Fundamentals of Astrophysics and Cosmology

Academic year 2025/2026

- Prof. Nicola Bartolo: nicola.bartolo@unipd.it
- Prof. Giulia Rodighiero: giulia.rodighiero@unipd.it
- Tutor Dr. Ripalta Amoruso: ripalta.amoruso@phd.unipd.it

Concepts of spherical astronomy and celestial mechanics

- Spherical trigonometry, altazimuthal and equatorial systems, ecliptic system.
- Galactic coordinates. Absolute and relative positions. Precession. Parallax.
- Measurement of time.
- Kepler’s laws. Orbital elements.

Radiative mechanisms and magnitudes

- Electromagnetic radiation. Intensity and radiation density, luminosity.
- Blackbody law. Stellar spectra. Stefan–Boltzmann law.
- Magnitudes and photometric systems.
- Interstellar absorption.

Elements of stellar evolution

- Spectral classes. Hertzsprung–Russell diagram.
- Main sequence, supergiants, giants, white dwarfs.
- Scaling relations: temperature, radius, luminosity, distance.
- Stellar structure equations. Radiative transport.
- Stellar nucleosynthesis.
- Evolutionary timescales of stellar evolution.
- Formation of a protostellar system (Jeans theory).
- Evolution of the Sun. Final stages for massive stars.

The Milky Way and the interstellar medium

- Components and structure of the Milky Way.
- Gas cycle in our galaxy. Components of the interstellar medium.
- Spiral arm kinematics, evidence of Dark Matter.
- The Galactic Center

External galaxies

- Galaxies in the local Universe. Hubble morphological sequence.
- Correlations between morphology and physical properties.
- Galaxies across different wavelengths.
- Spectral energy distributions (SEDs). Introduction to stellar population synthesis. IMF, stellar mass, star formation rate (SFR).
- Overview of galaxy evolution. Galaxy clusters.

From the local Universe to cosmological distances: the distance scale

- Parallax, standard candles.
- Cepheids and Supernovae.
- Introduction to Hubble's expansion law and the concept of redshift.

Fundamental concepts of Cosmology

- Main components of the Universe. Observational evidence for dark matter and dark energy.
- Expanding Universe and the Cosmological Principle.
- Robertson–Walker metric. Geometrical properties.
- Hubble constant and deceleration parameter.
- Definitions of distance in Cosmology; derivation of Hubble's law (approximate treatment at low redshift).
- Newtonian derivation of Friedmann equations and relativistic corrections.
- Friedmann models.
- The cosmological constant: Einstein's static solution and de Sitter model.
- Solutions for the flat case and Universes with non-zero spatial curvature.
- Exact treatment of Hubble's law.

Thermal history and the early Universe

- Number density, energy density, and pressure for a system of particles in thermal equilibrium.
- Conservation of entropy in a comoving volume.
- Temperature–time relation in the early epochs.
- Problems of the standard model: horizon, flatness, etc.
- “Inflation” in the early Universe. Solution to the horizon and flatness problems (overview).
- Kinematics and dynamics of inflation: the “inflaton” (overview).
- Old and new inflation, chaotic inflation; slow-roll dynamics (overview).
- Baryon asymmetry. Baryogenesis (overview).
- General definition of “decoupling.”
- Primordial nucleosynthesis of light elements.
- Hydrogen recombination: Saha equation. Radiation decoupling. Cosmic Microwave Background (CMB).

Dark matter: general properties

- Basic concepts of the Boltzmann equation in cosmology and “cosmic relics.”
- Hot and cold dark matter: definition, calculation of present abundance, and general properties in cosmology (overview).

Formation of cosmic structures

- Linear evolution of perturbations in the expanding Universe (fundamental principles).
- Spherical collapse of a protostructure.
- Mass function of cosmic structures: Press–Schechter theory.

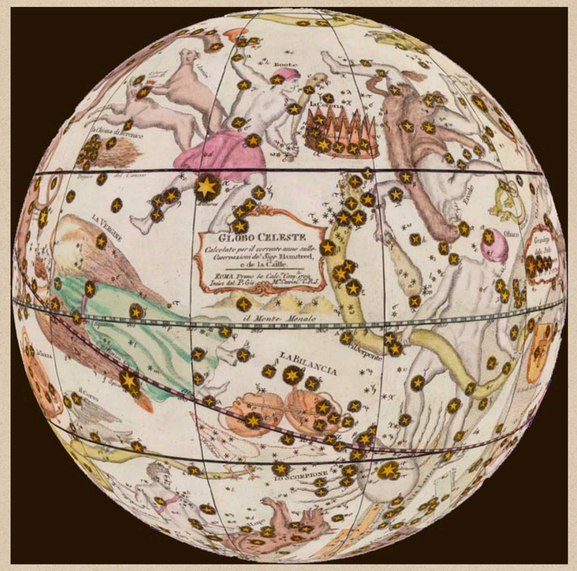
Study materials:
Relevant sections of the reference texts will be indicated during lectures (and can be inferred from the programme topics). Lecture notes prepared by the instructors on many topics will also be made available via the teaching website, accessible from the e-learning platform of the Department of Physics and Astronomy “G. Galilei” (https://stem.elearning.unipd.it/). Reference textbooks can be consulted at the Department library.

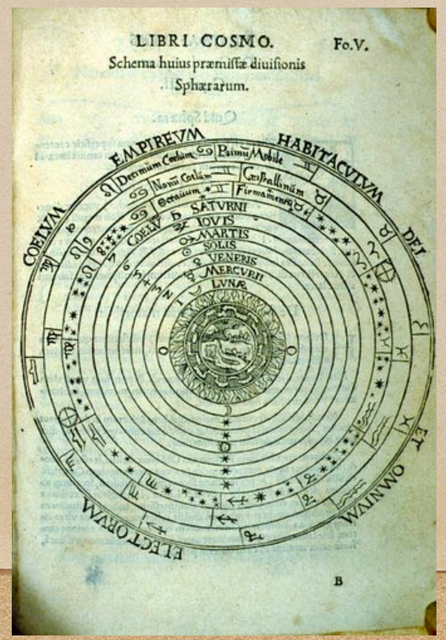
Reference texts:

- *Fundamental Astronomy*, by H. Karttunen, H. Oja, M. Poutanen, K. J. Donner
- *The Cosmic Perspective*, by J. O. Bennett, M. O. Donahue, N. Schneider & M. Voit
- *Extragalactic Astronomy and Cosmology*, by P. Schneider, 2nd edition
- Coles P., Lucchin F., *Cosmology: The Origin and Evolution of Cosmic Structure*, Wiley & Sons, 2002
- Kolb E.W., Turner M., *The Early Universe*, Addison-Wesley, 1990

The celestial sphere







The stars appear displaced on a sphere; therefore the study of their position and their motion is based on spherical trigonometry

We are only interested in the directions in which the stars appear to us, we do not know (and here we are not interested) in the distances

So we only need two coordinates to identify the points on the celestial sphere

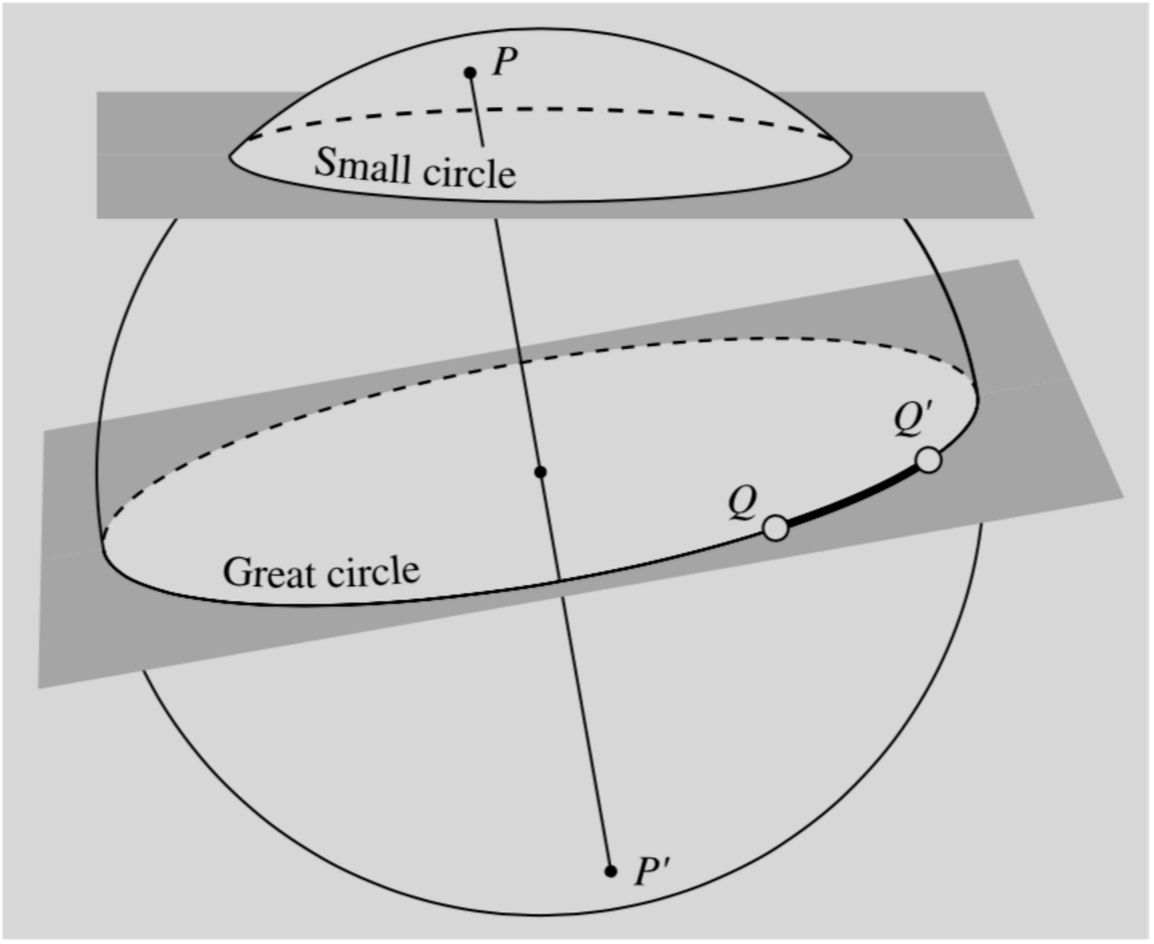
Spherical astronomy (1)

Topic 1

Material from

Chap. 2 "Fundamental Astronomy" edition, by H. Karttunen, H. Oja, M. Poutanen, K. J. Donner

Chap. 2&S1 "The cosmic perspective", by J. O. Bennett, M. O. Donahue, N. Schneider & M. Voit



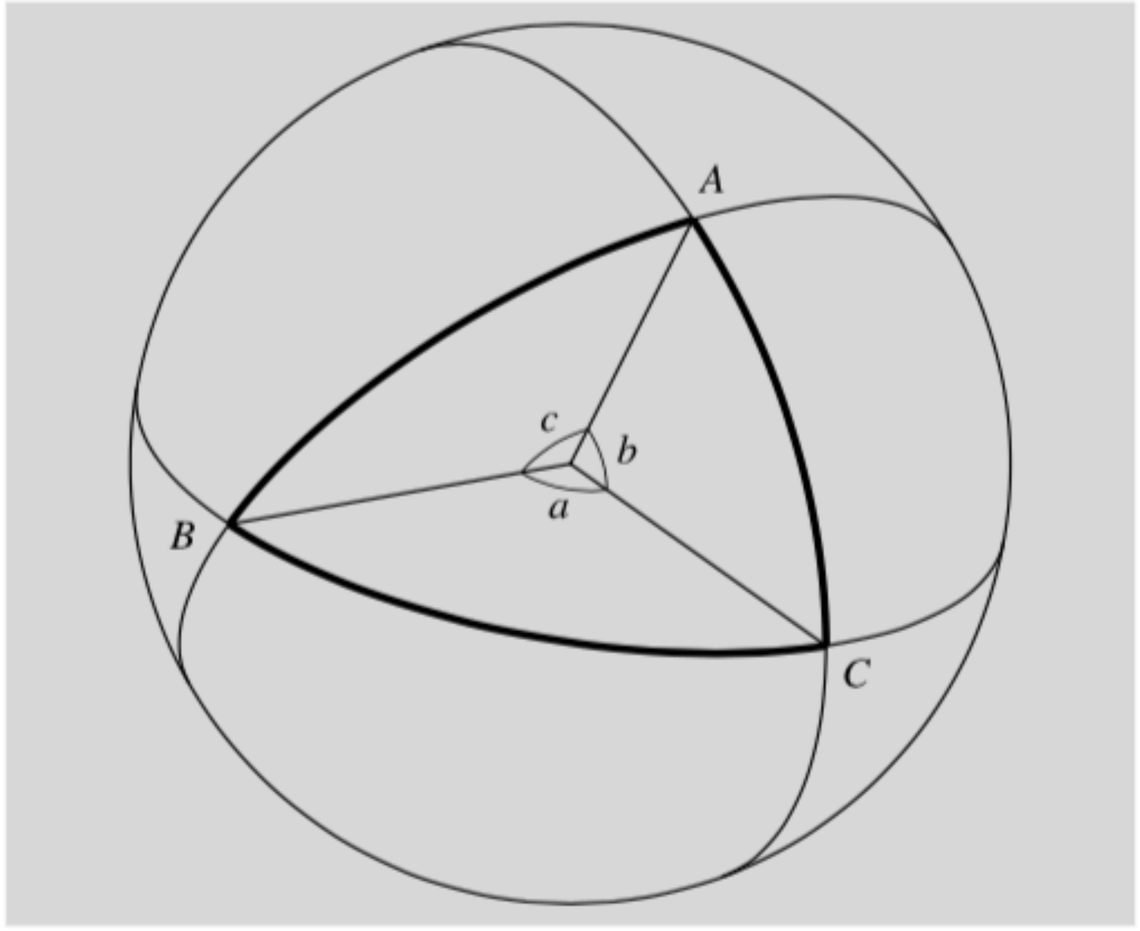
Maximum circle

Intersection between the sphere and the plane passing through its center

A straight line perpendicular to this passing through the center intercepts the sphere in the poles

Minor circle

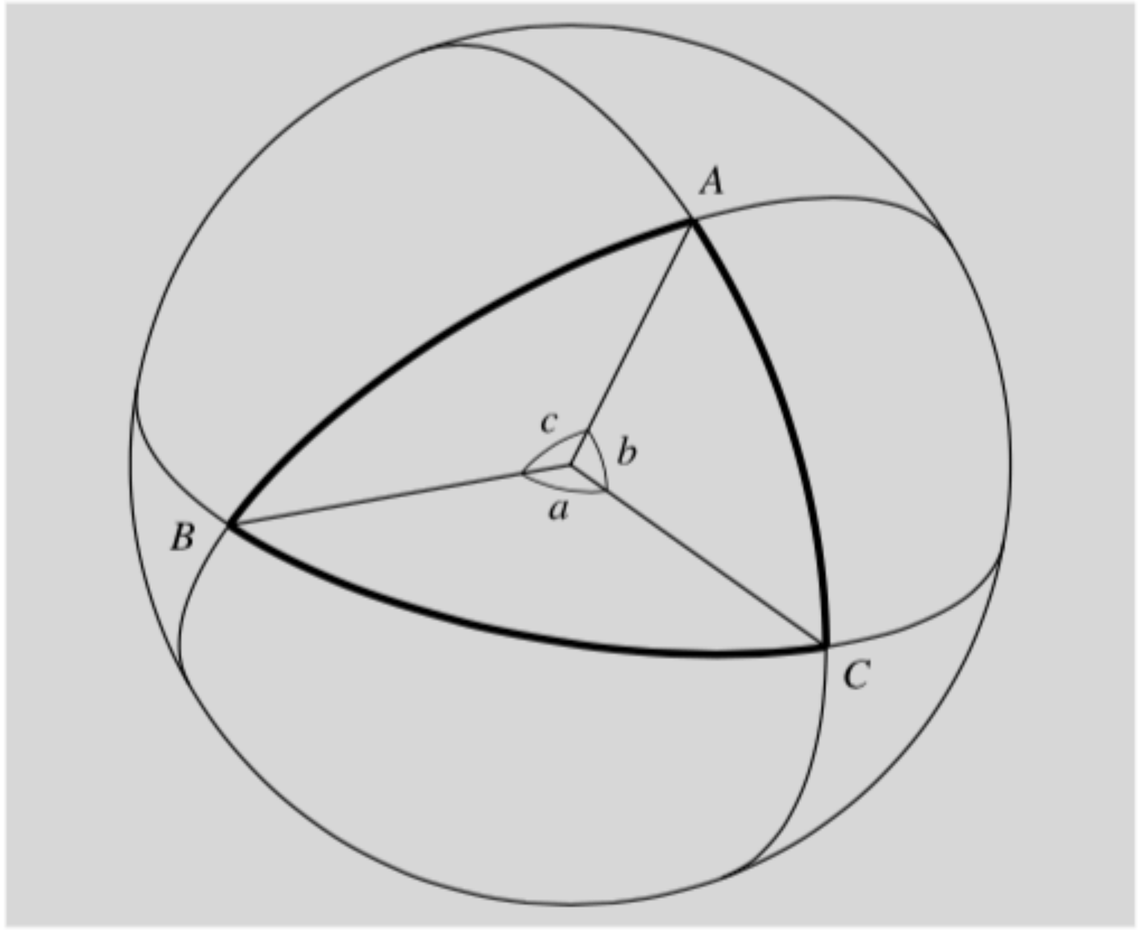
Intersection between sphere and plane not containing its center



Spherical triangle
Portion of sphere defined by 3 maximum circles

$$|AB| = rc \qquad [c] = rad$$

C central angle, r radius of the sphere
Bi-unique correspondence between |AB| and c; therefore the angle is usually used to indicate the distance between A and B; r radius of the sphere does not enter the equations



Spherical triangle

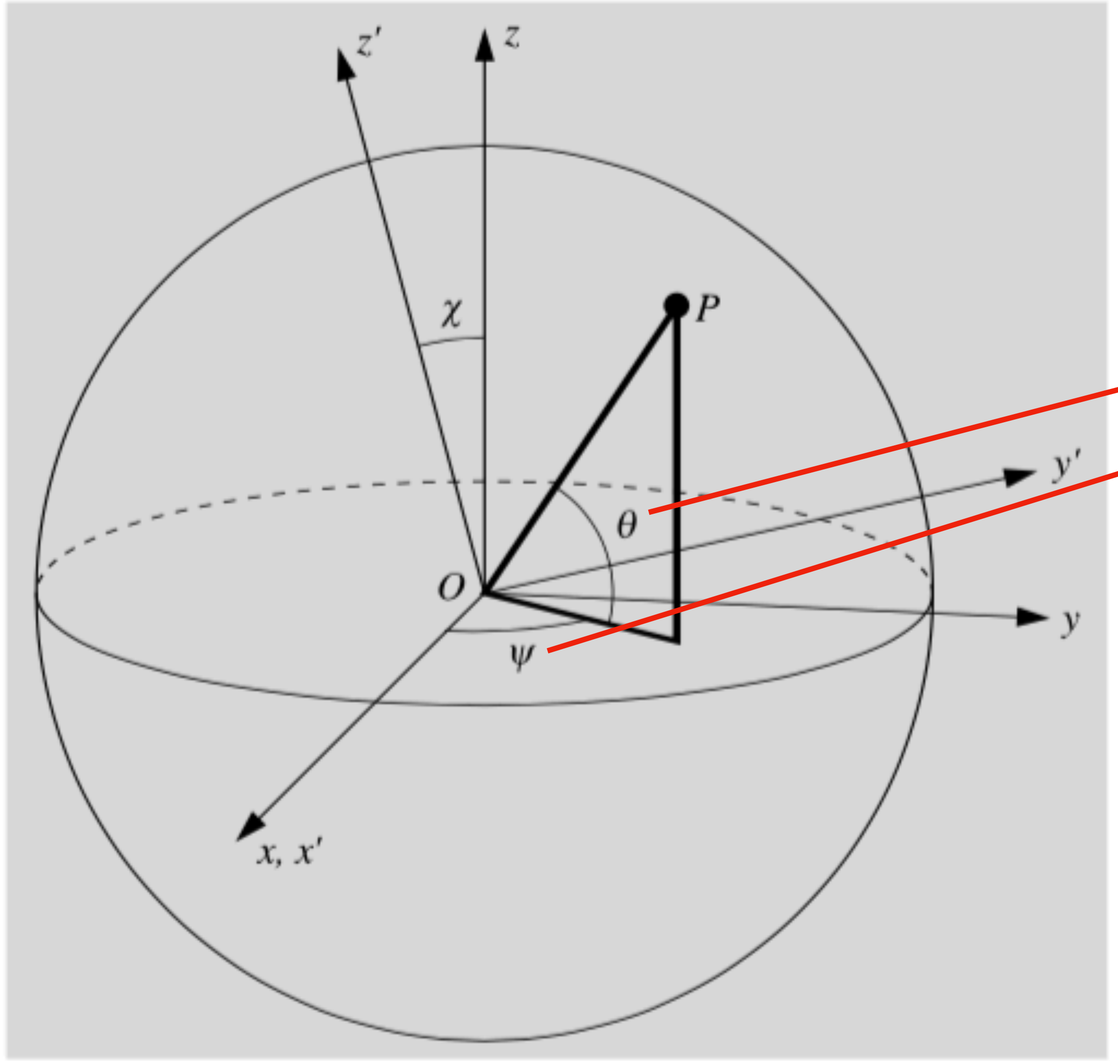
A, B, C: Spherical triangle angles; a, b, c: corresponding central corners

$$A + B + C > 180°;$$

$$E = A + B + C - 180°$$

E eccesso sferico

$$\text{Area}(ABC) = Er^2 \qquad [E] = rad$$



Position of a point P on the sphere

- cartesian coordinates x, y, z;
- 2 angles ψ, θ

θ Distance from the plane xy

ψ counter-clockwise starting from the positive axis x;

Let's consider the system x', y', z', Rotated around the x-axis of an angle χ

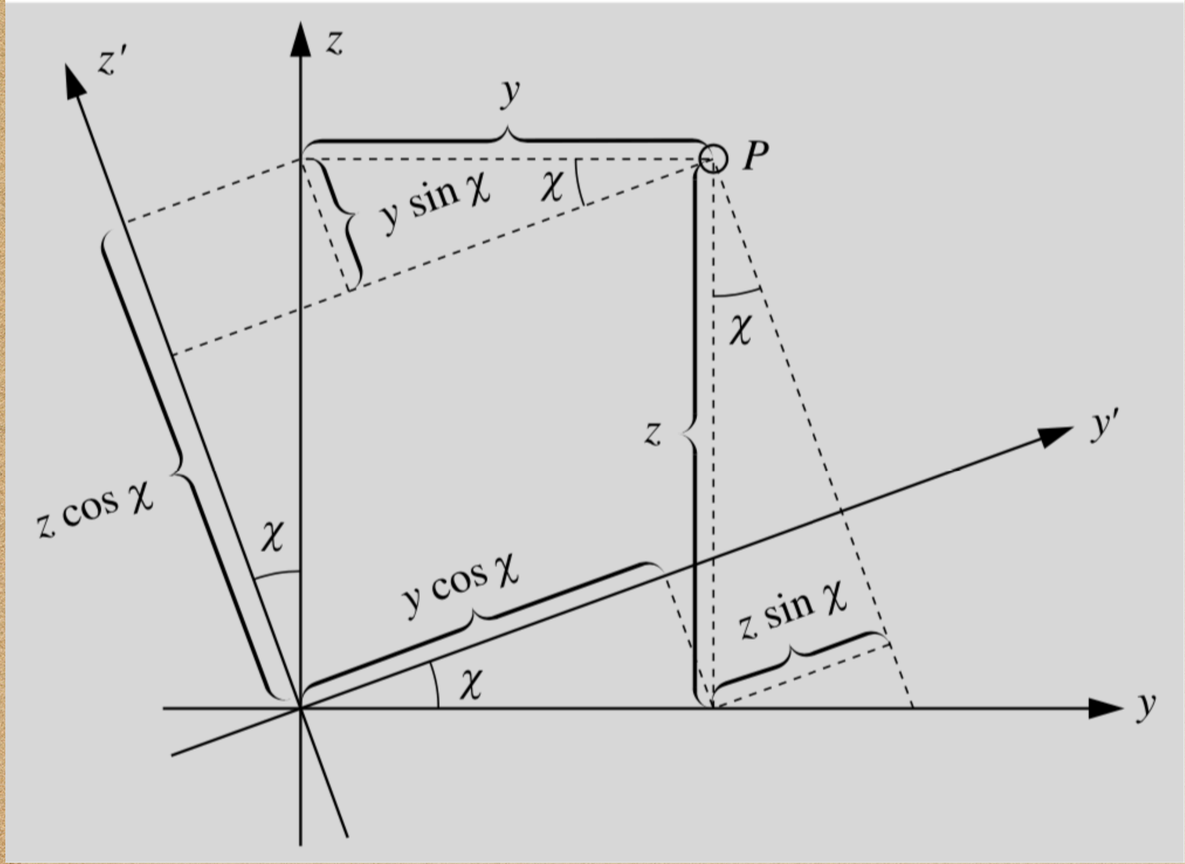
- Cartesian coordinates x', y', z';
- 2 angles ψ', θ'

$$x = \cos\psi\cos\theta \qquad x' = \cos\psi'\cos\theta'$$

$$y = \sin\psi\cos\theta \qquad y' = \sin\psi'\cos\theta'$$

$$z = \sin\theta \qquad z' = \sin\theta'$$

Spherical trigonometry



Transformation between system x,y,z and x',y',z'

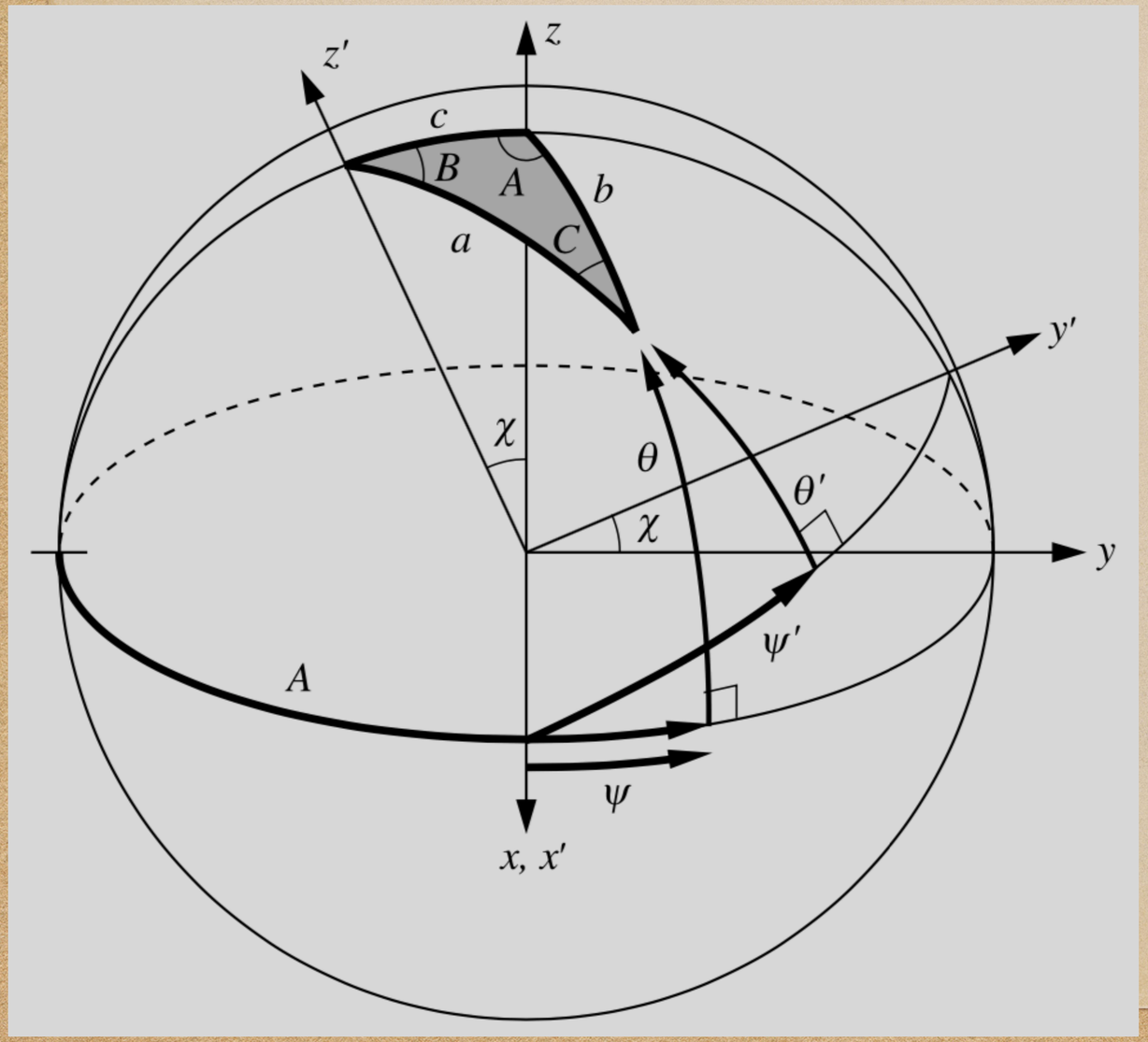
$$x = x'$$
$$y' = y\cos\chi + z\sin\chi$$
$$z' = z\cos\chi - y\sin\chi$$

$$x = \cos\psi\cos\theta \qquad x' = \cos\psi'\cos\theta'$$
$$y = \sin\psi\cos\theta \qquad y' = \sin\psi'\cos\theta'$$
$$z = \sin\theta \qquad z' = \sin\theta'$$

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = \sin\theta\cos\chi - \sin\chi\sin\psi\cos\theta$$

Useful for coordinate transformations that we will see later

Trigonometria sferica



As before, let's consider 2 coordinate systems x,y,z and x',y',z'

z points towards A, z' points towards B

We have:

$\psi = A - 90^\circ$ $\theta = 90^\circ - b$ $\chi = c$

$\psi' = 90^\circ - B$ $\theta' = 90^\circ - a$

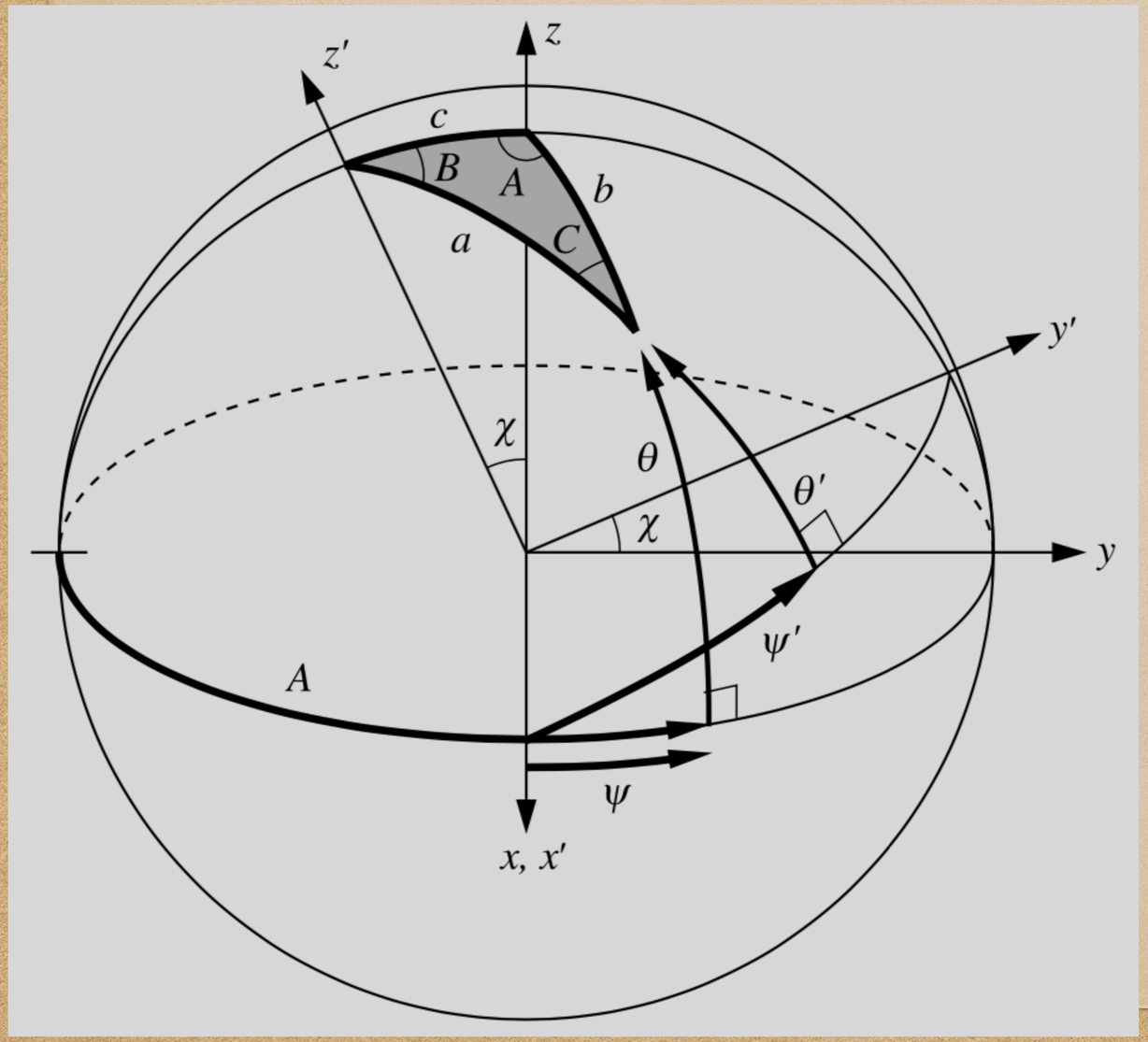
We replace in

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$

$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$

$$\sin\theta' = \sin\theta\cos\chi - \sin\chi\sin\psi\cos\theta$$

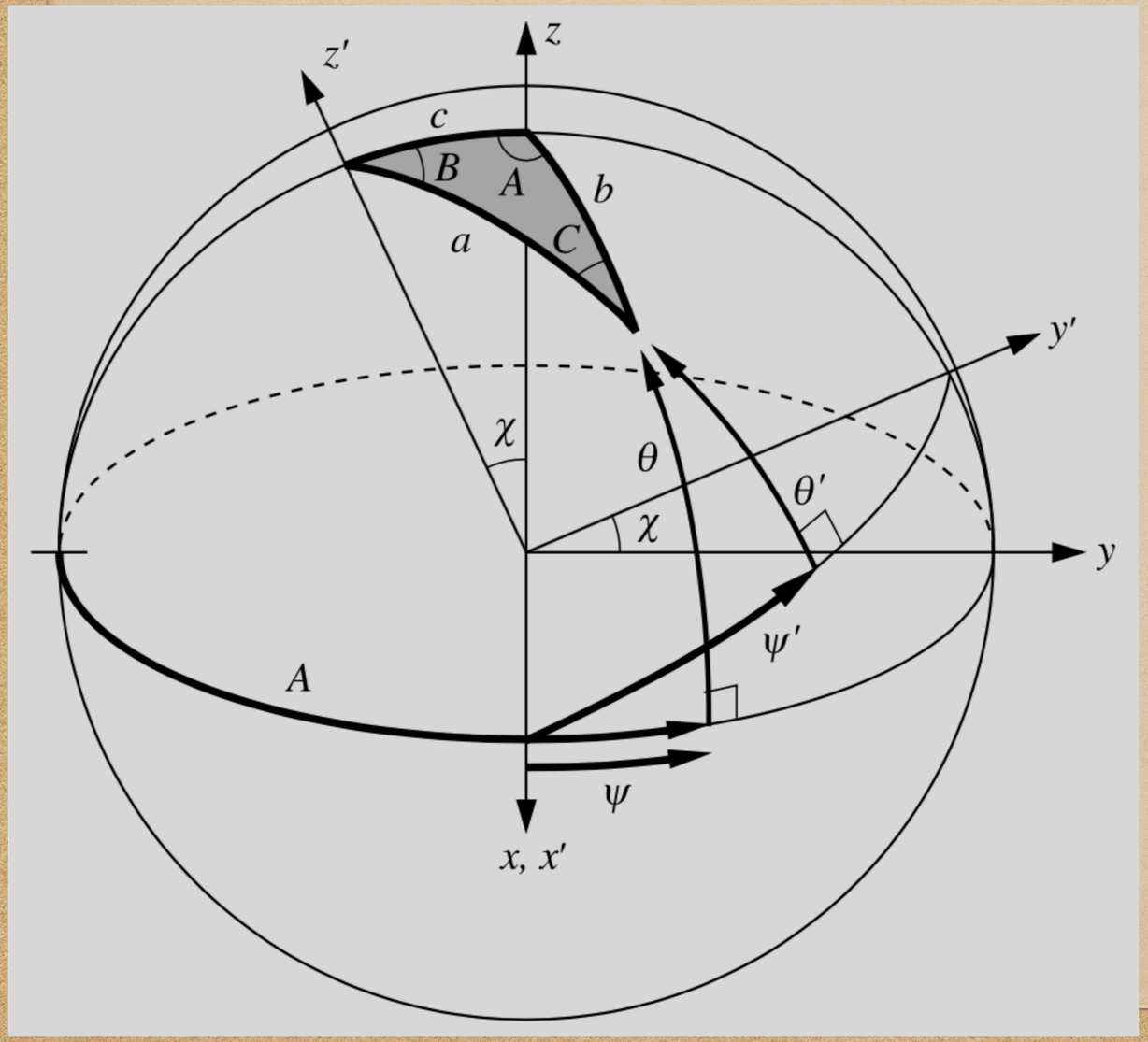
Spherical trigonometry



We obtain:

$$\cos(90^\circ - B)\cos(90^\circ - a)$$
$$= \cos(A - 90^\circ)\cos(90^\circ - b),$$
$$\sin(90^\circ - B)\cos(90^\circ - a)$$
$$= \sin(A - 90^\circ)\cos(90^\circ - b)\cos c$$
$$+ \sin(90^\circ - b)\sin c,$$
$$\sin(90^\circ - a)$$
$$= -\sin(A - 90^\circ)\cos(90^\circ - b)\sin c$$
$$+ \sin(90^\circ - b)\cos c,$$

Spherical trigonometry



Thus:

$$\sin B \sin a = \sin A \sin b,$$

$$\cos B \sin a = -\cos A \sin b \cos c + \cos b \sin c,$$

$$\cos a = \cos A \sin b \sin c + \cos b \cos c.$$

By permuting the angles we get, for example, from the first equation:

$$\sin C \sin b = \sin B \sin c,$$

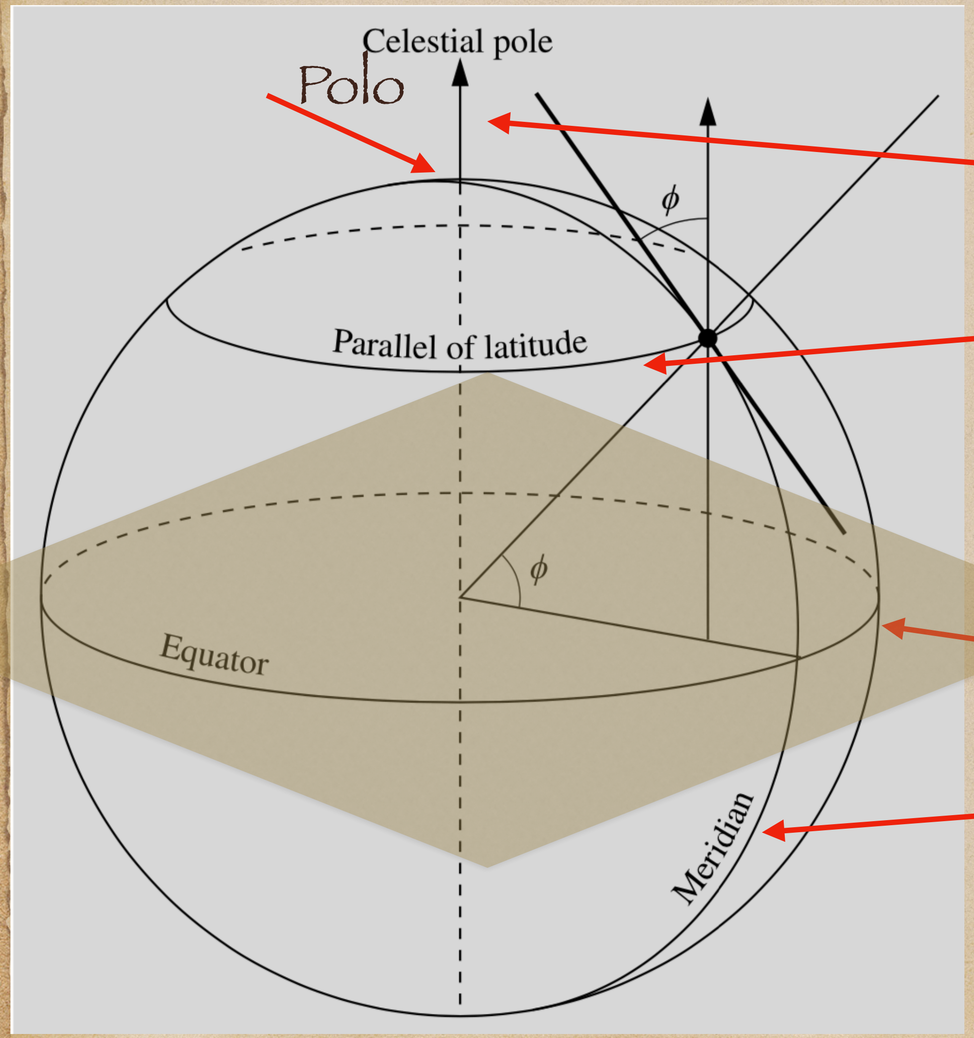
$$\sin A \sin c = \sin C \sin a.$$

and so:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}.$$

Useful to calculate distances between points on the sphere

The Earth



rotation axis

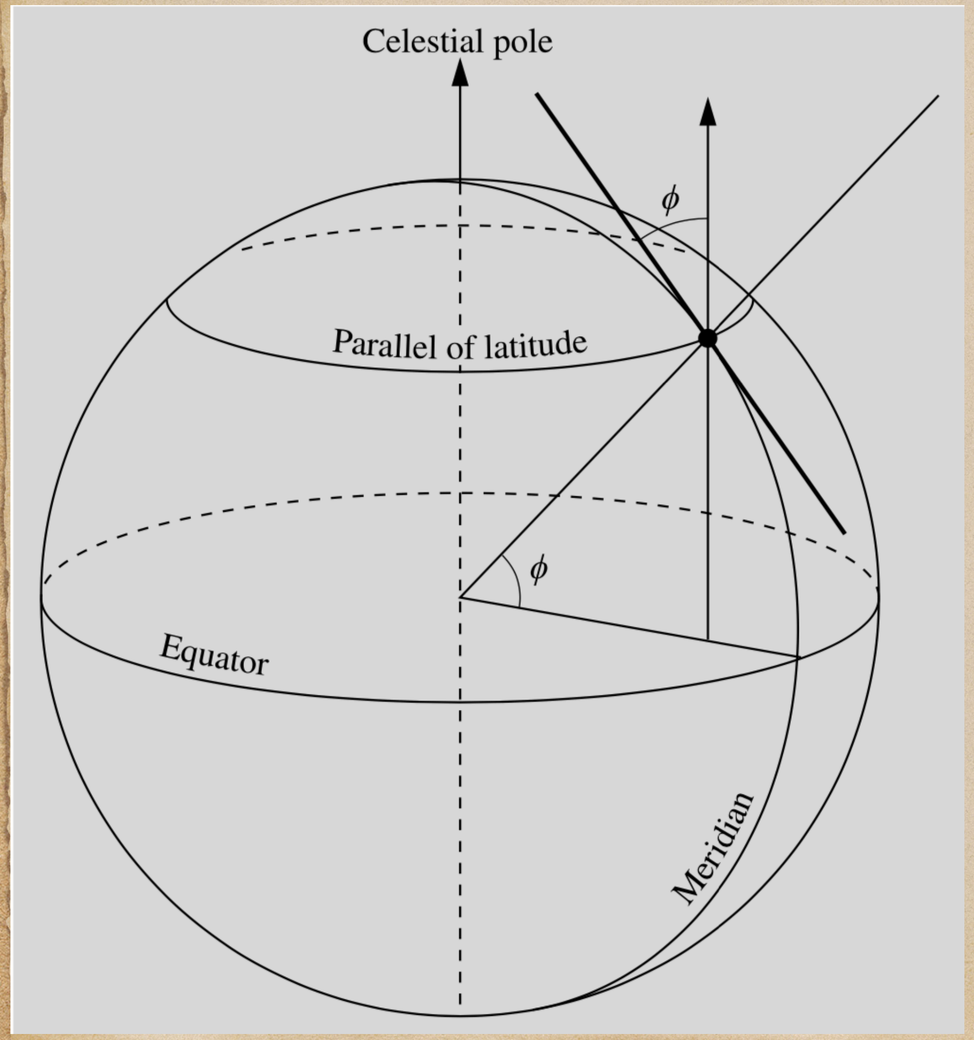
Parallel: minor circle // equator

Equatorial plane, ⊥ to the rotation axis.

Equator

Meridian, Semicircle that passes through the two poles

The Earth

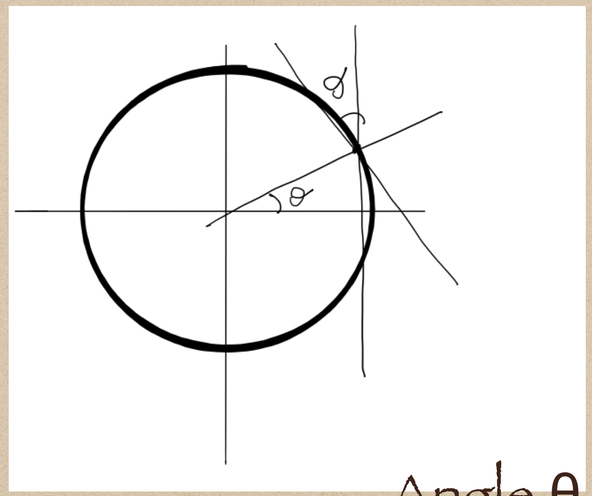


Position of a point P on Earth

2 Spherical coordinates

Latitude (geographical)

Angle ϕ between the local vertical and the equatorial plane (+ northern hemisphere, - southern hemisphere)



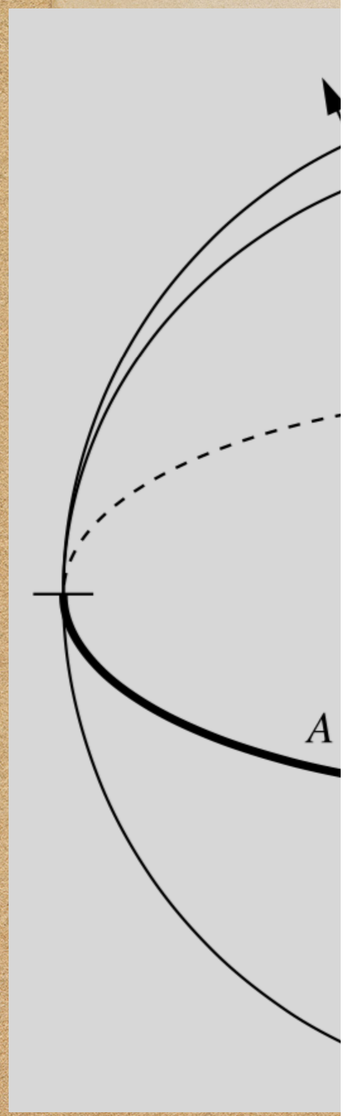
(It also corresponds to the height in the sky on the horizon of the pole)

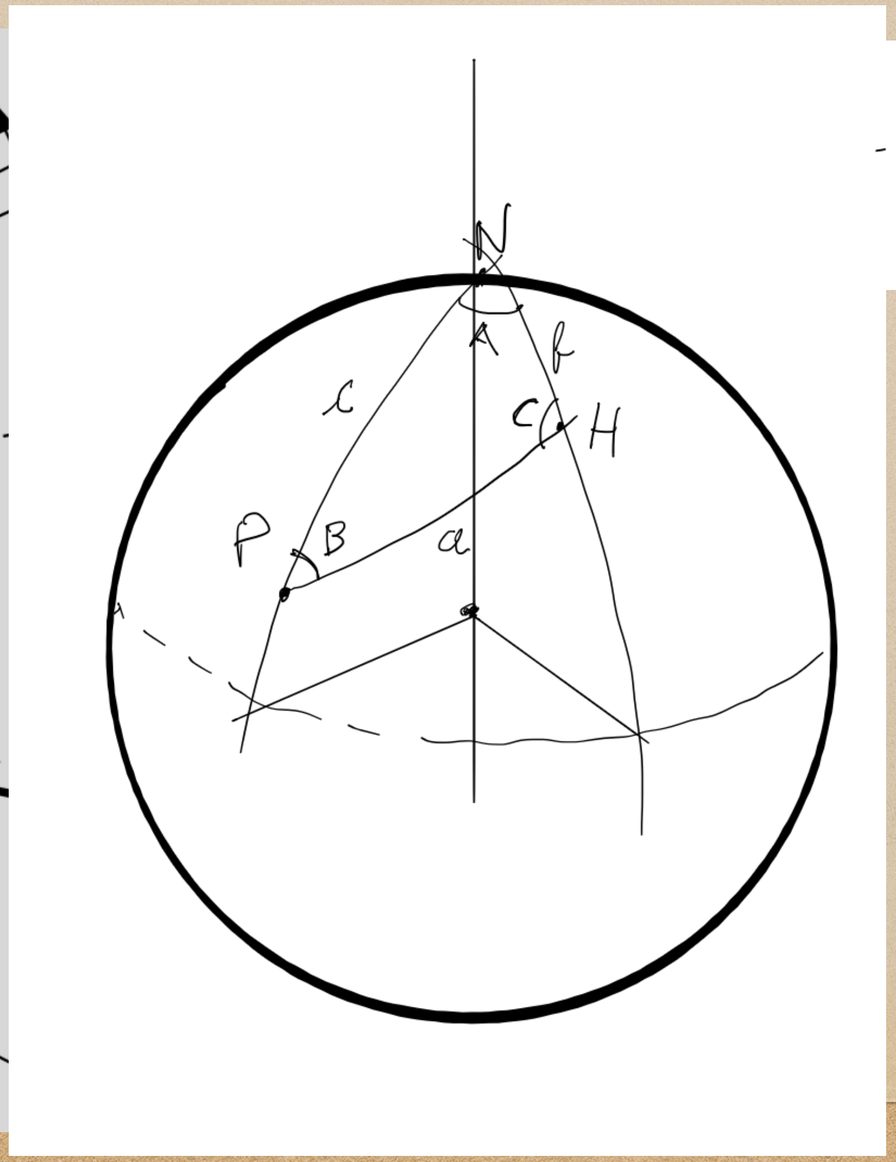
Geographical longitude

Angle θ between the meridian that contains the point and the meridian 0 passing through Greenwich (+ a East, - a West)

Spherical trigonometry

Distance between 2 locations on Earth:





$\phi_1 = 60^\circ \quad \lambda_1 = 25^\circ$ Helsinki

$\phi_2 = 28.7^\circ \quad \lambda_2 = -17.9^\circ$ La Palma

$$\cos a = \cos A \sin b \sin c + \cos b \cos c.$$

$$\cos a = \cos A \sin(90 - \phi_1) \sin(90 - \phi_2) + \cos(90 - \phi_1) \cos(90 - \phi_2)$$

$$A = \lambda_1 - \lambda_2$$

$\cos a = 0.732 \cdot 0.5 \cdot 0.877 + 0.866 \cdot 0.48$

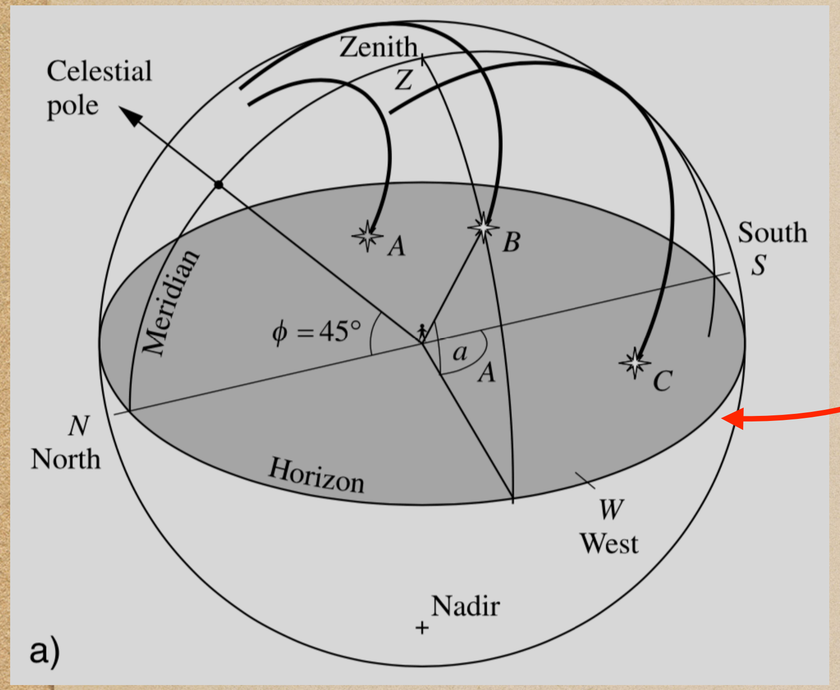
$0.32 + 0.416 = 0.732$

$a = 42.5^\circ = 0.74$ rad

$R = 6400$ km

$d = a \cdot R = 4748$ km

The horizontal system (Alt-azimutal)



Reference plane: plane tangent to the earth that passes through the observer

It crosses the celestial sphere on the horizon

Zenith: point on the celestial sphere above the observer; nadir: antipod to the zenith

Vertical circle: maximum circle passing through the zenith

Meridian: vertical circle passing through the celestial pole (and therefore through the cardinal points N and S)

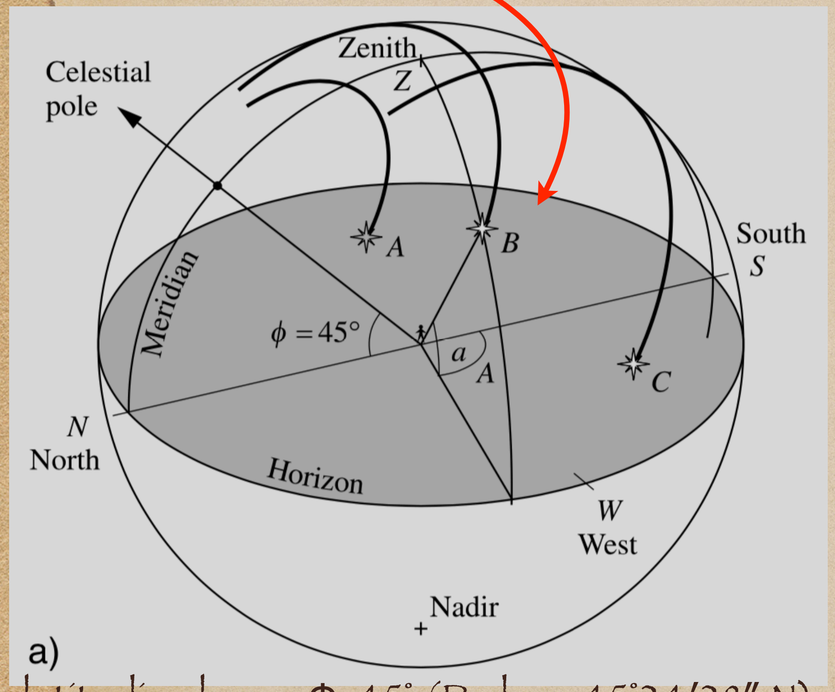
Position celestial bodies:

- Azimut A: Angle between the vertical of the body and the cardinal point S, clockwise; A=[0º, 360º]
- Height a: Angle between the horizon and the body, on the vertical circle; a=[-90º,+90º]
- Distance from the zenith: z = 90º- a

The horizontal system (Alt-azimutal)

Motion of 3 stars A, B, C in the sky

They rise in the East, culminate in the meridian, and set in the west

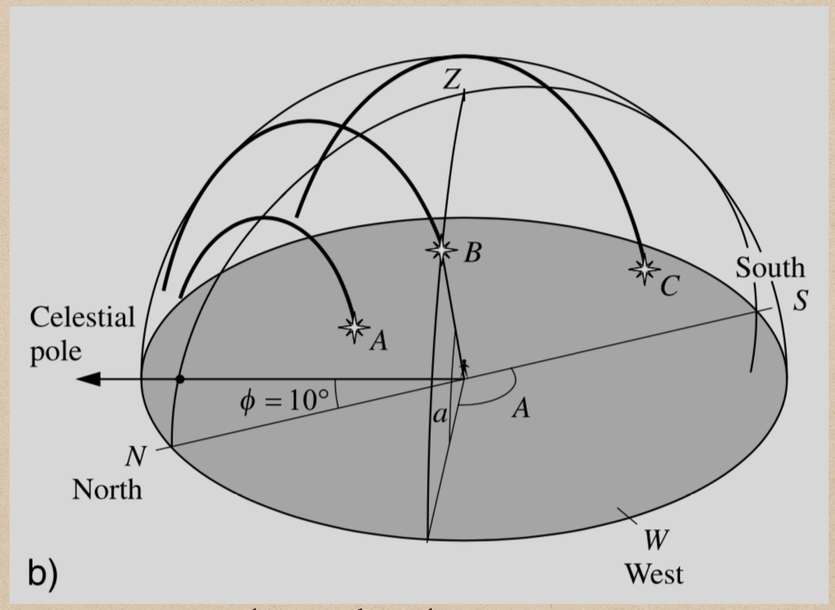


latitudine luogo Φ=45° (Padova 45°24′28″ N)

The horizontal coordinates depend on time

Same stars A, B, C from latitude 10° N

Their coordinates are different

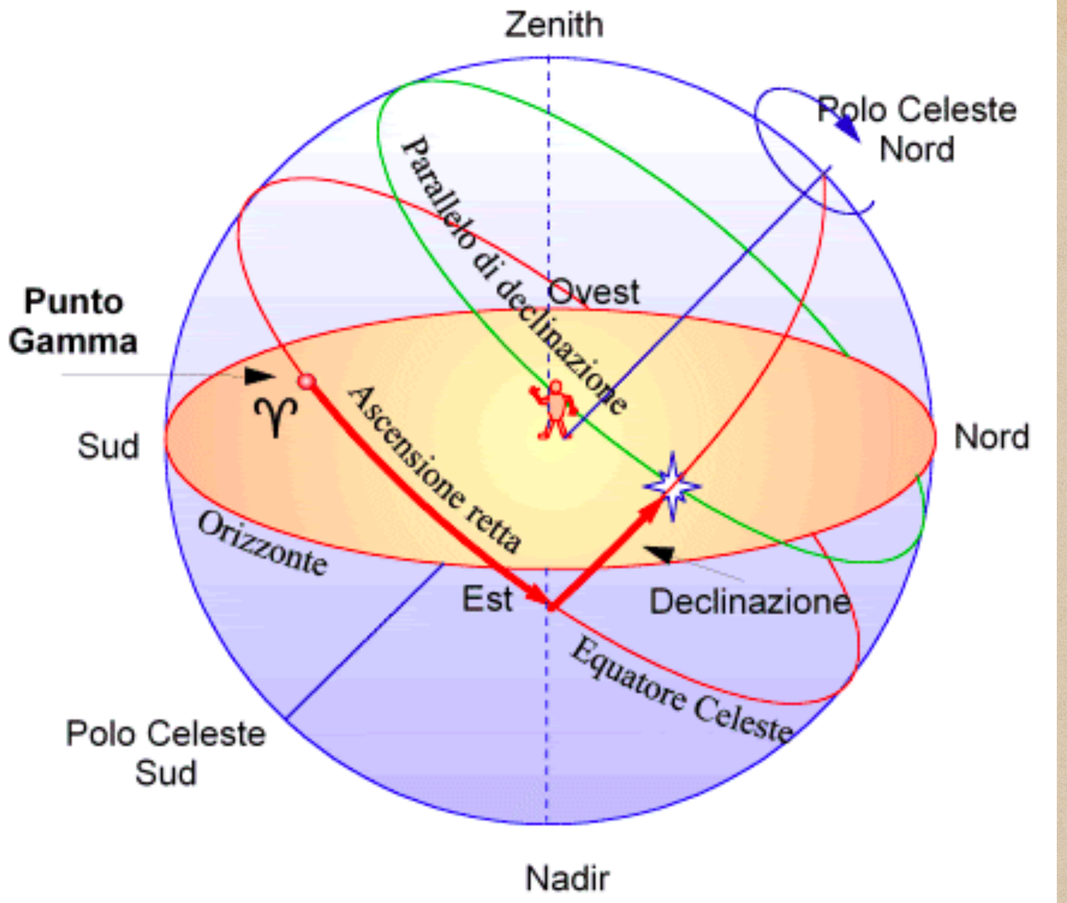


latitudine luogo Φ=10°

The coordinates of a celestial body in the horizontal system depend on the place

They are not universal, they cannot be used in catalogs

The equatorial system



The Earth's axis of rotation is (almost) constant over time, and therefore the position of the pole and the celestial equator are fixed

→ Reference plane

Astronomical meridian: it passes through zenith and pole

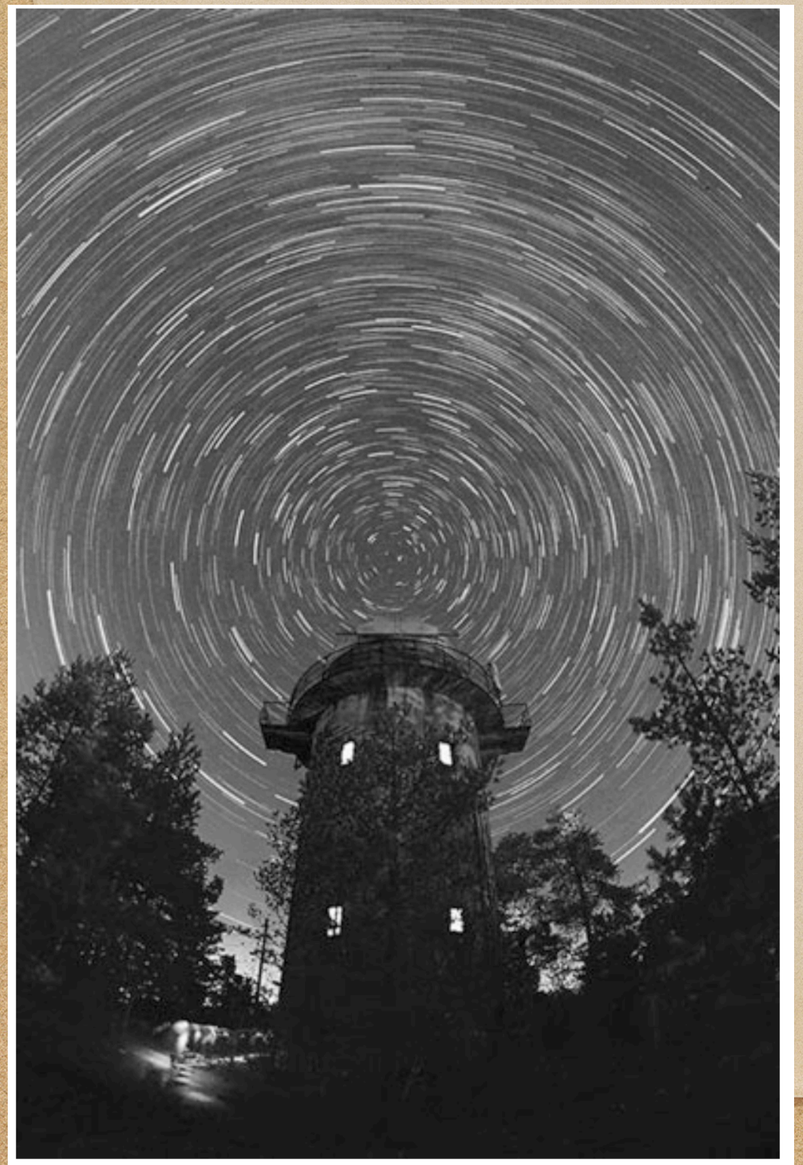
γ point: Spring intersection between celestial equator and ecliptic (apparent position of the Sun on the sphere, traveled in a year)

Equatorial coordinates:

- Right Ascension RA or α: Angular distance between the star and the point γ
- Declination δ: Star height on the celestial equator

Independent of the Earth's rotation and the position of the observer!!

The equatorial system



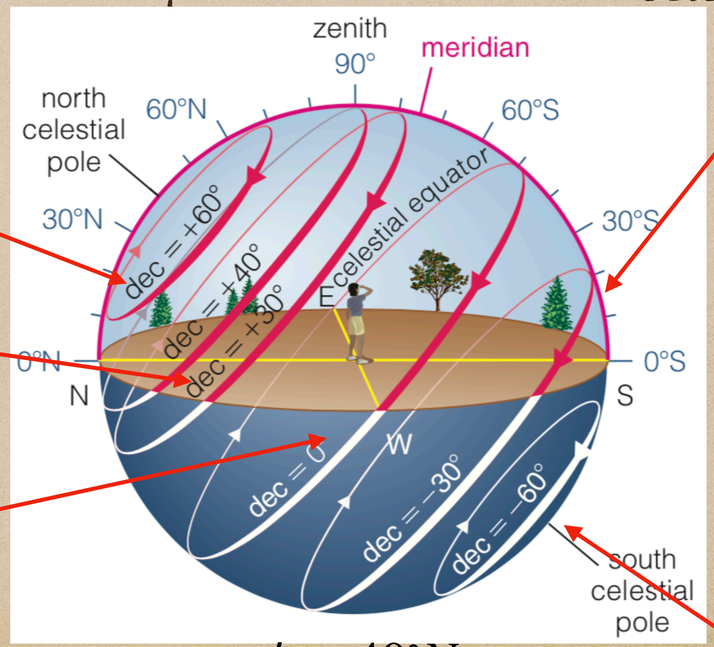
During the night the stars appear to rotate around the celestial north pole

The height of the pole on the horizon is equal to the latitude of the place

Circumpolar star: always visible

Star that rises and sets, above the horizon for > 12h

Star that rises exactly at E and sets at O, above the horizon for 12h

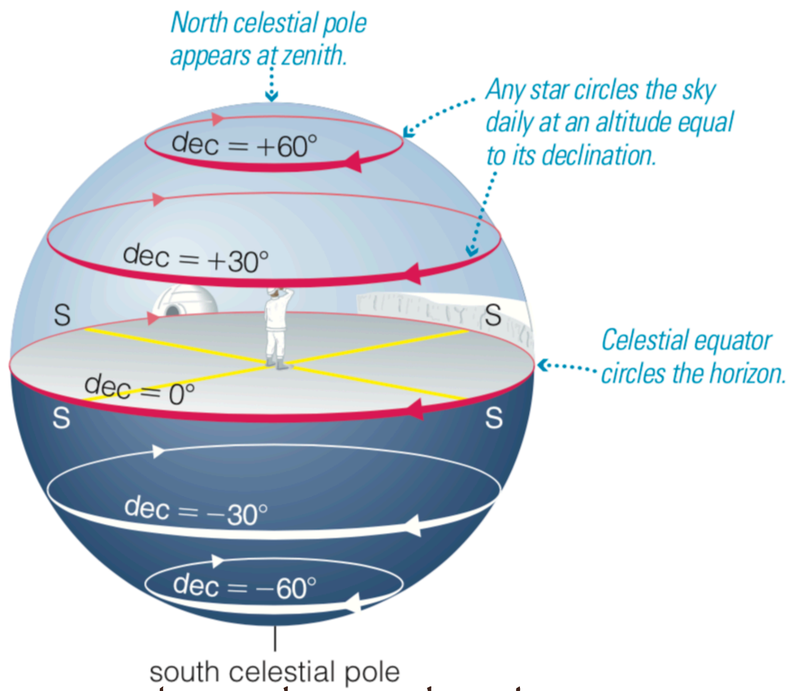


$\phi = 40°N$

Star that rises and sets, above the horizon for < 12h

Star never visible

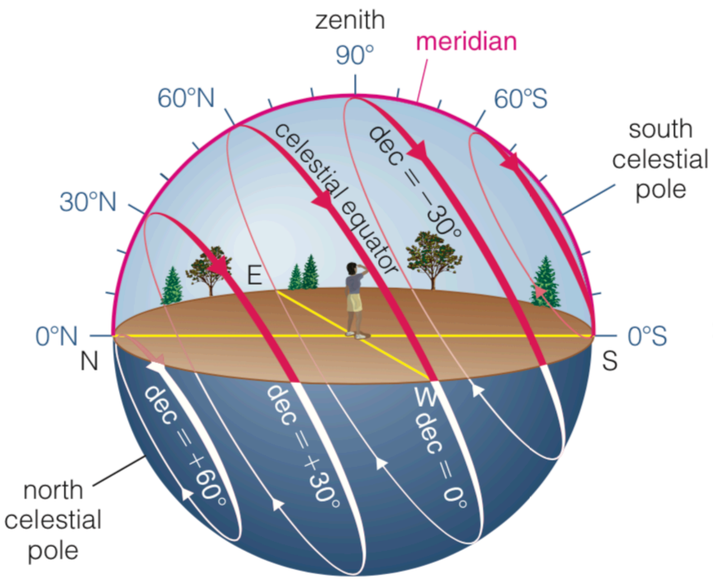
The equatorial system



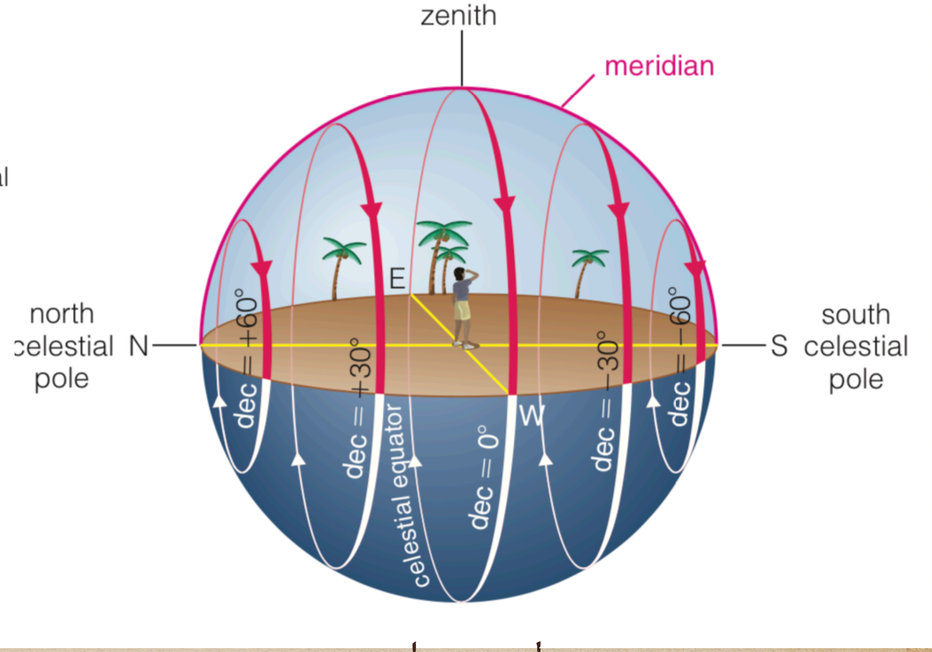
Sky at the north pole

Only half a celestial sphere is visible

All visible stars are circumpolar, visible for 24 hours



Sky at 30° S

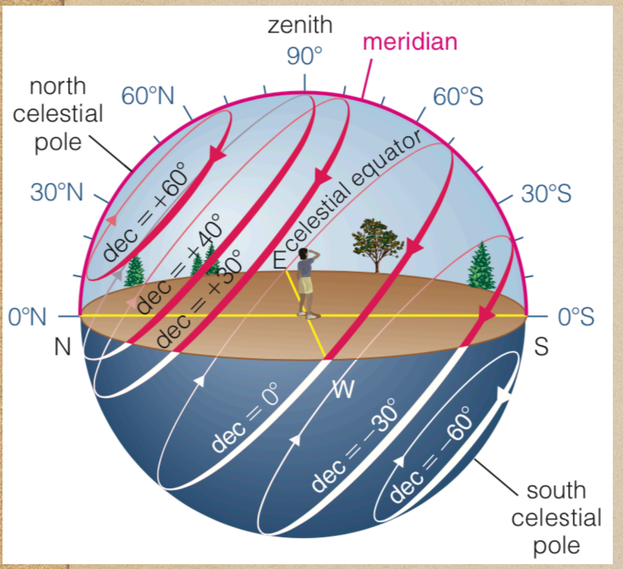


Sky at the equator

You can see the whole celestial sphere

There are no circumpolar stars. All the stars are visible for 12 hours

The equatorial system



The δ declination of an object allows us to know on which circle this object moves

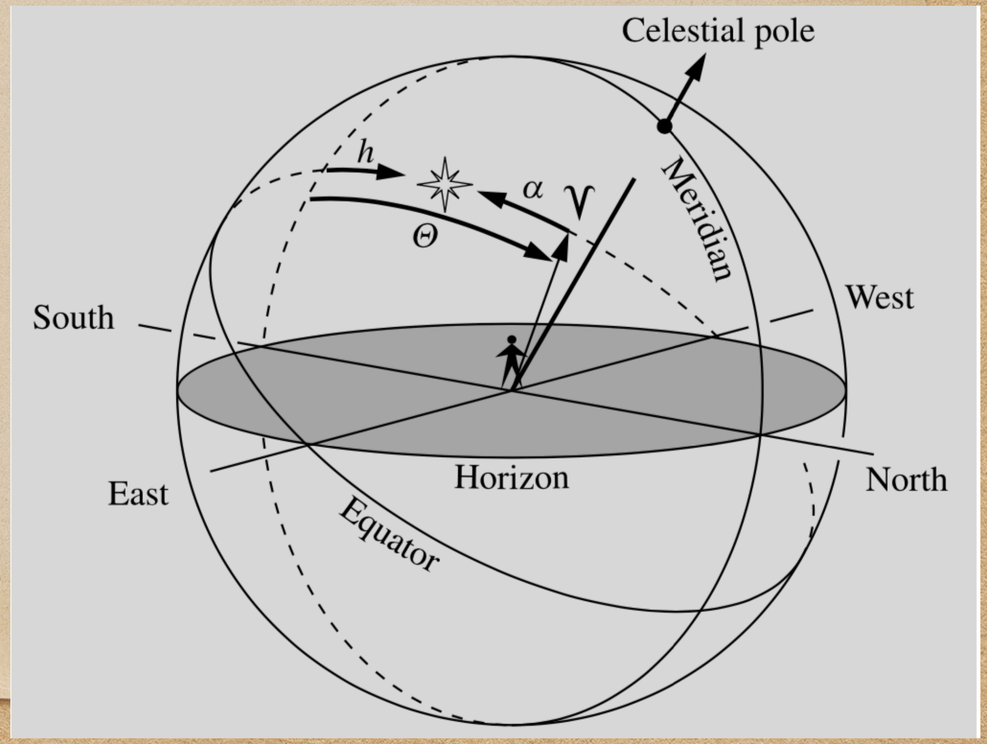
The point γ, however, moves in the sky during the night due to the Earth's rotation, so we can't use the RA to find the object

We need a local coordinate related to RA:

We use the south meridian to define the hour angle h, measured clockwise by the meridian

The hour angle of the point γ is called sidereal time Θ (or LST)

$$\Theta = h + \alpha$$



The equatorial system

Both sidereal time and time angle change at the same pace: it is practical to express them in units of time; consequently α (or RA) is also expressed in time

24h corresponds to 360°; 1h → 15°; 1m → 15'

Sidereal time can be obtained by pointing a star of known α, δ and reading h on the hours wheel of the telescope; then I apply

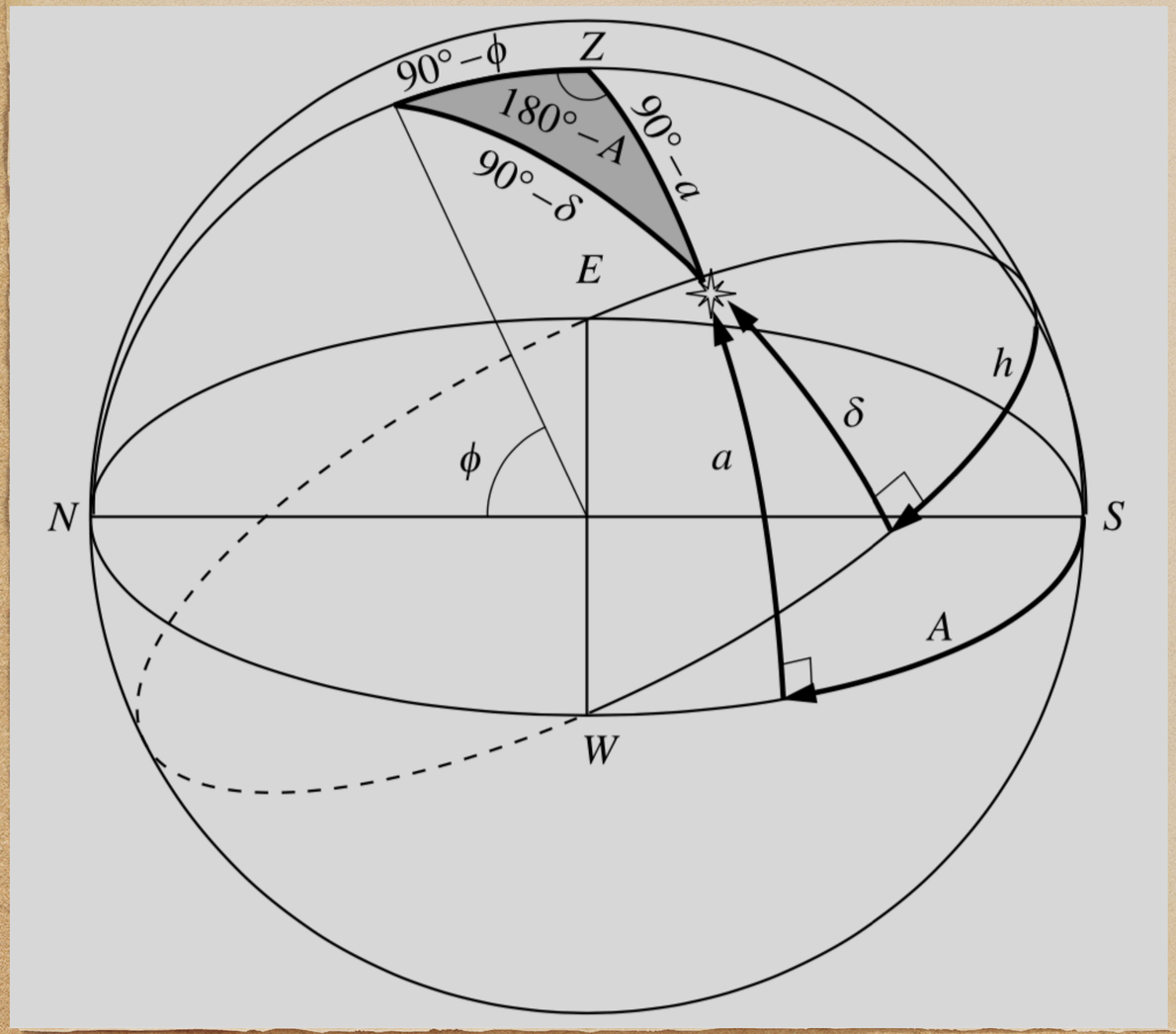
$$\Theta = h + \alpha$$

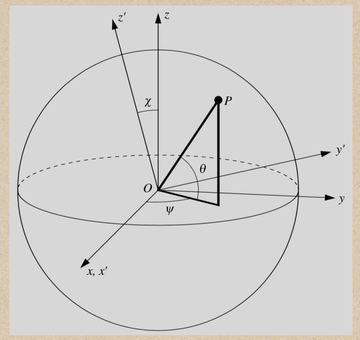
Sidereal day: interval after which the stars return to the same position in the sky

Solar day: interval after which the sun returns to the same position in the sky

Solar time ≠ space time; space time goes 3m56s faster than solar time in one day

Transformations between system Alt-azimuthal and equatorial (1)



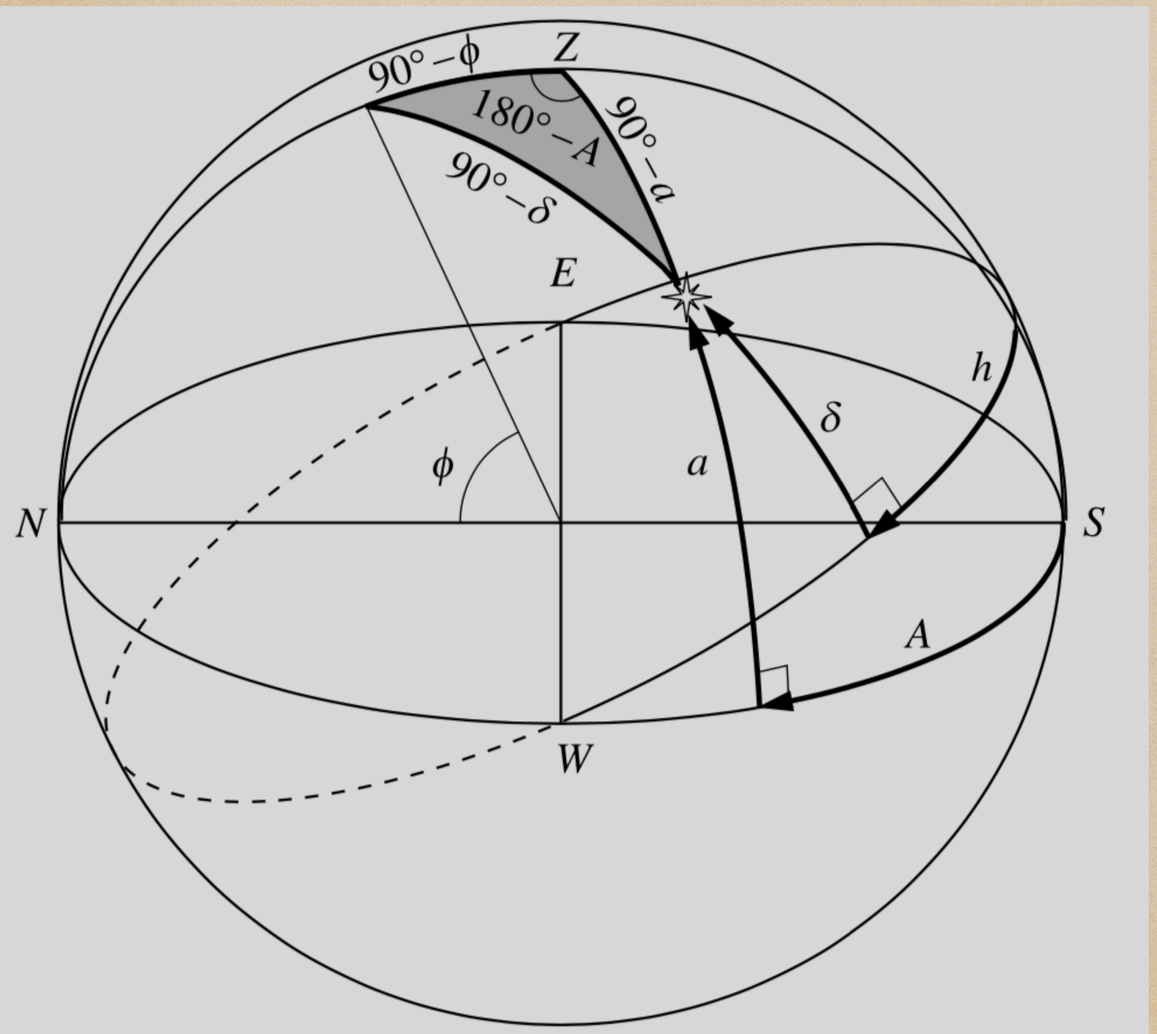


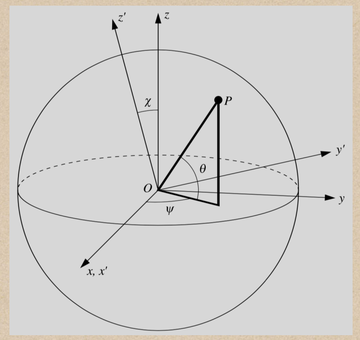
$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = \sin\theta\cos\chi - \sin\chi\sin\psi\cos\theta$$

$$\psi = 90^\circ - A \qquad \theta = a$$
$$\psi' = 90^\circ - h \qquad \theta' = \delta$$
$$\chi = 90^\circ - \phi$$

$$\sin h\cos\delta = \sin A\cos a$$
$$\cos h\cos\delta = \cos A\cos a\sin\phi + \sin a\cos\phi$$
$$\sin\delta = -\cos A\cos a\cos\phi + \sin a\sin\phi$$

Transformations between system Alt-azimuthal and equatorial (2)



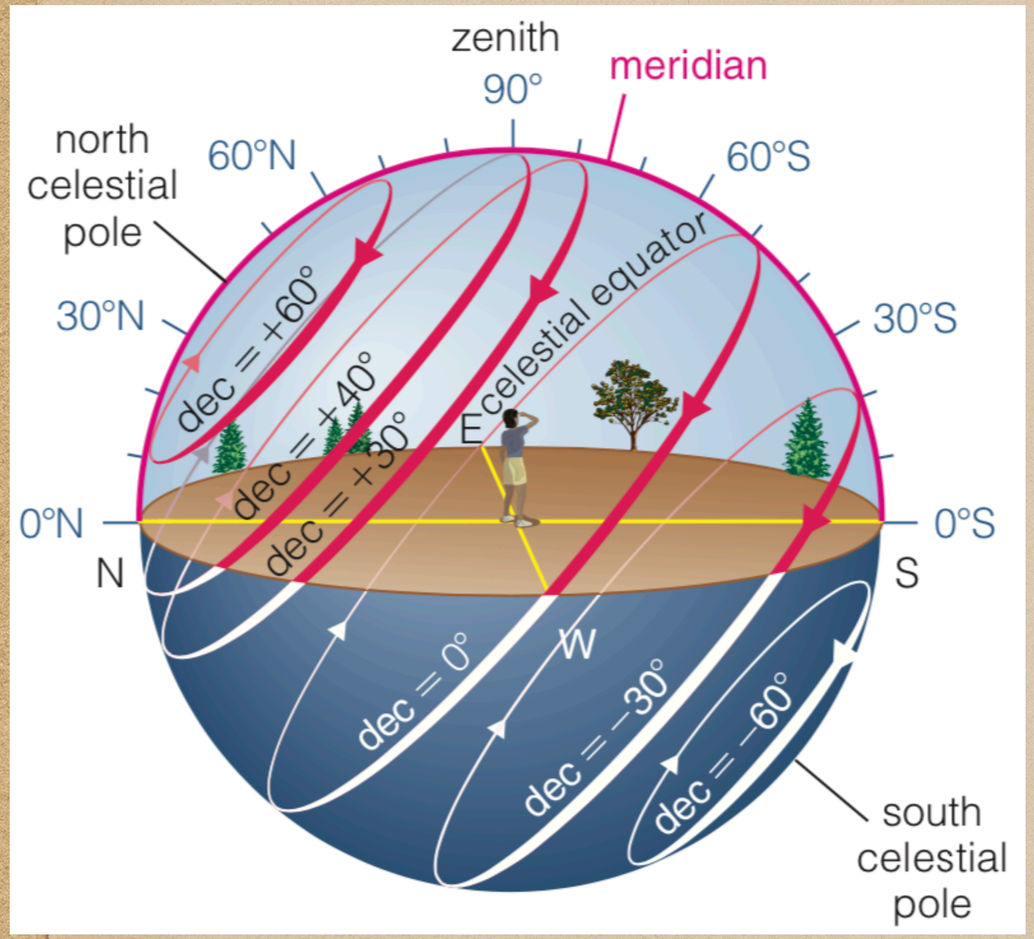


$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = \sin\theta\cos\chi - \sin\chi\sin\psi\cos\theta$$

$$\psi = 90^\circ - h \qquad \theta = \delta$$
$$\psi' = 90^\circ - A \qquad \theta' = a$$
$$\chi = -(90^\circ - \phi)$$

$$\sin A\cos a = \sin h\cos\delta$$
$$\cos A\cos a = \cos h\cos\delta\sin\phi - \sin\delta\cos\phi$$
$$\sin a = \cos h\cos\delta\cos\phi + \sin\delta\sin\phi$$

The equatorial system



$\phi = 40°N$

A star reaches its maximum height when it passes through the meridian: h=0 h → higher culmination

If h =0 in $\sin a = \cos h \cos\delta \cos\phi + \sin\delta \sin\phi$

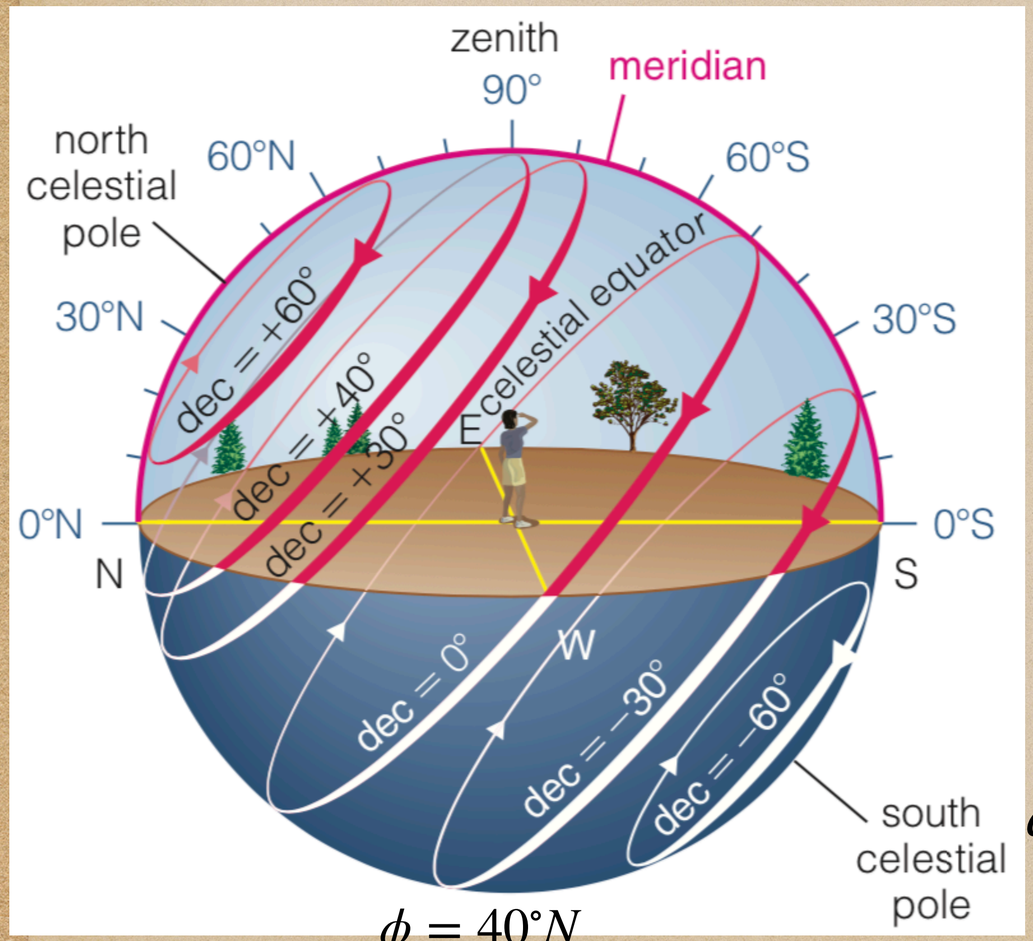
$$\sin a = \cos\delta\cos\phi + \sin\delta\sin\phi =$$

$$\begin{cases} = \cos(\phi - \delta) = \sin(90° - \phi + \delta) \\ = \cos(\delta - \phi) = \sin(90° - \delta + \phi) \end{cases}$$

$$a_{max} = \begin{cases} 90° - \phi + \delta & \text{The star culminates south of the zenith} \\ 90° + \phi - \delta & \text{The star culminates north of the zenith} \end{cases}$$

a_{max}>0 if δ>ϕ-90°; bodies with δ<ϕ-90° are not visible from that location

The equatorial system



$\phi = 40°N$

A star reaches the minimum height when h=12 h

→ lower culmination (for non-circumpolar stars, this height will be negative)

If h = 12h in $\sin a = \cos h \cos\delta \cos\phi + \sin\delta \sin\phi$

$$\sin a = -\cos\delta\cos\phi + \sin\delta\sin\phi$$

$$= -\cos(\phi + \delta) = \sin(\phi + \delta - 90°)$$

$$= -\cos(-\phi - \delta) = \sin(-\phi - \delta - 90°)$$

$$a_{min} = \begin{cases} \phi + \delta - 90° \\ -\phi - \delta - 90° \end{cases}$$

The star "anti-culminates" to the north of the zenith

The star "anti-culminates" to the south of the zenith

a_{min}>0 if δ+φ>90°; bodies with δ+φ>90° are circumpolar

The equatorial system

How to determine the coordinates α and δ from observations

For a circumpolar star which culminates to the N of the zenith:

$$a_{min} = \phi + \delta - 90^\circ \qquad a_{max} = 90^\circ + \phi - \delta$$

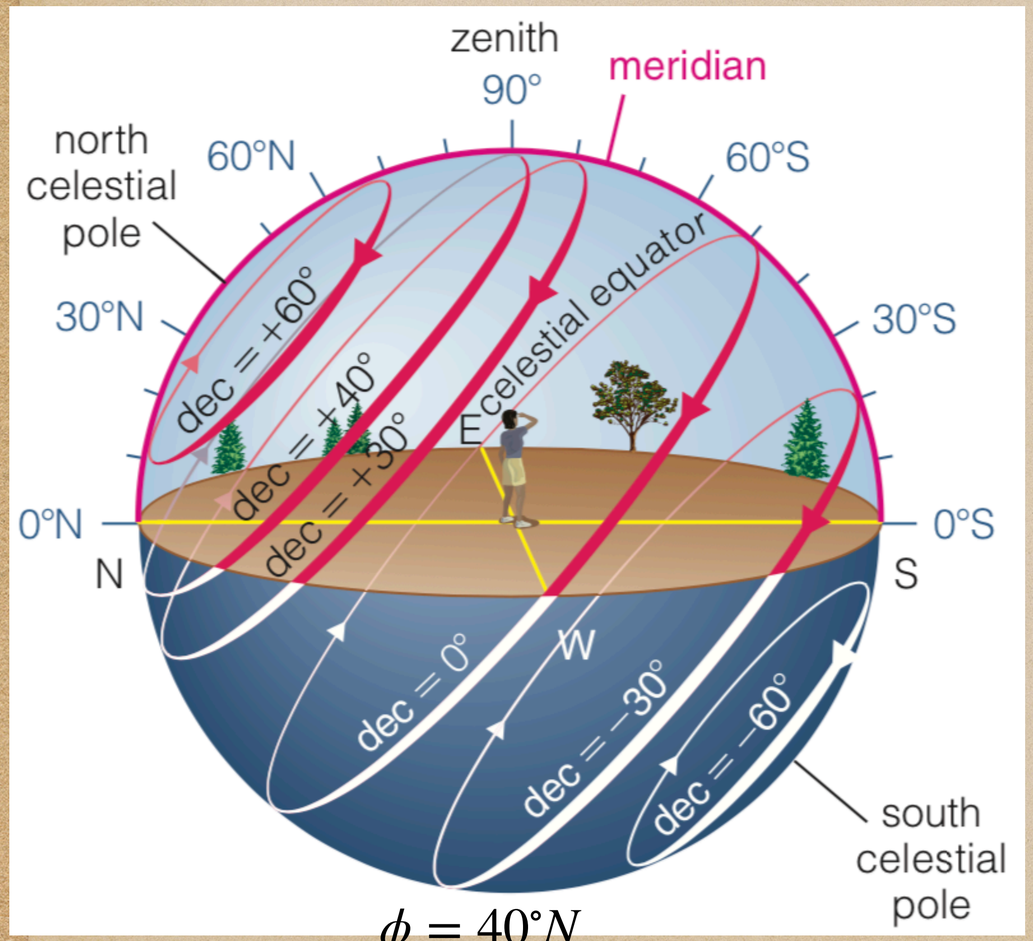
$$\Rightarrow \quad \delta = \frac{1}{2}(a_{min} - a_{max}) + 90^\circ$$

For a circumpolar star culminating to the S of the zenith:

$$\delta = \frac{1}{2}(a_{min} + a_{max})$$

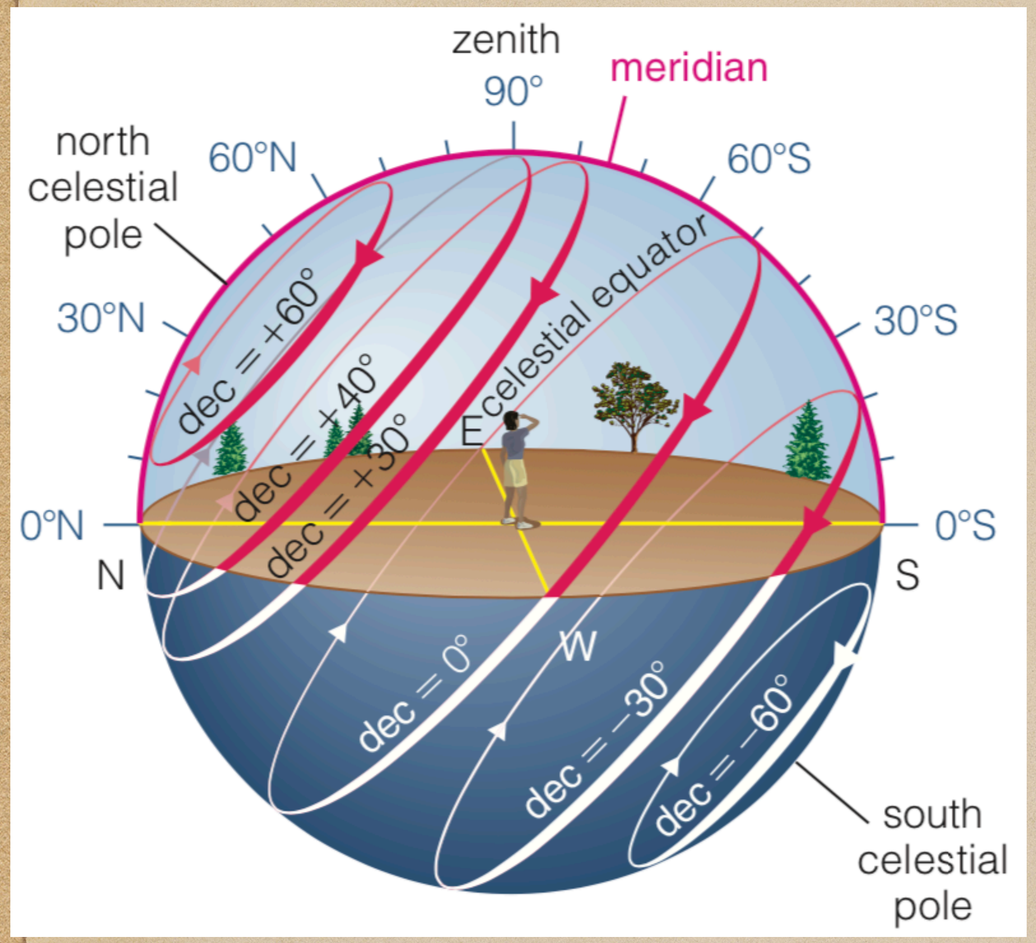
At the same time, we get the position of the pole, and we can get the δ of any object as distance from pole

The position of the equator is also defined, and as zero point of the RA or α we use the point where the Sun crosses the equator from south to north



$\phi = 40^\circ N$

Rise and set



From $\sin a = \cos h \cos\delta \cos\phi + \sin\delta \sin\phi$

$\sin a = \cos h \cos\delta \cos\phi + \sin\delta \sin\phi$

$\cos h \cos\delta \cos\phi = \sin a - \sin\delta \sin\phi$

$$\cos h = \frac{\sin a}{\cos\delta \cos\phi} - \tan\delta \tan\phi$$

$$\cos h = \frac{\sin a}{\cos\delta \cos\phi} - \tan\delta \tan\phi$$

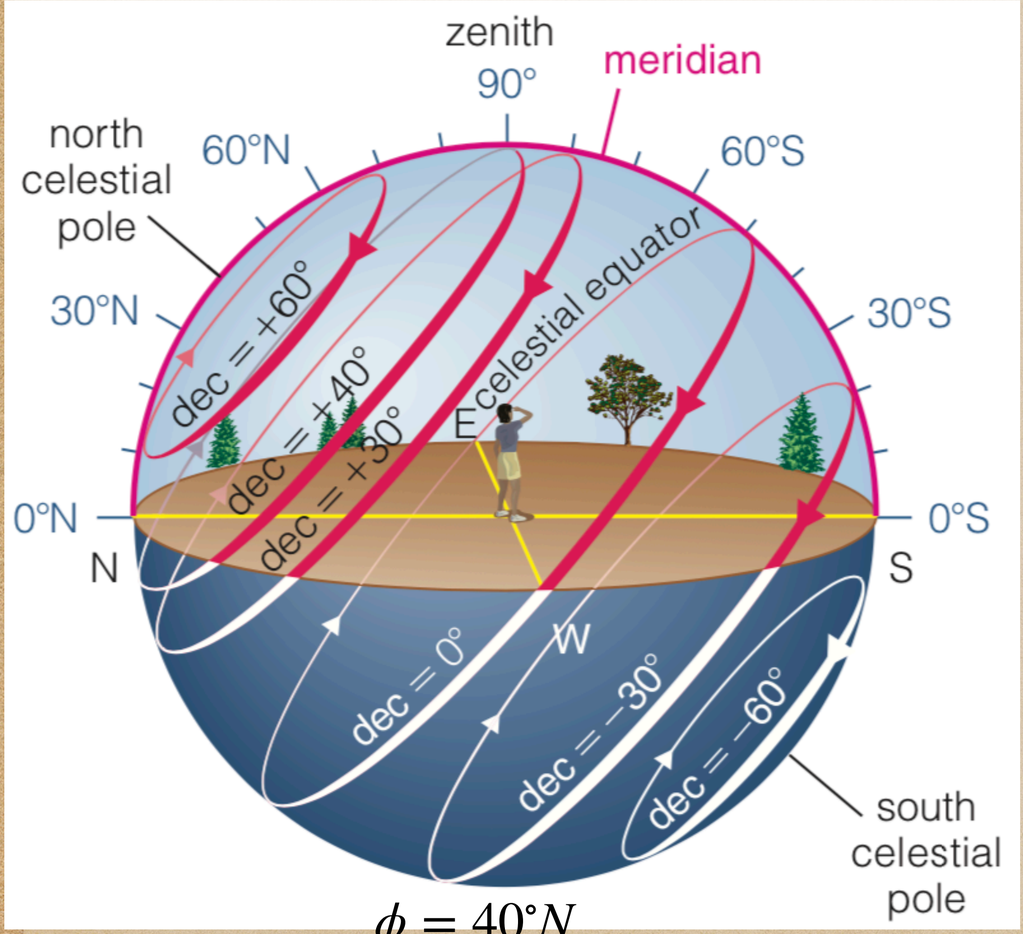
a ≈ 0 rise, set

$$\cos h_{s,t} = -\tan\delta \tan\phi \qquad \Theta_{s,t} = h_{s,t} + \alpha$$

$\phi = 40°N$

They do not take into account atmospheric refraction (about 34')

Rise and set



$\phi = 40°N$

$$\cos h_{s,t} = -\tan\delta \tan\phi \qquad \Theta_{s,t} = h_{s,t} + \alpha$$

$\cos h = -\tan\delta \tan\phi$

$\phi = 40°$ $\quad \cos h = -0.839 \cdot 0.577$

$\delta = -30°$ $\quad h = \pm 1.065\ \mathrm{rad} = \pm 61°,\ 4{,}1\,h$

$\delta = 30°$ $\quad \cos h = 0.839 \cdot 0.577 = \ldots$ $\quad h = \pm 119°,\ 7{,}93\,h$

$\delta = 50°$ $\quad \cos h = -1$ $\quad h = 180°,\ 12\,h$

$\delta = -50°$ $\quad \cos h = 1$ $\quad h = 0°,\ 0\,h$

$\delta = 0°$ $\quad \cos h = 0$ $\quad h = \pm 90°,\ \pm 6\,h$