Spherical astronomy (2)

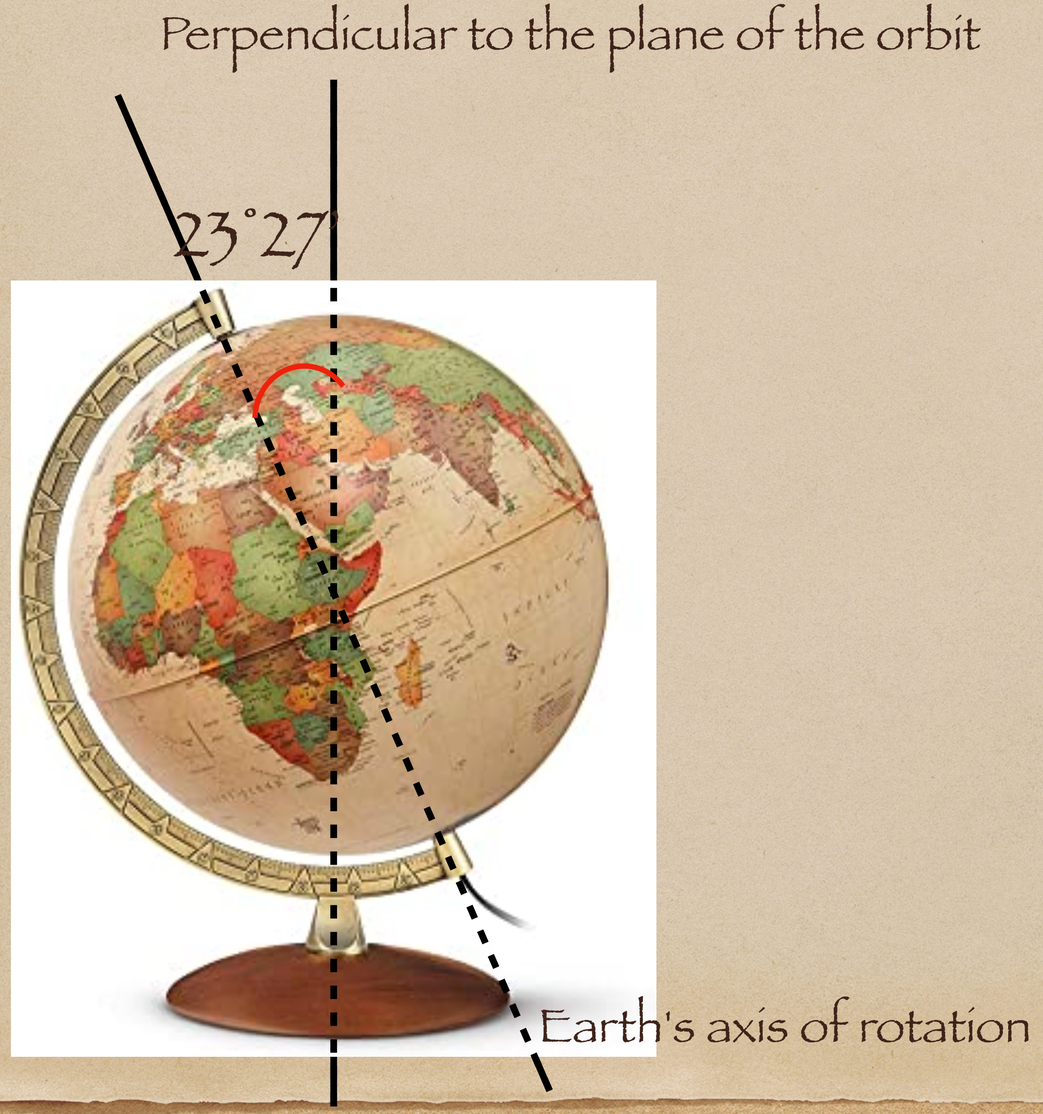
Topic 1

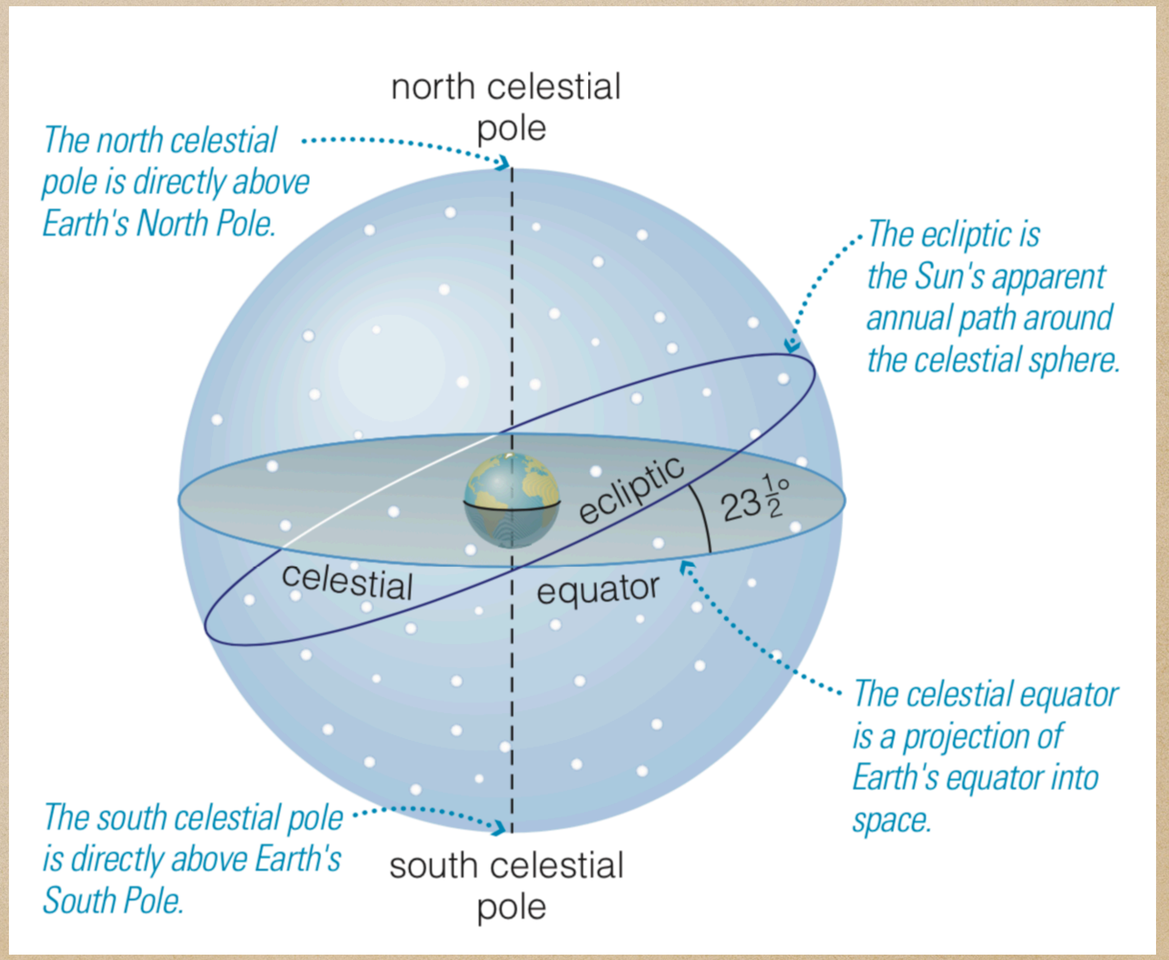
Materiale da

Chap. 2 "Fundamental Astronomy" edition, by H. Karttunen, H. Oja, M. Poutanen, K. J. Donner

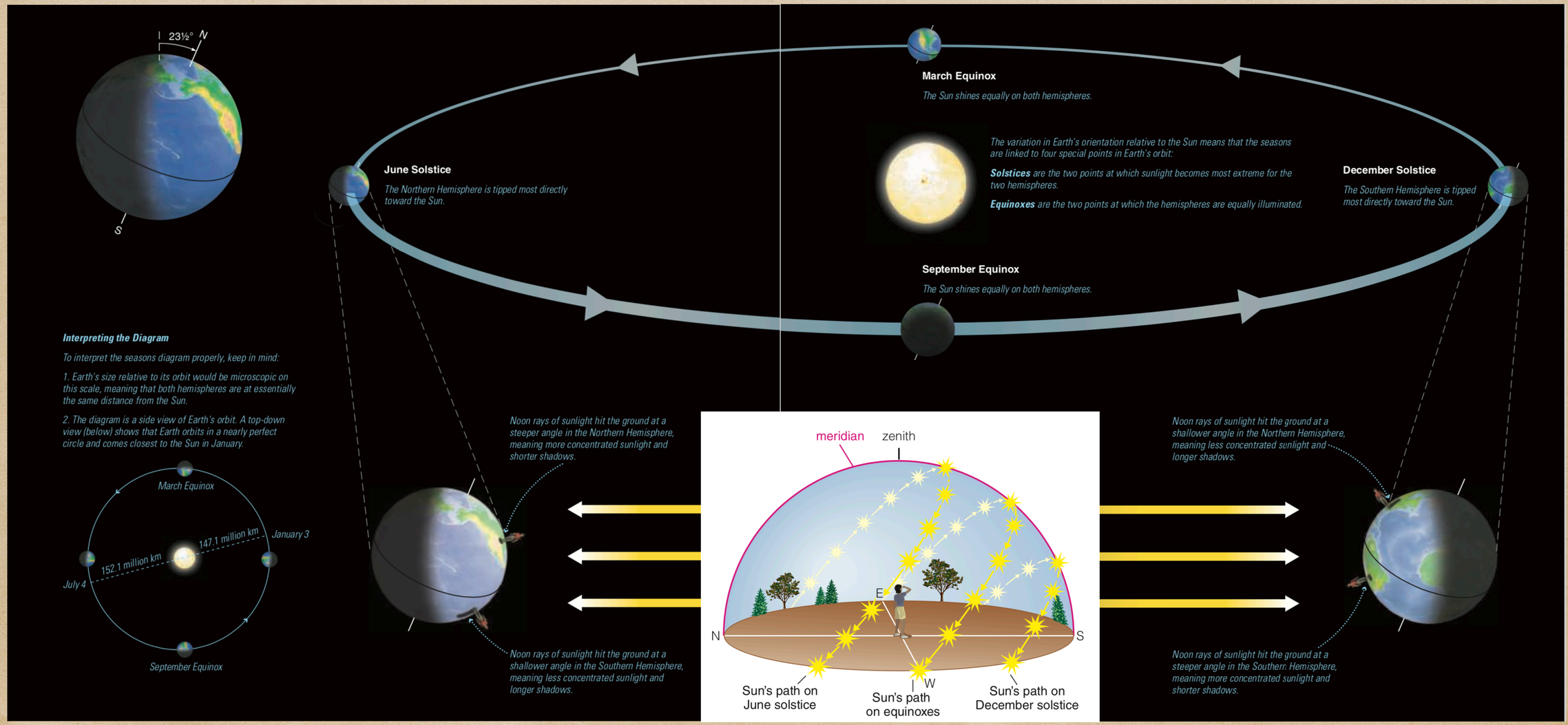
Chap. 2&S1 "The cosmic perspective", by J. O. Bennett, M. O. Donahue, N. Schneider & M. Voit

Ecliptic system

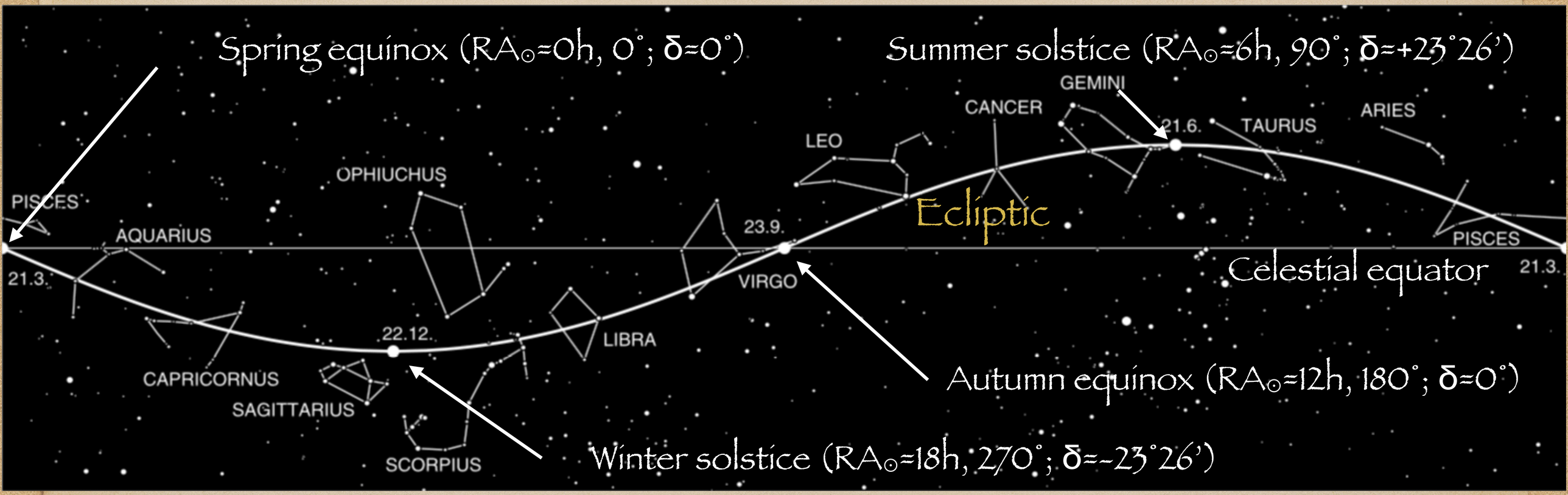




Seasons


23½°
N
S
March Equinox
The Sun shines equally on both hemispheres.
June Solstice
The Northern Hemisphere is tipped most directly toward the Sun.
The variation in Earth's orientation relative to the Sun means that the seasons are linked to four special points in Earth's orbit:
Solstices are the two points at which sunlight becomes most extreme for the two hemispheres.
Equinoxes are the two points at which the hemispheres are equally illuminated.
December Solstice
The Southern Hemisphere is tipped most directly toward the Sun.
September Equinox
The Sun shines equally on both hemispheres.
Interpreting the Diagram
To interpret the seasons diagram properly, keep in mind:
1. Earth's size relative to its orbit would be microscopic on this scale, meaning that both hemispheres are at essentially the same distance from the Sun.
2. The diagram is a side view of Earth's orbit. A top-down view (below) shows that Earth orbits in a nearly perfect circle and comes closest to the Sun in January.
March Equinox
147.1 million km
January 3
152.1 million km
July 4
September Equinox
Noon rays of sunlight hit the ground at a steeper angle in the Northern Hemisphere, meaning more concentrated sunlight and shorter shadows.
Noon rays of sunlight hit the ground at a shallower angle in the Southern Hemisphere, meaning less concentrated sunlight and longer shadows.
meridian
zenith
E
N
S
W
Sun's path on June solstice
Sun's path on equinoxes
Sun's path on December solstice
Noon rays of sunlight hit the ground at a shallower angle in the Northern Hemisphere, meaning less concentrated sunlight and longer shadows.
Noon rays of sunlight hit the ground at a steeper angle in the Southern Hemisphere, meaning more concentrated sunlight and shorter shadows.

Ecliptic system



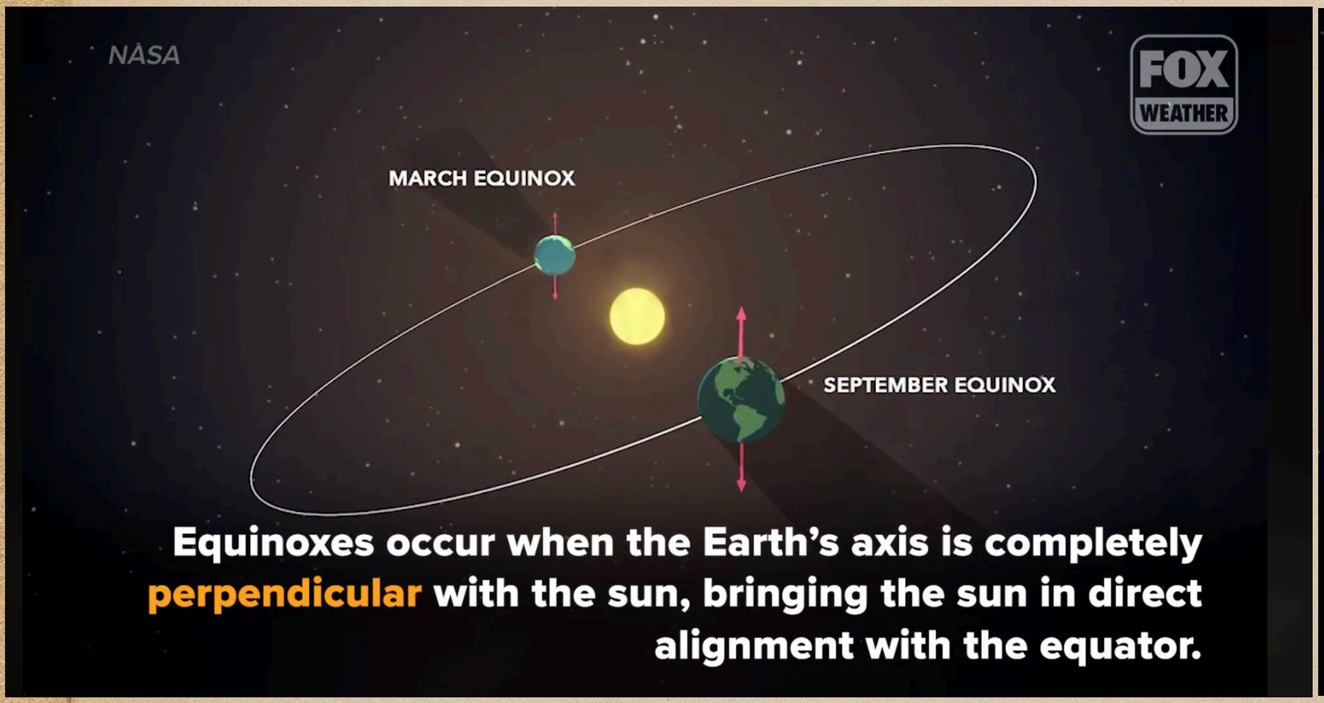
Ecliptic

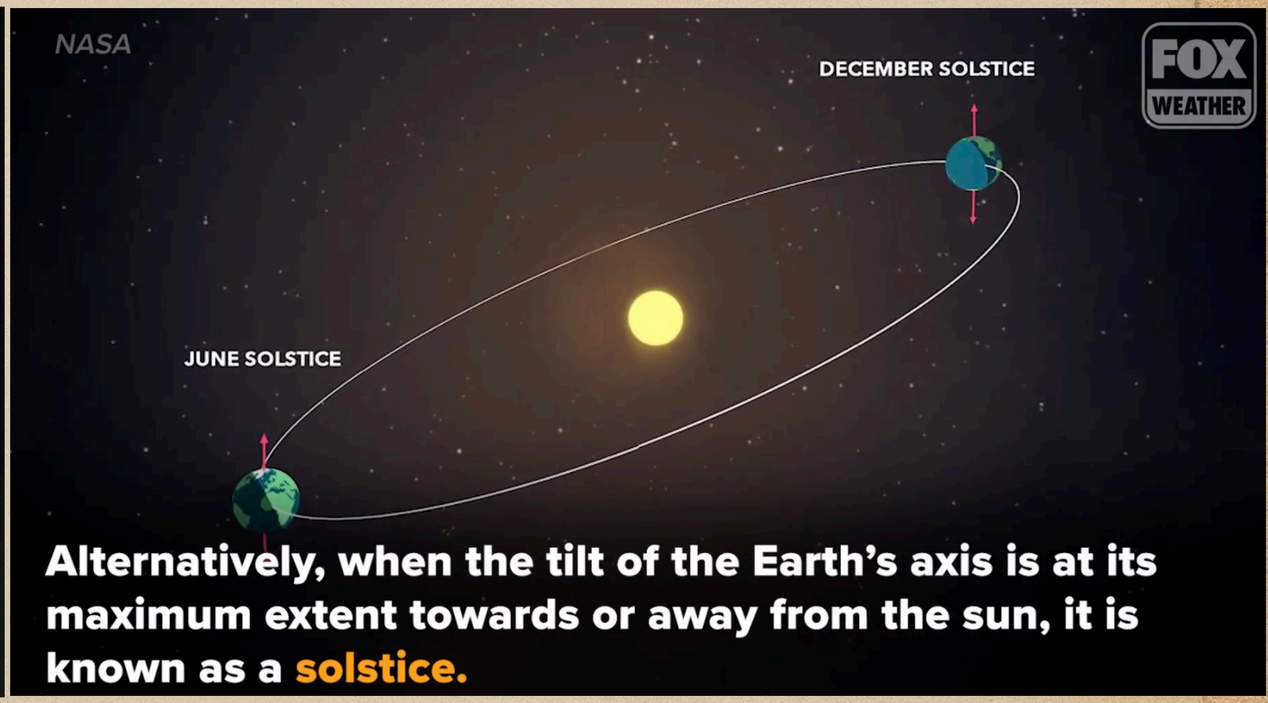
Apparent orbit of the Sun in the sky

It crosses 13 constellations (12 form the zodiac) (due to precession, the constellations are no longer aligned with the zodiac signs)

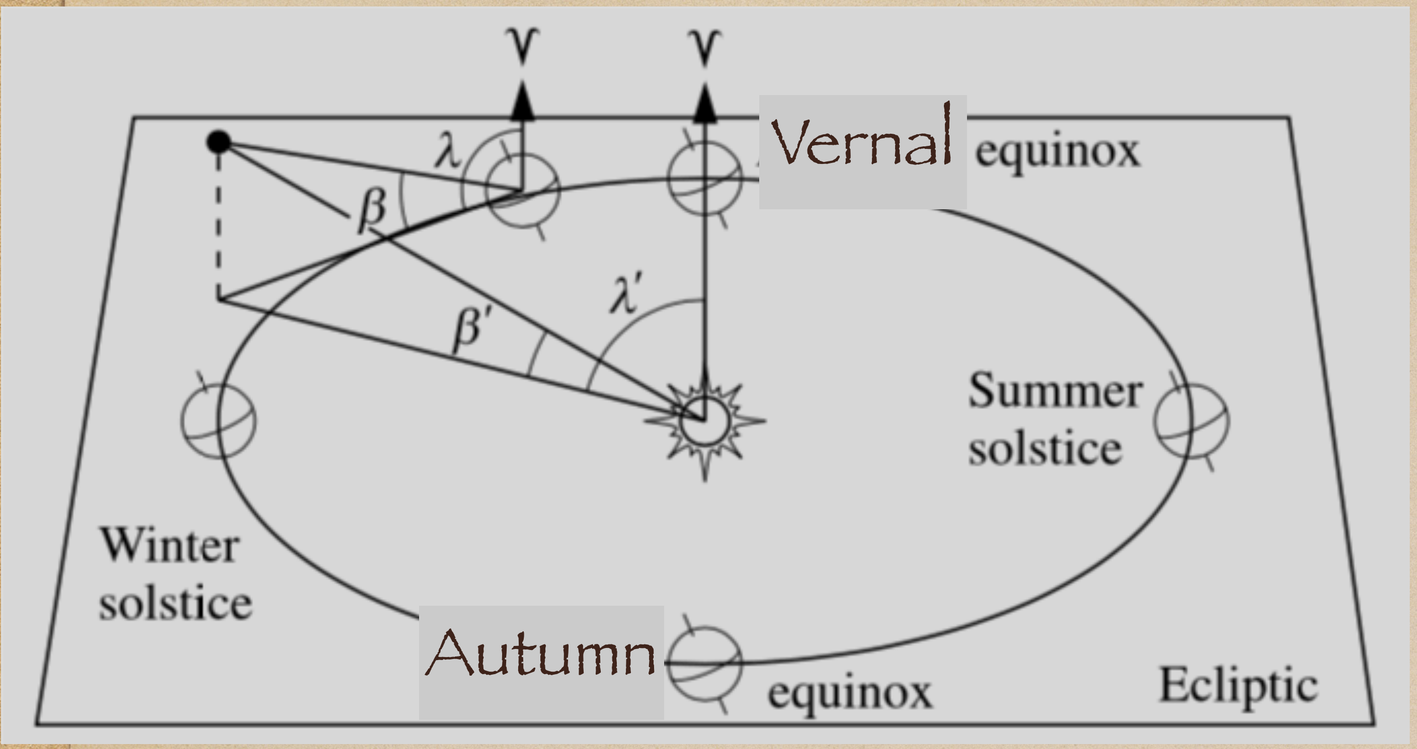
In spring, the Sun crosses the equator from the southern hemisphere to the northern hemisphere. That instant is called spring equinox; then RA and dec (or α and δ) are =0.

Seasons


NASA
FOX WEATHER
MARCH EQUINOX
SEPTEMBER EQUINOX
Equinoxes occur when the Earth's axis is completely perpendicular with the sun, bringing the sun in direct alignment with the equator.


NASA
FOX WEATHER
DECEMBER SOLSTICE
JUNE SOLSTICE
Alternatively, when the tilt of the Earth's axis is at its maximum extent towards or away from the sun, it is known as a solstice.

Ecliptic system



Ecliptic latitude β: angular distance from the ecliptic

Ecliptic longitude λ: counter-clockwise from the spring/vernal equinox

ε angle between the equator and the ecliptic, or Earth axis inclination

N.B.: Earth or heliocentric ecliptic coordinates are generally different; small differences for distant objects (but not for solar system bodies)

Transformations from equatorial system to ecliptic

$$\sin\lambda\cos\beta = \sin\delta\sin\varepsilon + \cos\delta\cos\varepsilon\sin\alpha,$$

$$\cos\lambda\cos\beta = \cos\delta\cos\alpha,$$

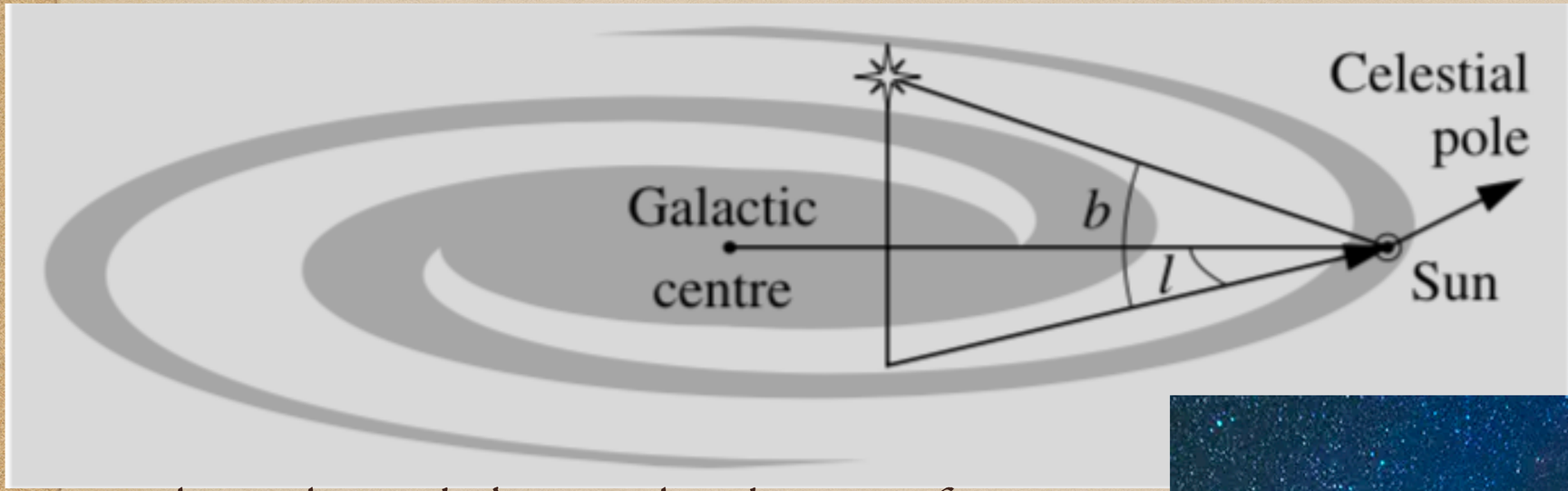
$$\sin\beta = \sin\delta\cos\varepsilon - \cos\delta\sin\varepsilon\sin\alpha,$$

$$\sin\alpha\cos\delta = -\sin\beta\sin\varepsilon + \cos\beta\cos\varepsilon\sin\lambda,$$

$$\cos\alpha\cos\delta = \cos\lambda\cos\beta,$$

$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda.$$

Galactic coordinates

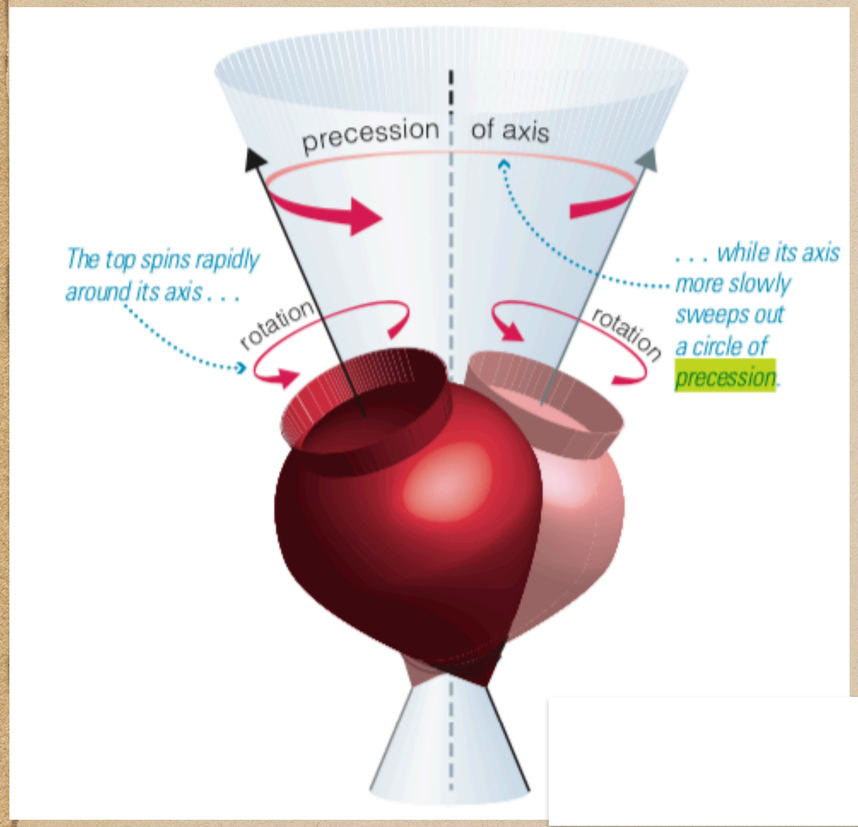


Galactic latitude b: angular distance from the galaxy plane

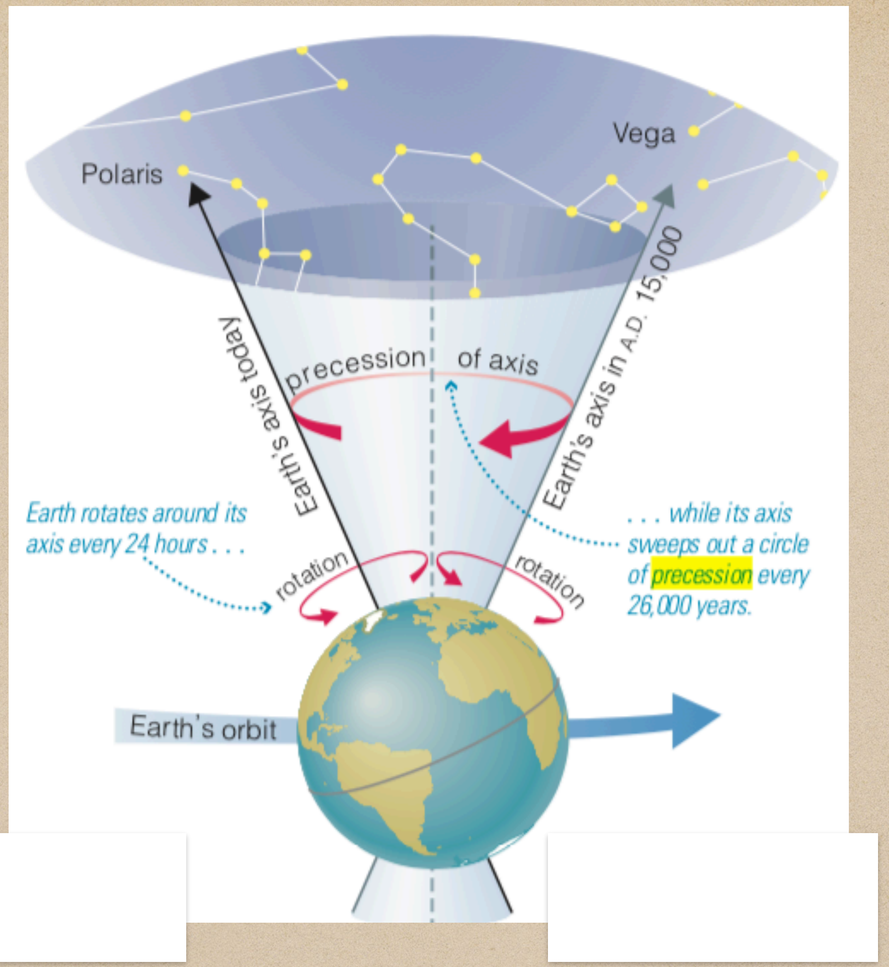
Galactic longitude l: counter-clockwise from the direction of the galaxy center (Sagittarius constellation, α=17h 45.7min; δ=-29°00')



Coordinate disturbances



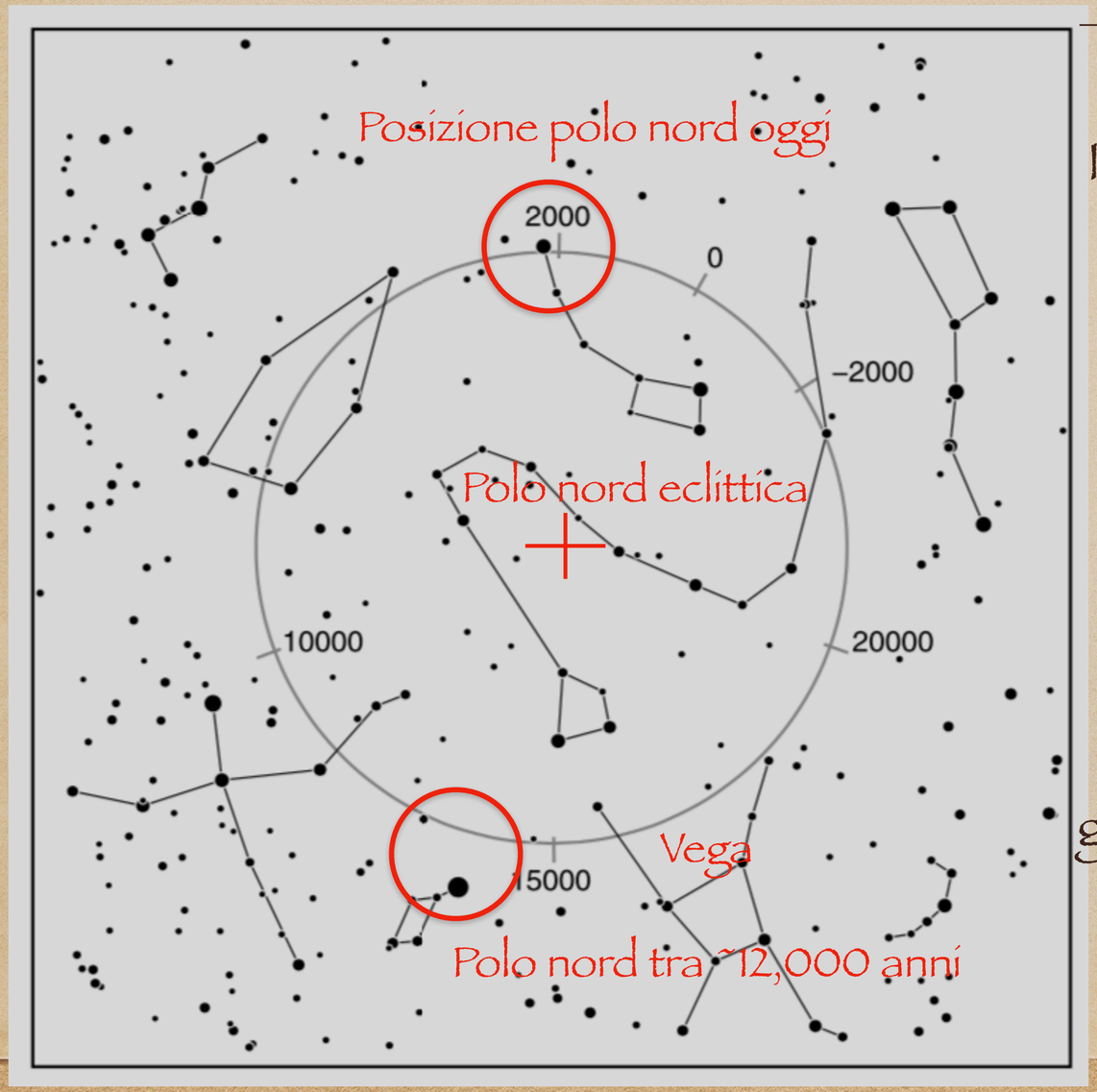
A spinning top rotates around its axis, but the force of gravity applied to the (non-vertical) axis changes direction to the axis itself (precession in the same direction of rotation)



The same happens to the Earth, subjected to the forces due to the Sun and Moon (precession in the opposite direction to the rotation)

The direction of the axis changes over time (but the angle of inclination of the axis does not change, which remains 23° 26')

Precession



It does not change the rotation angle of the Earth axis

→ the equator will never coincide with the ecliptic

In its precession motion, the axis drags the point γ:

Full cycle ~26,000 years, ~50"/year

→ the ecliptic longitude λ increases by ~50"/year

→ α and δ of objects change accordingly

We need to know for what era the coordinates are given: usually, J2000.0 (which means the beginning of the year 2000, or rather noon on January 1, 2000)

Precession

From the transformation equations between the equatorial system and the ecliptic

$$\sin\delta = \cos\epsilon \sin\beta + \sin\epsilon \cos\beta \sin\lambda$$

Differentiate (N.B. $\epsilon = cost, \beta = cost$) :

$$\cos\delta d\delta = \sin\epsilon \cos\beta \cos\lambda d\lambda$$

Take $\cos\lambda \cos\beta = \cos\delta \cos\alpha$

Substitute on the right

$$\cos\delta d\delta = \sin\epsilon \cos\delta \cos\alpha d\lambda$$

And thus:

$$d\delta = \sin\epsilon \cos\alpha d\lambda$$

Variation of the declination for dλ

$$\cos\alpha \cos\delta = \cos\beta \cos\lambda$$

Differentiate (N.B. $\beta = cost$) :

$$-\sin\alpha \cos\delta d\alpha - \cos\alpha \sin\delta d\delta = -\cos\beta \sin\lambda d\lambda$$

We use: $d\delta = \sin\epsilon \cos\alpha d\lambda$

$$\cos\beta \sin\lambda = \sin\delta \sin\epsilon + \cos\delta \cos\epsilon \sin\alpha$$

$$-\sin\alpha\cos\delta\, d\alpha - \cos\alpha\sin\delta\sin\varepsilon\cos\alpha\, d\lambda = -\sin\delta\sin\varepsilon\, d\lambda - \cos\delta\cos\varepsilon\sin\alpha\, d\lambda$$

$$\sin\alpha\cos\delta\, d\alpha = \left(-\cos^2\alpha\,\sin\delta\sin\varepsilon + \sin\delta\sin\varepsilon + \cos\delta\cos\varepsilon\sin\alpha\right) d\lambda$$

$$\sin\alpha\, d\alpha = \left(-\cos^2\alpha\,\tan\delta\sin\varepsilon + \tan\delta\sin\varepsilon + \cos\varepsilon\sin\alpha\right) d\lambda$$

$$\sin\alpha\, d\alpha = \tan\delta\sin\varepsilon\left(1-\cos^2\alpha\right) d\lambda + \cos\varepsilon\sin\alpha\, d\lambda$$

$$d\alpha = d\lambda(\sin\epsilon \sin\alpha \tan\delta + \cos\epsilon)$$

Variation of the right ascension for dλ

Precession

$$d\delta = \sin\epsilon \cos\alpha d\lambda$$

Variation of the declination for dλ

$$d\alpha = d\lambda(\sin\epsilon \sin\alpha \tan\delta + \cos\epsilon)$$

Variation of the right ascension for dλ

dλ ≈ 50"/anno

sin ε ≈ cost

cos ε ≈ cost

$$d\delta = n\cos\alpha$$
$$d\alpha = m + n\sin\alpha\tan\delta$$

con

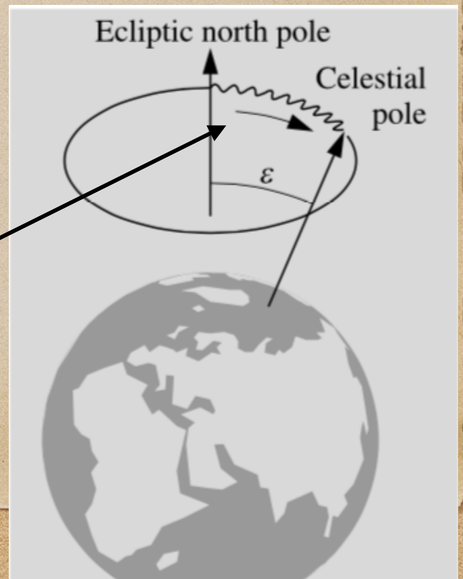
$$m = d\lambda\cos\epsilon$$
$$n = d\lambda\sin\epsilon$$

Precession constants

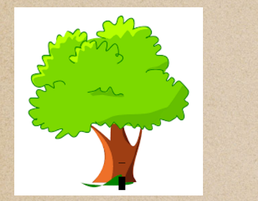
Table 2.1 Precession constants m and n. Here, "a" means a tropical year

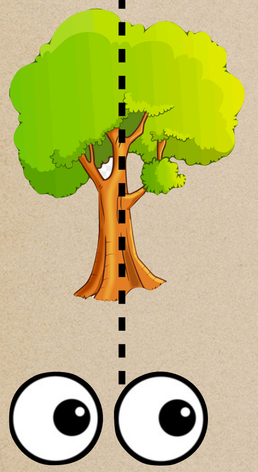
Epoch	m	n	
1800	3.07048 s/a	1.33703 s/a =	20.0554″/a
1850	3.07141	1.33674	20.0511
1900	3.07234	1.33646	20.0468
1950	3.07327	1.33617	20.0426
2000	3.07419	1.33589	20.0383

Not exactly constant due to the nutation



Parallax

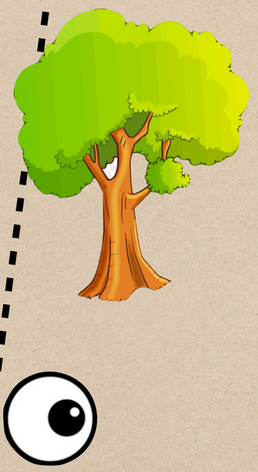


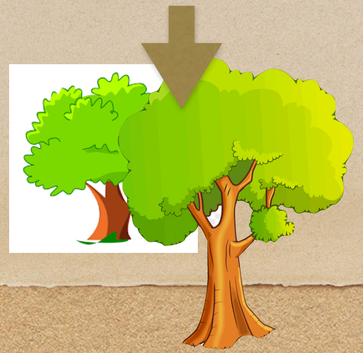




What do I see

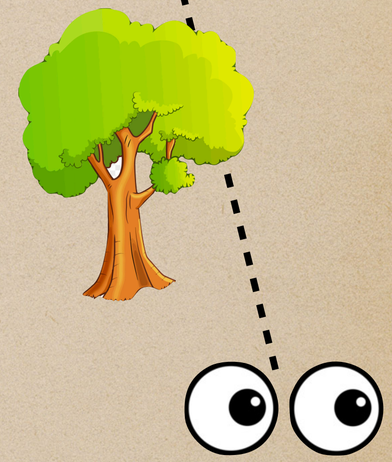


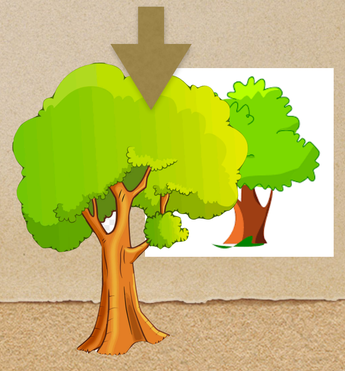




What do I see







What do I see

Parallax

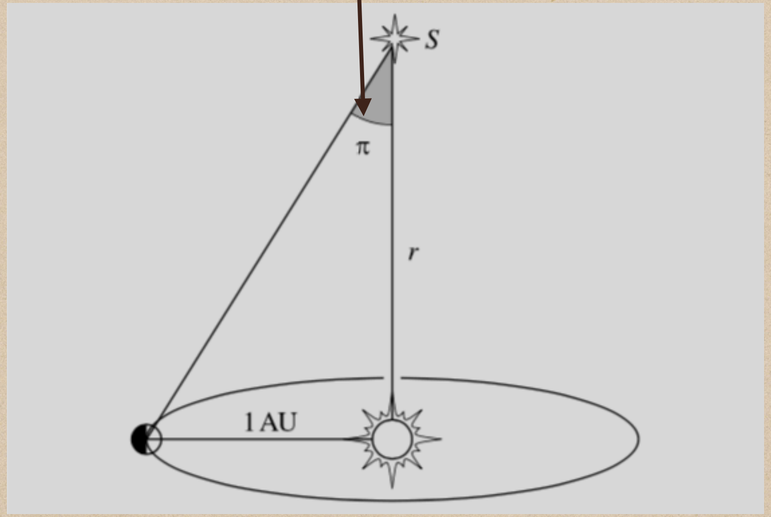
The effects of parallax change the coordinates of objects

The closer the object, the greater the effect

Annual parallax: due to the motion of revolution of the Earth around the Sun (useful for measuring distances from the nearest stars)

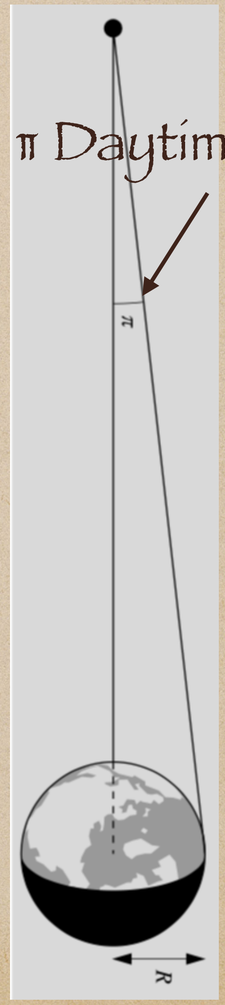
Daytime parallax: due to the motion of rotation of the Earth around its axis (also depends on the latitude of the observer)

π Annual parallax



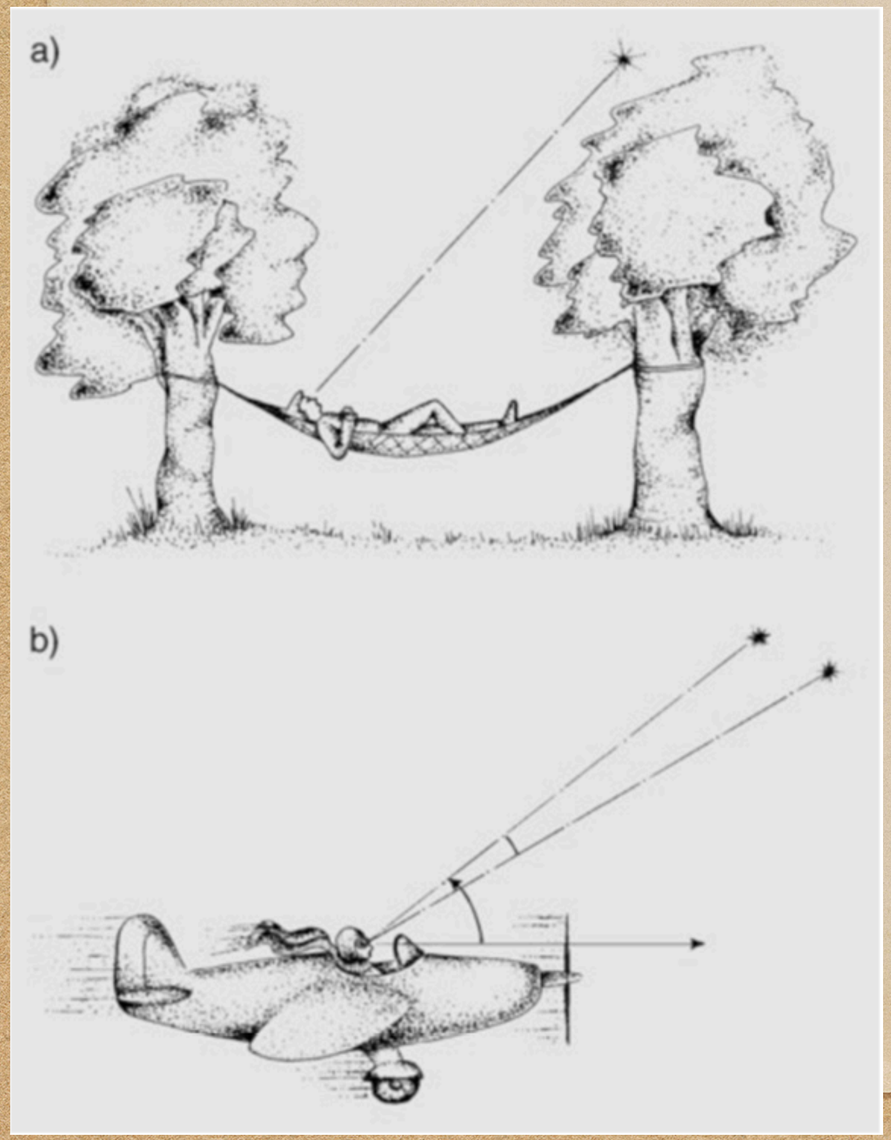
R = 1 AU = 1.5×10^{11} m

π Daytime parallax



R = 6378 km

Aberration



Due to the fact that the speed of light is limited, an observer in motion sees an object moving towards the direction of its motion

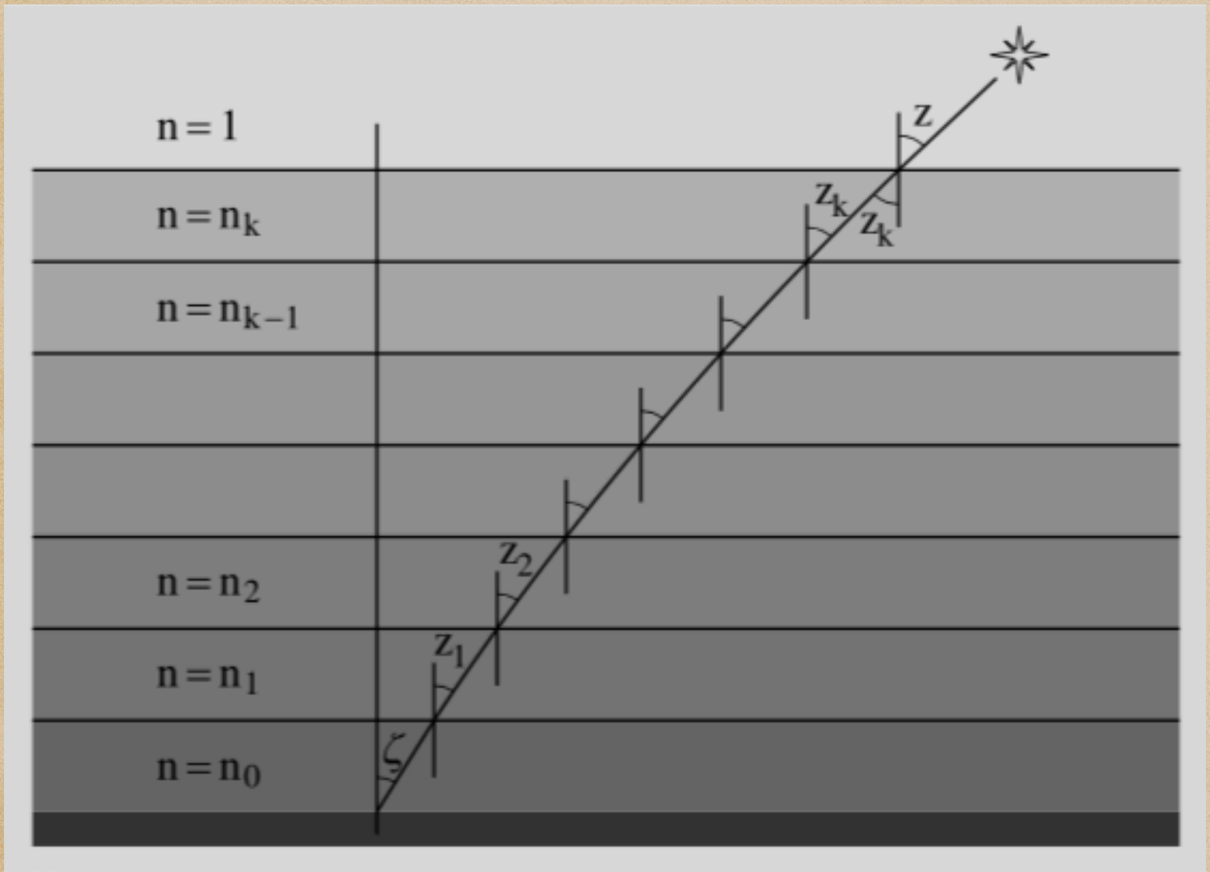
$$a = \frac{v}{c} \sin\theta, \qquad [a] = \text{rad},$$

θ True direction of the object

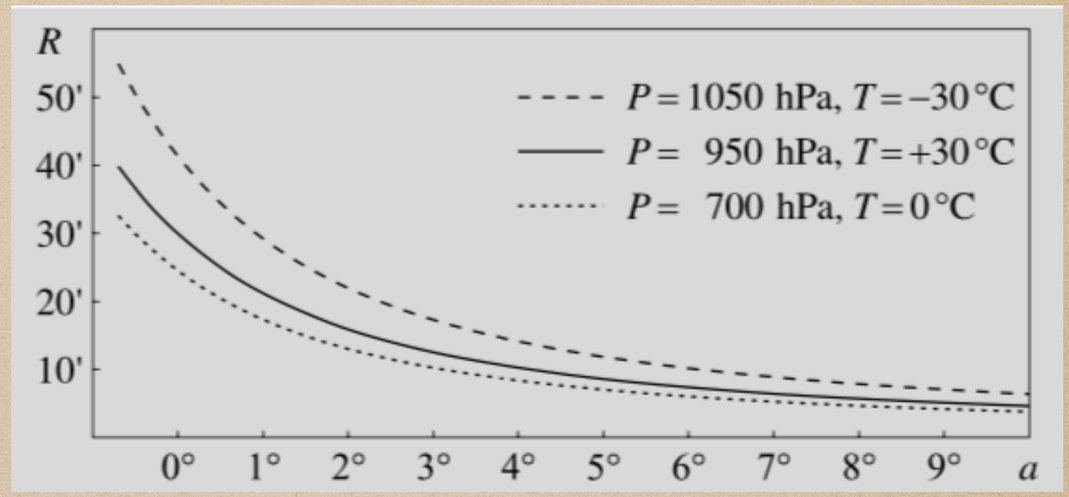
Earth revolution speed: 30 km/s → a_{max} ~ 20”

Earth rotation speed: 1668 km/h → a_{max} ~ 0.3”

Atmospheric refraction



The light enters the atmosphere with an angle z with respect to the vertical, and reaches the ground with an angle ζ < z (the height always increases)



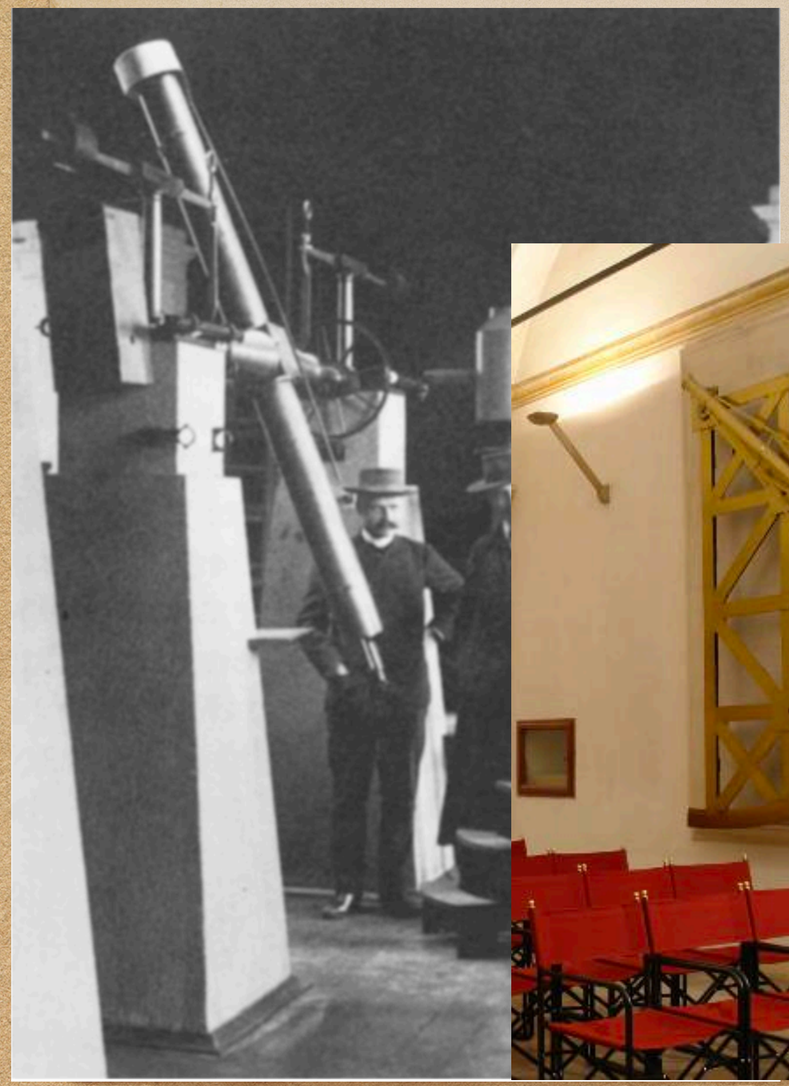
Let's assume for simplicity that the atmosphere is a superposition of layers of different densities (and therefore different refractive indices ni)

The effect is greater as the height a decreases

Absolute position

Meridian

Telescope that moves only in the meridian plane





When a star culminates, the sidereal time Θ of the transit is marked Considering that at the passage from the meridian h=0, from:

$$\Theta = h + \alpha$$

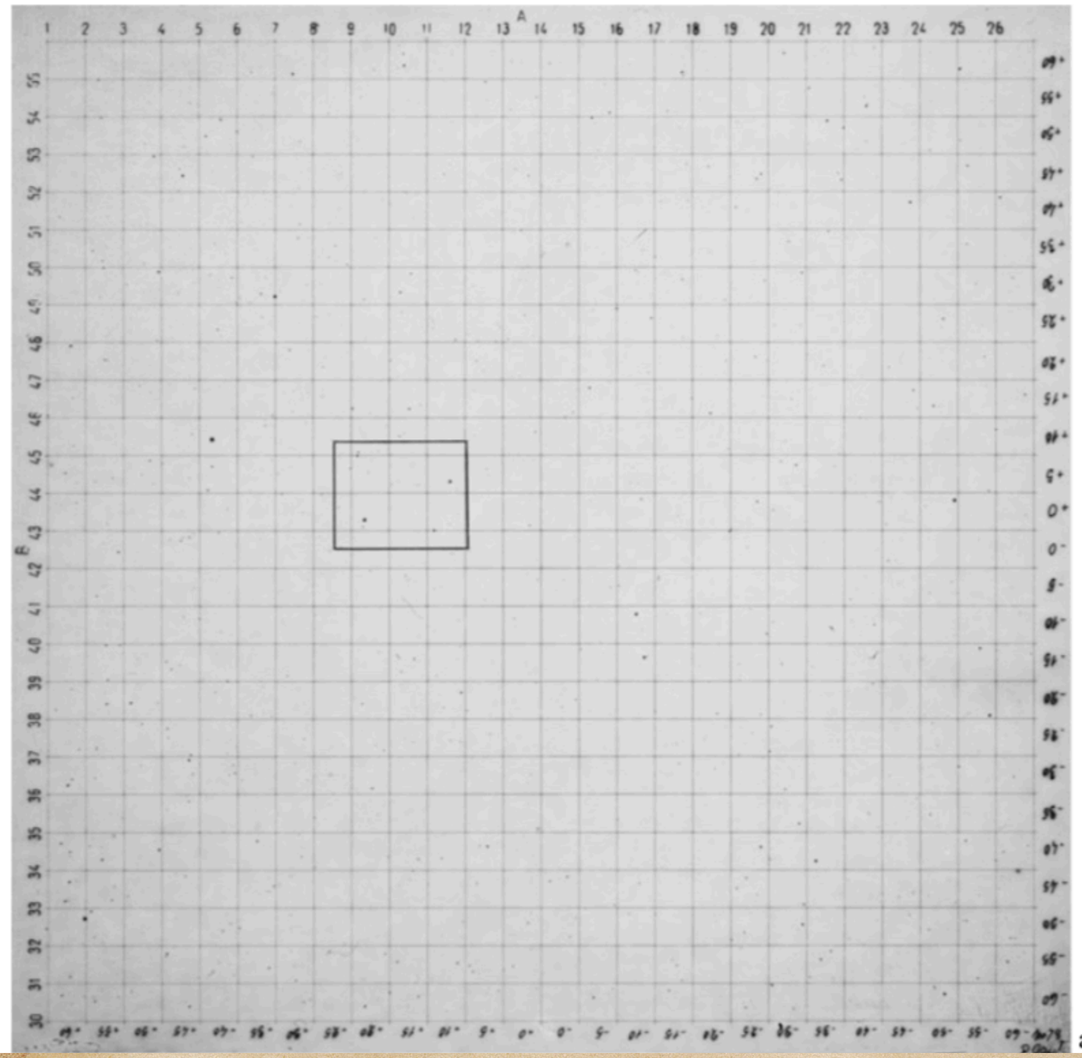
I get the right ascension α. The declination is obtained from

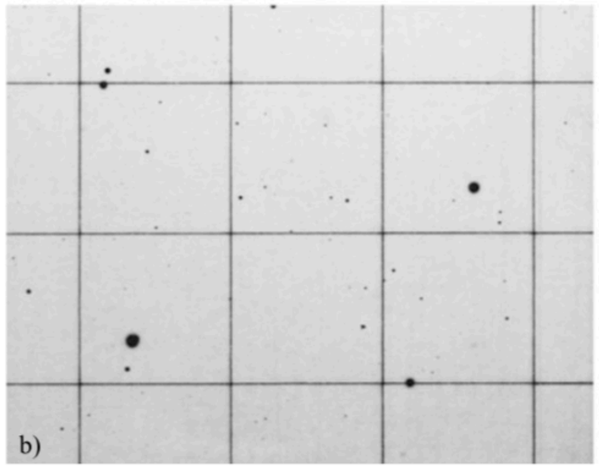
$$\delta = a - (90^\circ - \phi)$$

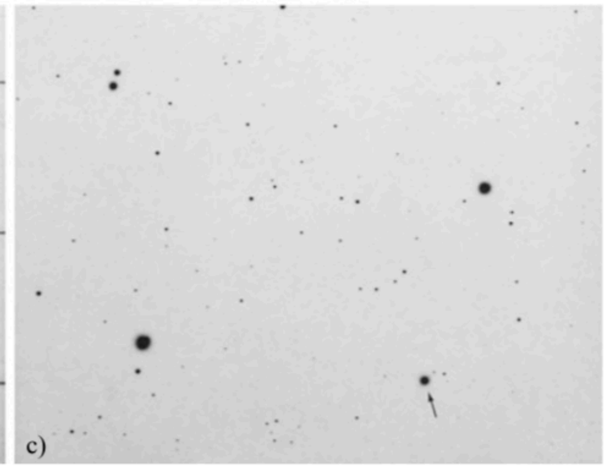
Where a is the observed height and ϕ is the latitude of the place

Absolute coordinates

Relative coordinates





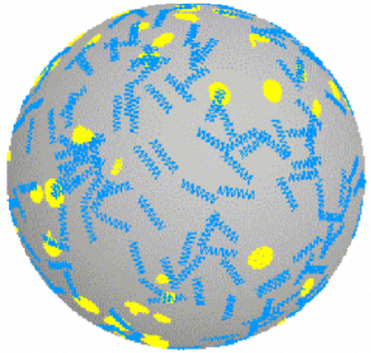


The relative coordinates are measured on photographic cards

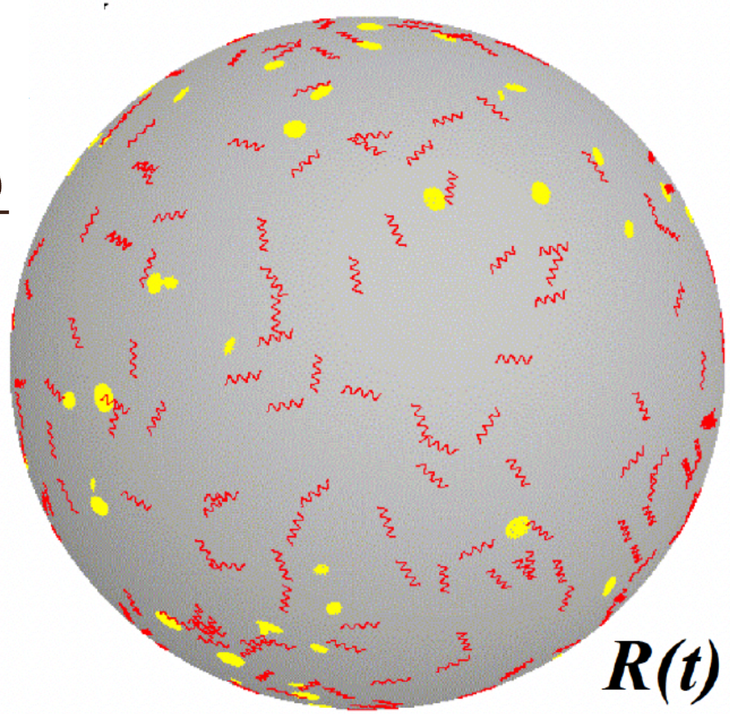
- The card is "calibrated" using known stars
- You get the positions of all the other objects

In cosmology you will be faced with an evolving (expanding) coordinate system

Universe dimensions are proportional to a time dependent function: the scale factor R(t) or a(t)



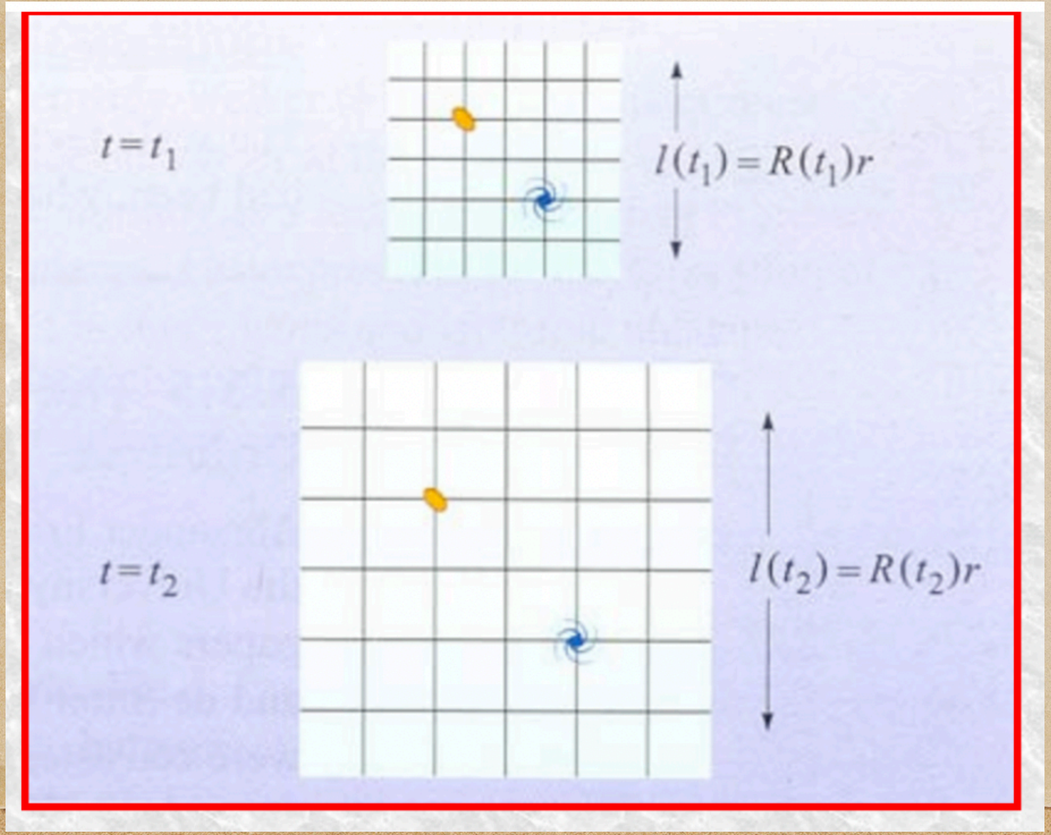
R(t)

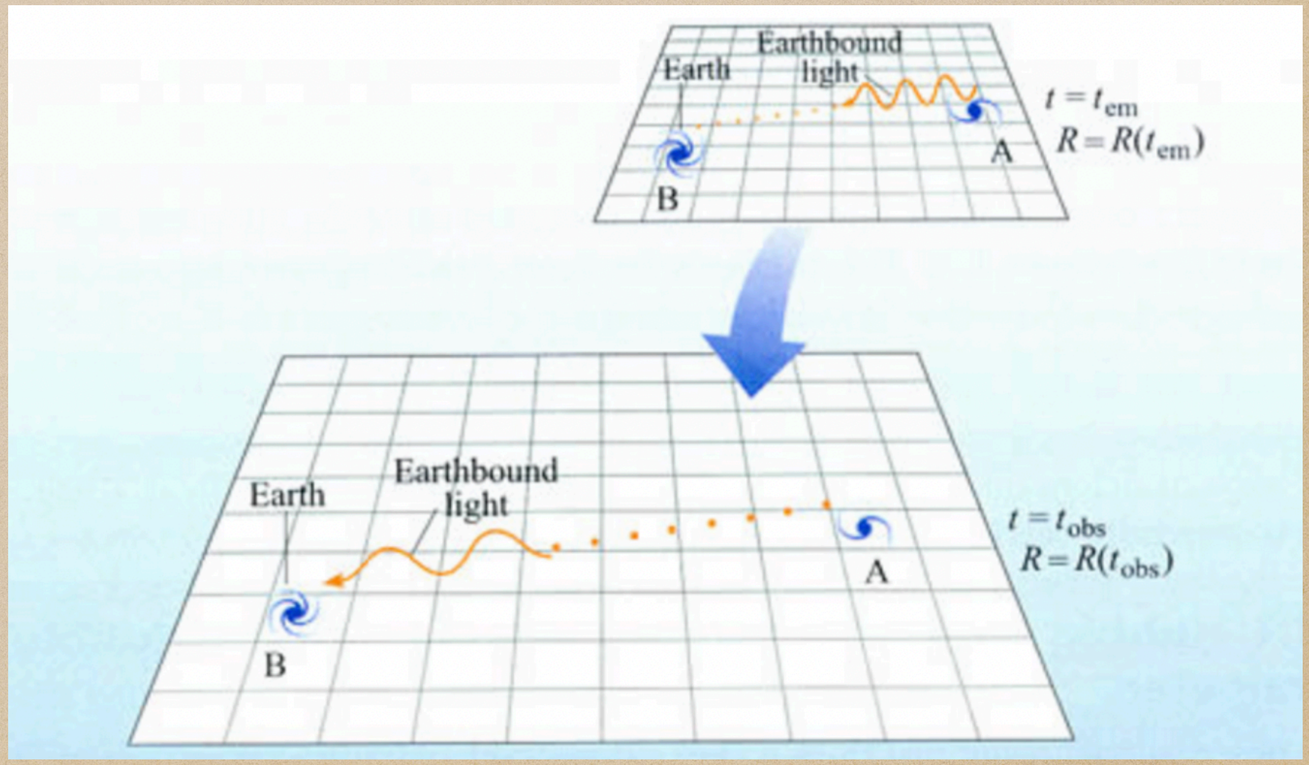


R(t)

Need to introduce a time independent system to simplify the metrics:

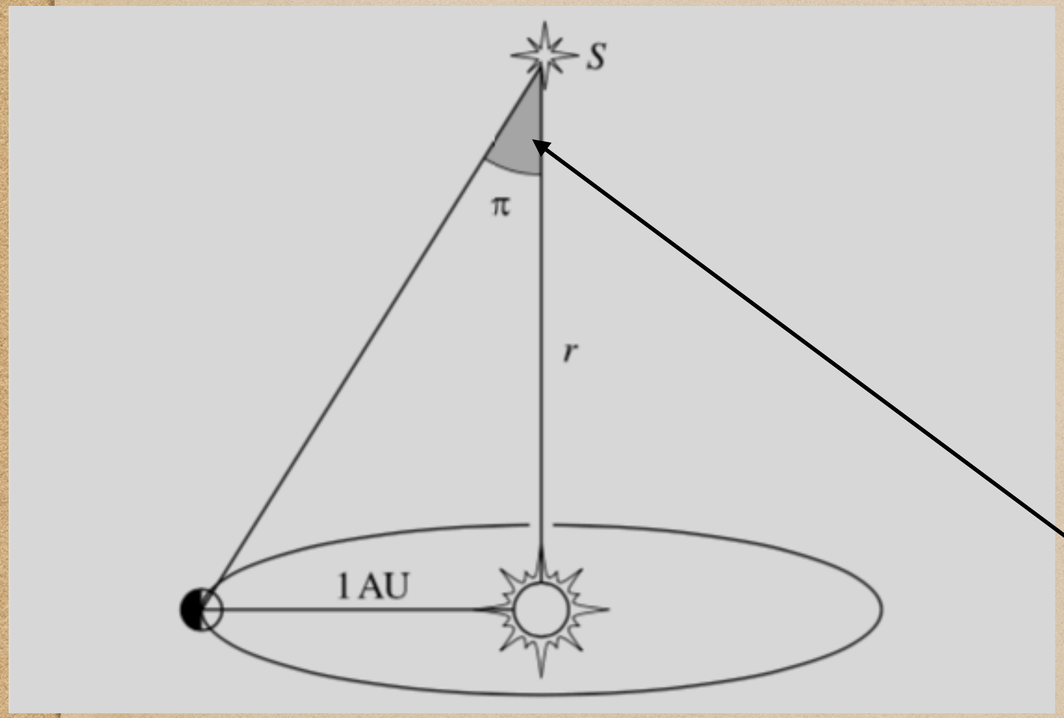
the COMOVING COORDINATES





Parallax: distance measures

π Annual parallax



R = 1 AU = 1.5x10^{11} m

It is the classic method for measuring the distance of neighboring stars, and is the basis of the "distance scale"

Over the course of a year:

- A star at the ecliptic pole makes a perfect circle;
- A star on the ecliptic moves on a segment;
- Another star moves on an ellipse

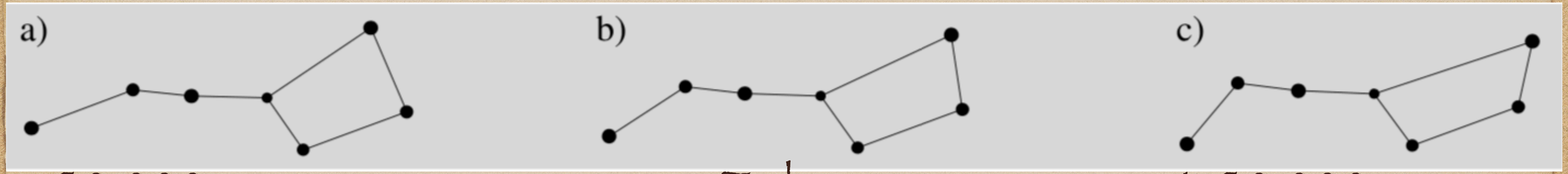
The major semi-axis of this ellipse is called parallax π; it is equal to the angle underlying the radius of the Earth's orbit seen from the star

Distances in astronomy are measured in parsec, pc:

1 pc is the distance r at which 1AU is seen at an angle of 1"

1 rad = 206265" → 1 pc = 206265 AU (3.086x10^{16} m)

Proper motions

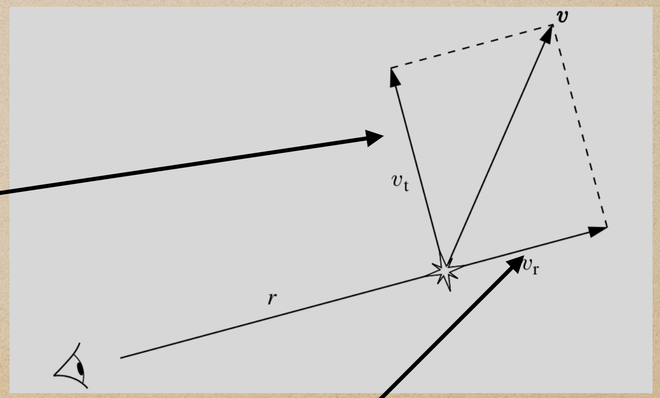


30,000 years ago | Today | In 30,000 years

Proper motion
In addition to the parallax motion, some stars move in a constant direction

This motion reflects the true motion of the star with respect to the Earth-Sun system, and in particular the transverse motion

It can be determined by monitoring the position of the stars over time



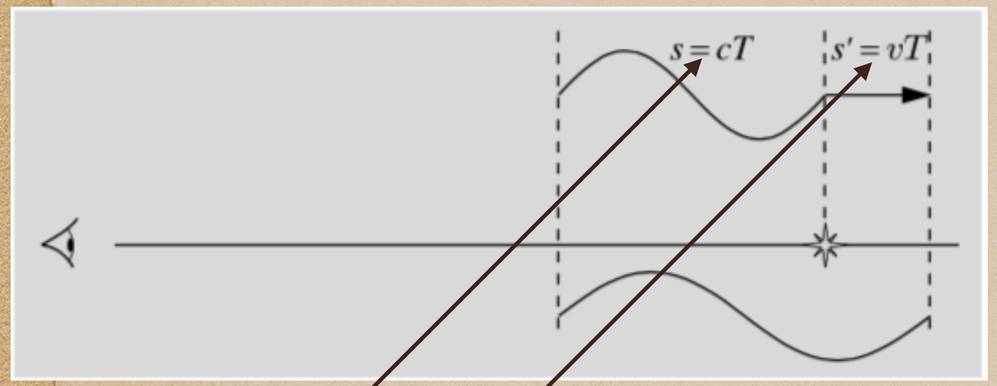
How do I determine the radial motion?

Doppler effect

A source emits radiation of temporal period T, and moves away from the observer with speed v

$$\lambda_0 = cT$$

λ_0 is the wavelength that would be measured if v=0



$$\Delta\lambda = \lambda - \lambda_0 = cT + vT - cT = vT$$

thus: $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$

Valid only in the non-relativistic case v<<c

$$\lambda = s + s' = cT + vT$$

λ is the wavelength measured by the observer

$$\frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

Valid for all speeds

$\Delta\lambda$

Positive is v>0, λ increases: redshift

Negative if v<0, λ decreases: blueshift

Time measurement

Time measurement

Rotation of the Earth → Solar time

Revolution of the Earth around the Sun → Sidereal time

Atomic clocks

Its flow is as constant as the earth's rotation speed is constant; this speed decreases, therefore the duration of the sidereal day increases

Apparent sidereal time: depends on the true point γ, it can be obtained from observations

Average sidereal time: determined by the position of γ if there was no nutation

<u>Sidereal day:</u>

Time needed for the stars to return to the same position in the sky

Practical definition: time between two successive culminations of the point γ at the meridian

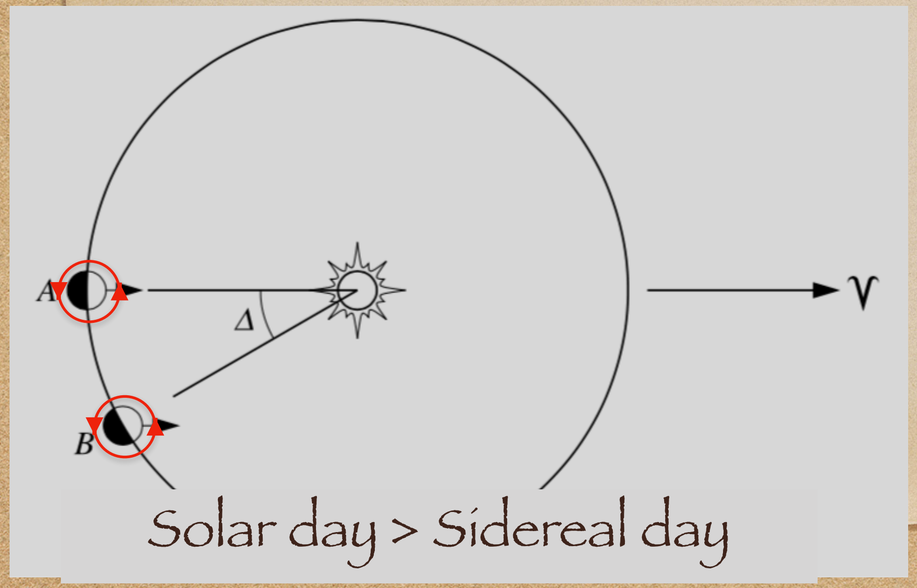
It lasts less than a solar day (the γ point takes less than the Sun between 2 successive culminations)

Time measurement

After a rotation, the Earth is in the same position with respect as the γ point

In that time interval, the Earth will have moved in its orbit around the sun by an angle Δ (exaggerated in the figure)

So you have to do a little more rotation to get back to the same position with respect to the Sun



Solar day > Sidereal day

Solar time

Our life follows the alternation of day and night, so it is more practical to use solar time

Problem: Solar time does not flow at a constant pace

- The orbit of the Sun is elliptical and not circular, therefore its speed of revolution is not constant
- The Sun moves along the ecliptic and not along the equator, so its RA does not increase steadily

Solution: We invent a "Mean Sun", which:

- Moves at a constant angular velocity
- Moves along the Equator
- Makes a tour in a tropical year (time needed to go from equinox to equinox, 365d 5h 48m 46s)*

Average solar time

Hour angle of the center of the average Sun +12h

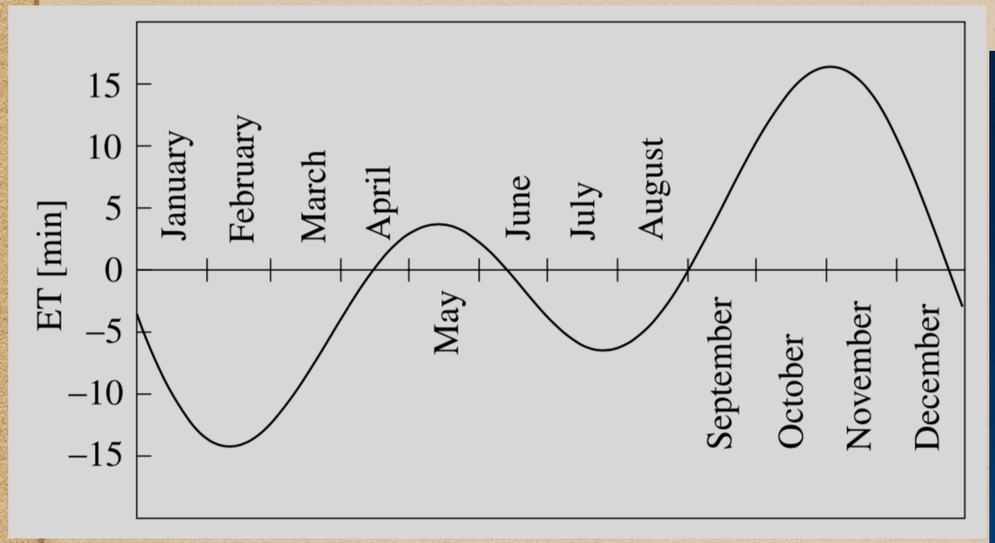
$T_M = h_M + 12h$

* Tropical year ≠ sidereal year due to precession; sidereal year = time when the Sun returns to the same position with respect to the stars

Time equation

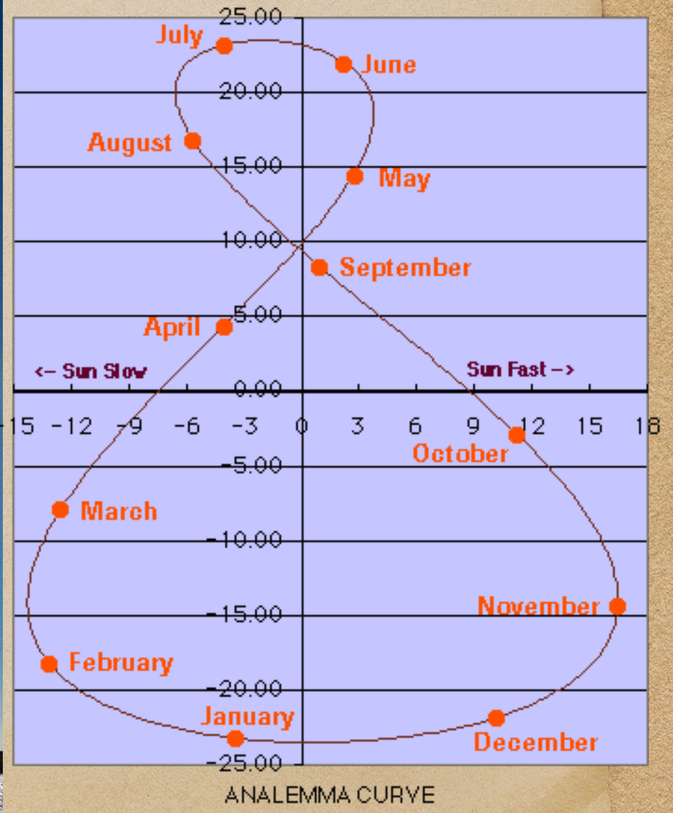
The difference between true solar time and average solar time is the time equation:

$$E.T. = T - T_M$$



Analemma: position of the Sun at the same average solar time over the course of a year





Local time and time zones

The time marked by digital clocks most of the time differs from both the true and average solar time (and even less from the sidereal one)

The time of daily use is divided into zones or time zones

