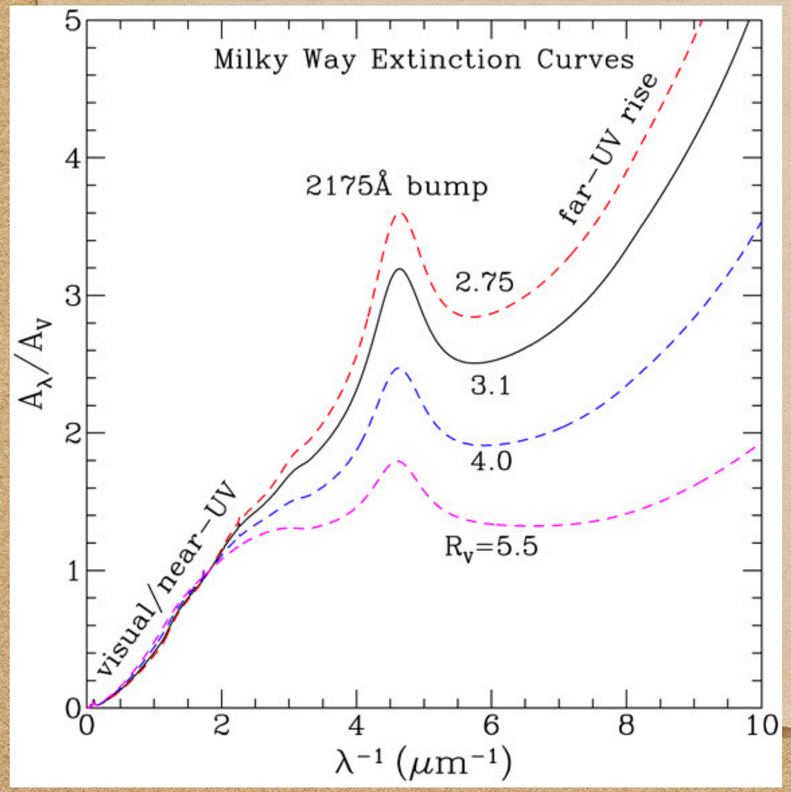
Interstellar absorption

Considering bands *B* e *V*:

$$A_V = R_V\, E(B-V)$$

For example, in our Milky Way we have the typical relationship:

$$A_V = (3.1 \pm 0.1) E(B-V)$$

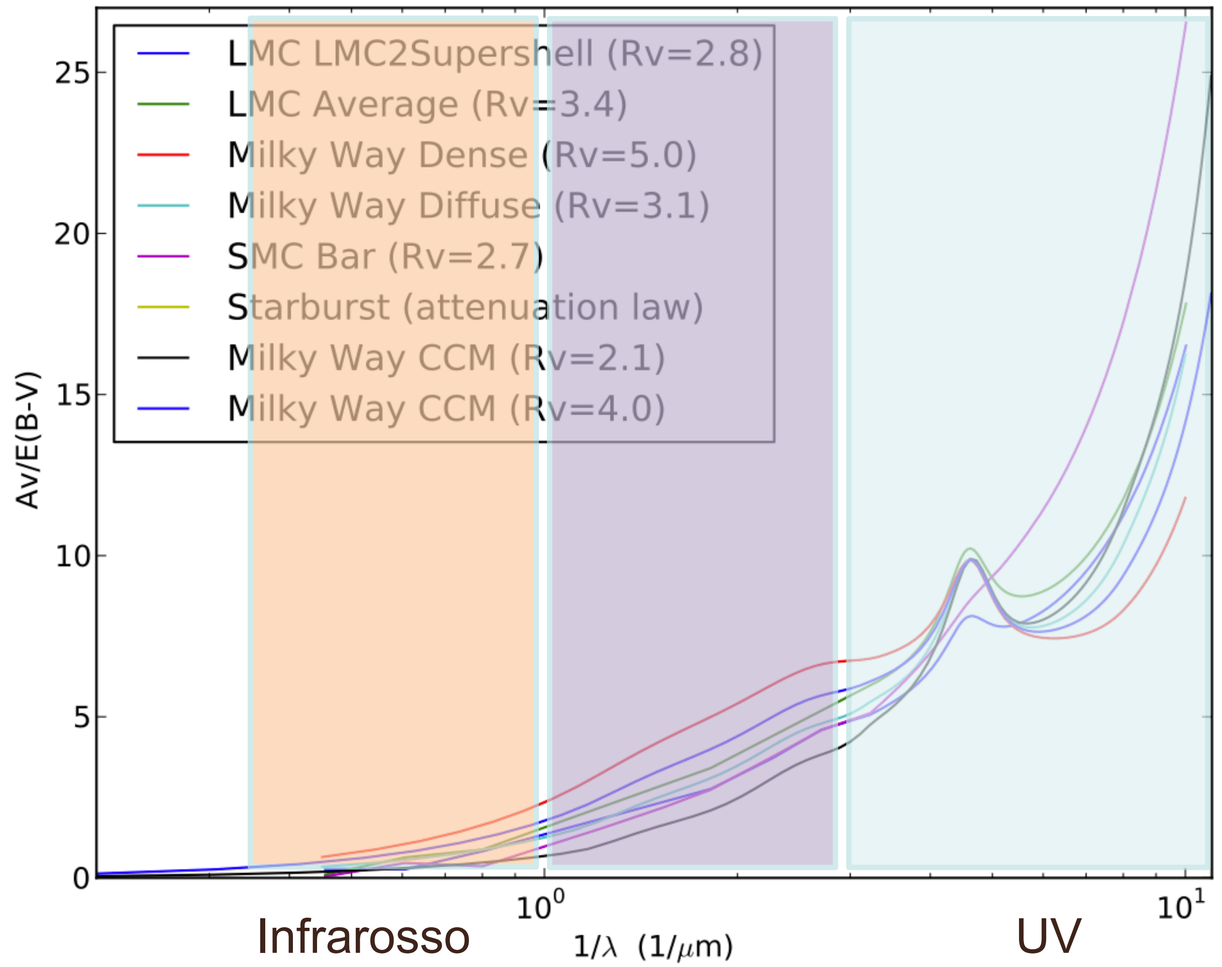


Which, however, is not universal:

But it depends on the characteristics of the dust (composition, geometry, distribution...)

A_V and *E(B-V)* They are directly proportional to the amount of dust along the line of sight

Extinction coefficient in the Milky Way and in the Magellanic Clouds



A_V decreases with λ

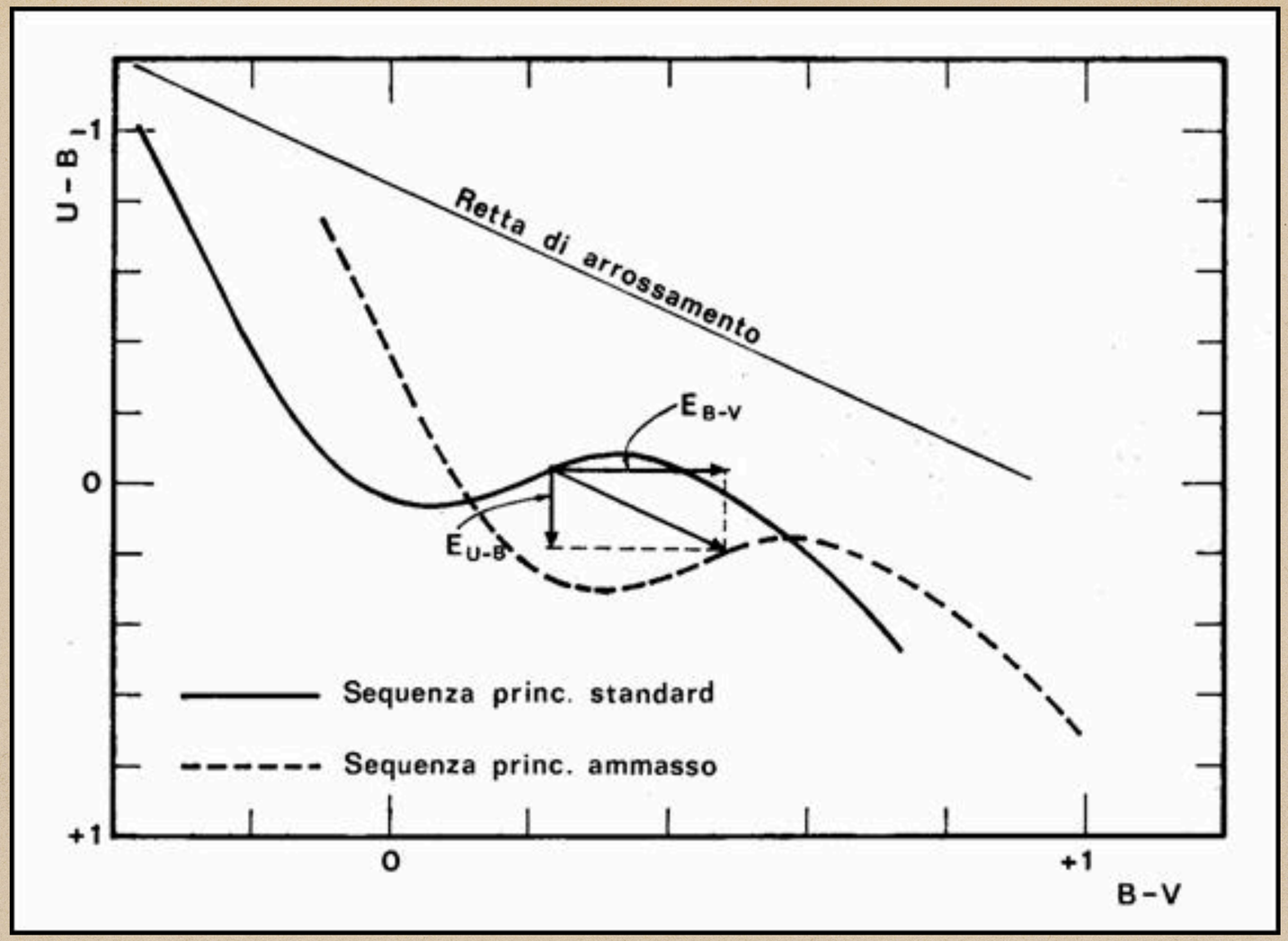
$A_Y/A_X<1$

$$E(X-Y)=A_X\left(1-\frac{A_Y}{A_X}\right)>0$$

$$(X-Y)=(X-Y)_0+E(X-Y)>0$$

The observed color is always greater (redder) than the one in the absence of absorption

Interstellar absorption



Redness line on the color-color diagram: all colors become redder (larger), so the redness rigidly moves all the stars along the reddening line

Extinction and attenuation

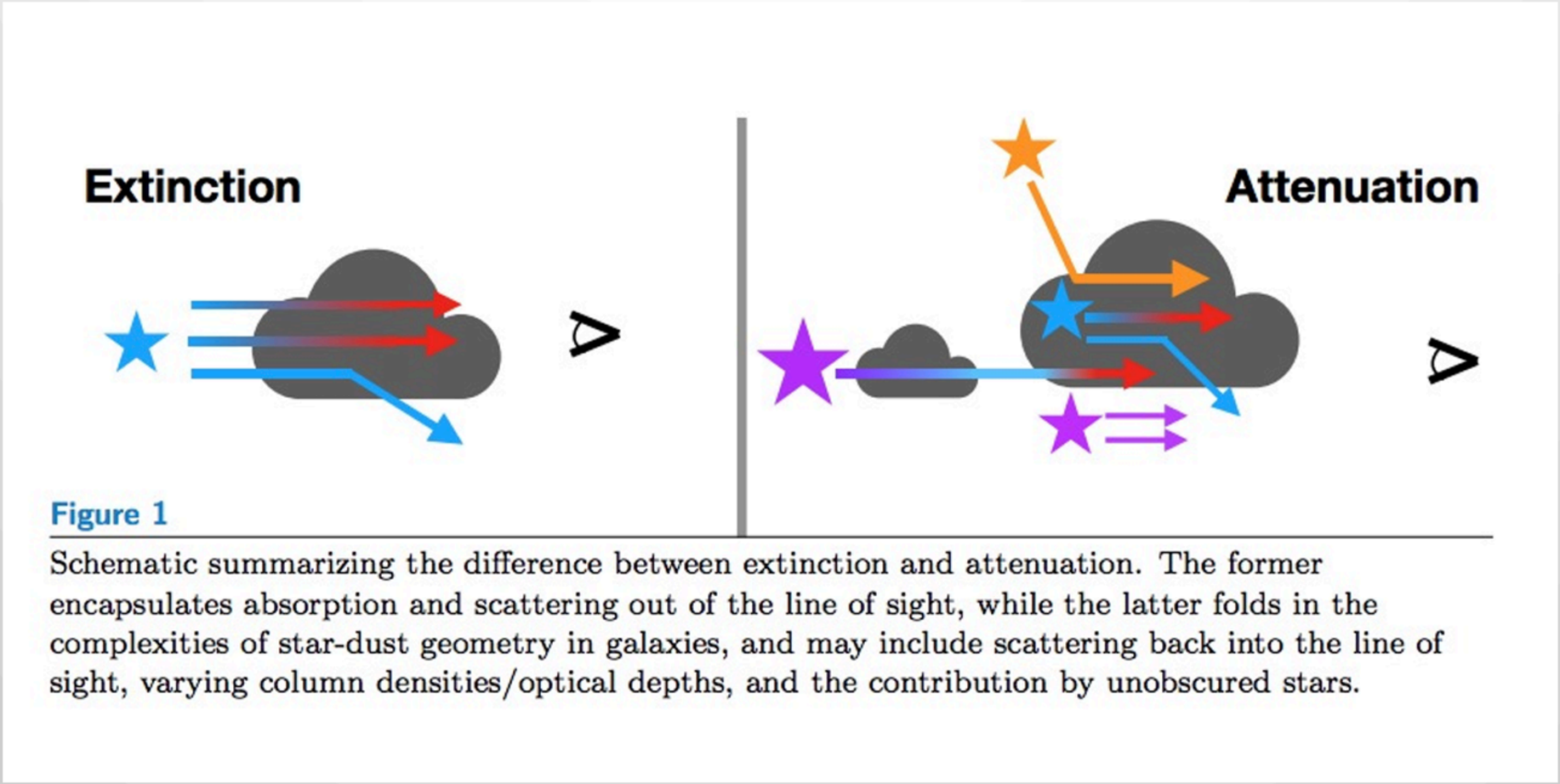


Figure 1

Schematic summarizing the difference between extinction and attenuation. The former encapsulates absorption and scattering out of the line of sight, while the latter folds in the complexities of star-dust geometry in galaxies, and may include scattering back into the line of sight, varying column densities/optical depths, and the contribution by unobscured stars.

Salim & Narayanan (2022)

Extinction is the difference in magnitudes between the intrinsic and observed emission:

$$A_\lambda = m_{\lambda,obs} - m_{\lambda,int}$$