



$(x \in]-\sqrt{2}, \sqrt{2}[)$.

1. $y(x) = \int (x-1) \log(2-x^2) dx = (\text{per parti})$

$$= \left(\frac{x^2}{2} - x\right) \log(2-x^2) - \int \left(\frac{x^2}{2} - x\right) \frac{-2x}{(2-x^2)} dx$$

inoltre

$\int \frac{x^3 - 2x^2}{2-x^2} dx$ ↓ divisione $= \int \left(2 - x + \frac{2x-4}{2-x^2}\right) dx =$

$$= 2x - \frac{x^2}{2} - 2 \int \frac{x-2}{x^2-2} dx$$
 ↓ scomposizione

$$= 2x - \frac{x^2}{2} - \int \left((1+\sqrt{2}) \frac{1}{x+\sqrt{2}} + (1-\sqrt{2}) \frac{1}{x-\sqrt{2}} \right) dx$$

$$= 2x - \frac{x^2}{2} - (1+\sqrt{2}) \log(x+\sqrt{2}) + (\sqrt{2}-1) \log(\sqrt{2}-x) + C$$

Quindi:

$$y(x) = \left(\frac{x^2}{2} - x\right) \log(2-x^2) + 2x - \frac{x^2}{2}$$

$$- (1+\sqrt{2}) \log(x+\sqrt{2}) + (\sqrt{2}-1) \log(\sqrt{2}-x) + C$$

e allora

$$3 = y(1) = 0 + 2 - \frac{1}{2} - (1+\sqrt{2}) \log(1+\sqrt{2}) + (\sqrt{2}-1) \log(\sqrt{2}-1) + C$$

$$\Rightarrow C = \frac{3}{2} + (1+\sqrt{2}) \log(1+\sqrt{2}) - (\sqrt{2}-1) \log(\sqrt{2}-1).$$

$$2. f(x) = (1-|x|) e^{\frac{1}{2x+2}}$$

$$D = \mathbb{R} \setminus \{-1\}.$$

$$\lim_{x \rightarrow +\infty} (1-x) e^{\frac{1}{2x+2}} = -\infty$$

$$\lim_{x \rightarrow -\infty} (1+x) e^{\frac{1}{2x+2}} = -\infty$$

$$\lim_{x \rightarrow (-1)^-} (1+x) e^{\frac{1}{2x+2}} = 0$$

$$\lim_{x \rightarrow (-1)^+} (1+x) e^{\frac{1}{2(1+x)}} = \lim_{x \rightarrow (-1)^+} \frac{e^{\frac{1}{2(1+x)}}}{\frac{1}{2(1+x)}} \cdot \frac{1}{2} =$$

$$= \frac{1}{2} \lim_{y \rightarrow +\infty} \frac{e^y}{y} = +\infty$$

$$y = \frac{1}{2(1+x)}$$

$x = -1$ is asymptote vertical oblique

$$\lim_{x \rightarrow +\infty} \frac{(1-x) e^{\frac{1}{2x+2}}}{x} = -1$$

$$\lim_{x \rightarrow +\infty} (1-x) e^{\frac{1}{2x+2}} + x =$$

$$= \lim_{x \rightarrow +\infty} e^{\frac{1}{2x+2}} + x \left(1 - e^{\frac{1}{2x+2}} \right) = \frac{1}{2}$$

$\rightarrow 1$
 $\sim -\frac{1}{2x+2}$

$y = -x + \frac{1}{2}$ è asintoto obliquo per $x \rightarrow +\infty$

$$\lim_{x \rightarrow -\infty} \frac{(1+x) e^{\frac{1}{2x+2}}}{x} = 1$$

$$\lim_{x \rightarrow -\infty} (1+x) e^{\frac{1}{2x+2}} - x =$$

$$= \lim_{x \rightarrow -\infty} e^{\frac{1}{2x+2}} + x \left(e^{\frac{1}{2x+2}} - 1 \right) = \frac{3}{2}$$

$\rightarrow 1$
 $\sim \frac{1}{2x+2}$

$y = x + \frac{3}{2}$ è asintoto obliquo per $x \rightarrow -\infty$.

Se $x \neq 0, -1$

$$f'(x) = e^{\frac{1}{2x+2}} \left(\frac{-1}{2(x+1)^2} (1-|x|) - \frac{x}{2x+2} \right)$$

$$= \begin{cases} e^{\frac{1}{2x+2}} \cdot \frac{1}{2(x+1)^2} \cdot (-2x^2 - 3x - 3) & \text{se } x > 0 \\ e^{\frac{1}{2x+2}} \cdot \frac{1}{2(x+1)^2} \cdot (2x^2 + 3x + 1) & \text{se } x < 0 \end{cases}$$

sgn f' : $\quad \quad \quad -1 \quad \quad -1/2 \quad \quad 0 \quad \quad \quad$

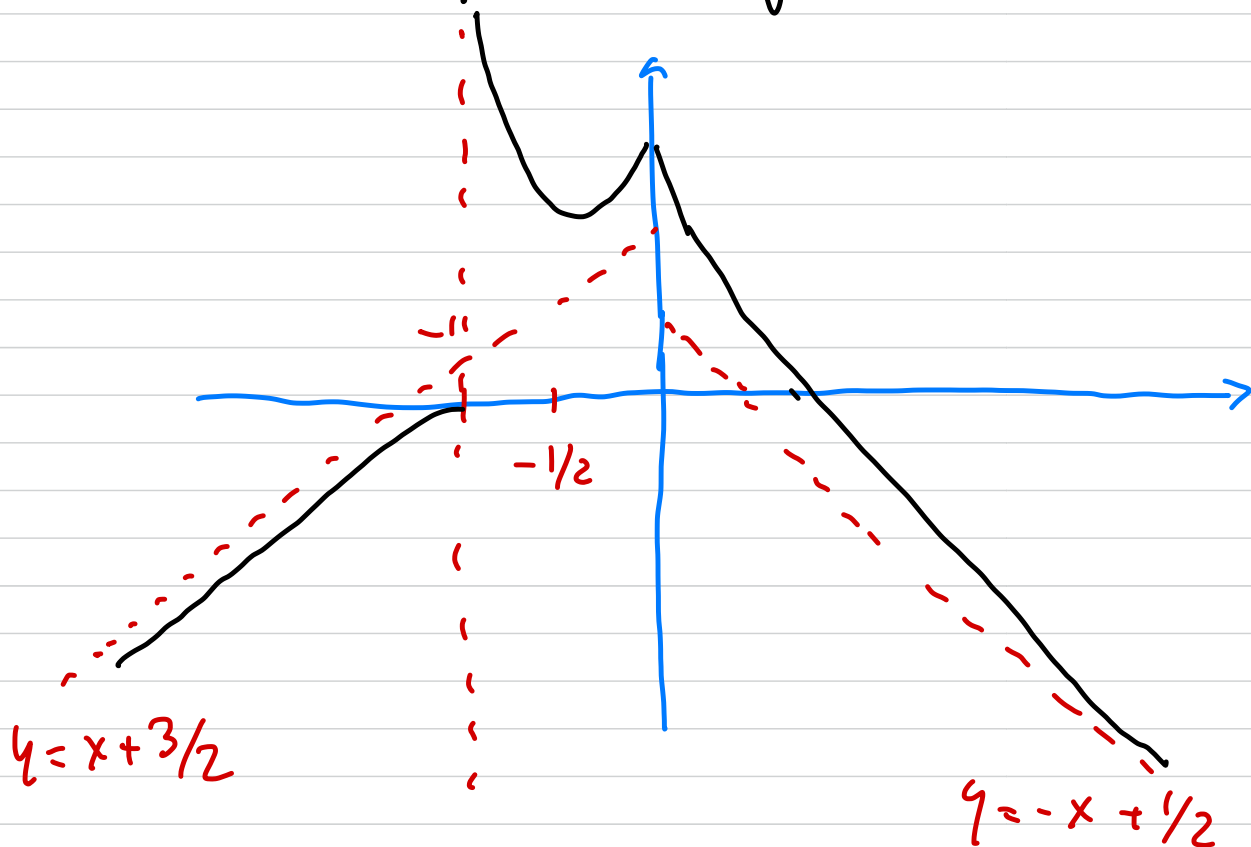
$x = -1/2$ è punto di minimo locale stretto

$x = 0$ è punto di massimo locale stretto.

$$\lim_{x \rightarrow (-1)^-} f'(x) = 0$$

$$\lim_{x \rightarrow 0^-} f'(x) = \frac{\sqrt{e}}{2} \quad ; \quad \lim_{x \rightarrow 0^+} f'(x) = -\frac{3}{2}\sqrt{e}$$

$x = 0$ è punto angoloso



$f(D) \cong f([-1, +\infty[) = \mathbb{R}$ per il teorema
dei valori intermedi

$$3. f(x) = \sqrt{4+2x^2} - \sqrt{4-2x^2}$$

$$= 2 \left(1 + \frac{x^2}{2}\right)^{1/2} - 2 \left(1 - \frac{x^2}{2}\right)^{1/2} = \quad y = \pm \frac{x^2}{2}$$

$$= 2 \left[1 + \frac{x^2}{4} - \frac{1}{8} \frac{x^4}{4} + \frac{3}{48} \frac{x^6}{8} - 1 + \frac{x^2}{4} + \frac{1}{8} \frac{x^4}{4} + \frac{3}{48} \frac{x^6}{8} + o(x^6) \right]$$

$$= x^2 + \frac{x^6}{32} + o(x^6)$$

per $x \rightarrow 0$

$$g(x) = \sin(x^3) - x^3 + 1 - \cosh x$$

$$= -\frac{x^9}{6} - \frac{x^2}{2} + o(x^2) \sim -\frac{x^2}{2}$$

$$\sin(x^\alpha) - x^\alpha = \frac{x^{3\alpha}}{6} + o(x^{3\alpha})$$

$$\lim_{x \rightarrow 0} \frac{\sin(x^\alpha) - x^\alpha + 1 - \cosh x}{\sqrt{4+2x^2} - \sqrt{4-2x^2}} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x^{3\alpha}}{6} + o(x^{3\alpha}) - \frac{x^2}{2} + o(x^2)}{x^2} =$$

$$= \lim_{x \rightarrow 0} \begin{cases} \frac{-x^2/2}{x^2} \\ \frac{-x^2/3}{x^2} \\ \frac{x^{3\alpha}/6}{x^2} \end{cases}$$

$$\text{se } 3\alpha > 2 \quad \text{se } 3\alpha = 2 \quad \text{se } 3\alpha < 2 \quad = \begin{cases} -\frac{1}{2}, & \alpha > 2/3 \\ -\frac{1}{3}, & \alpha = \frac{2}{3} \\ +\infty, & \alpha < 2/3 \text{ o} \\ \text{non esiste}, & \alpha \text{ intero dispari} \end{cases}$$

$$4. \begin{cases} y' = (1+y^2) \cos x \\ y(0) = y_0 \end{cases}$$

È equazione differenziale a variabili separabili.

Non ha soluzioni costanti.

Soluzioni non costanti

$$\int \frac{1}{1+y^2} dy = \int \cos x \, dx$$

$y = y(x)$

$$\arctan(y(x)) = \sin x + C = \sin x + \arctan y_0$$

imponendo le condizioni iniziali

$$\arctan y_0 = \arctan(y(0)) = C$$

In forma esplicita

$$y(x) = \tan(\sin x + \arctan y_0)$$

definite se $\sin x + \arctan y_0 \in]-\frac{\pi}{2}, \frac{\pi}{2}[$

Esplode per $x \rightarrow \frac{\pi}{4}$ se

$$\sin \frac{\pi}{4} + \operatorname{arctg} y_0 = \frac{\pi}{2}$$

$$(\Leftrightarrow) \operatorname{arctg} y_0 = \frac{\pi}{2} - \frac{\sqrt{2}}{2}$$

$$(\Leftrightarrow) y_0 = \operatorname{tg} \left(\frac{\pi}{2} - \frac{\sqrt{2}}{2} \right).$$

$$\int (x-1) \lg(2-x^2) = \left(\frac{x^2}{2} - x\right) \lg(2-x^2)$$

$$+ \int \left(\frac{x^2}{2} - x\right) \frac{2x}{2-x^2}$$

$$1+\sqrt{2} \cdot \frac{1}{\sqrt{2}-x} + \frac{(\sqrt{2}-1)}{\sqrt{2}+x}$$

$$\int \frac{x^3 - 2x^2}{2-x^2} = \int (-x+2) + \frac{2x-4}{2-x^2}$$

$$\begin{array}{r|l} x^3 - 2x^2 & -x^2 + 2 \\ x^3 - 2x & -x + 2 \\ \hline -2x^2 + 2x & \\ -2x^2 + 4 & \\ \hline 2x - 4 & \end{array}$$

$$\begin{aligned} & \frac{A}{\sqrt{2}-x} + \frac{B}{\sqrt{2}+x} \\ &= \frac{\sqrt{2}A + Ax + \sqrt{2}B - Bx}{2-x^2} \end{aligned}$$

$$A - B = 2$$

$$A + B = -2\sqrt{2}$$

$$A = 1 - \sqrt{2}$$

$$B = -1 - \sqrt{2}$$

$$-\frac{x^2}{2} + 2x + (\sqrt{2}-1) \lg(\sqrt{2}-x) - (\sqrt{2}+1) \lg(\sqrt{2}+x)$$