

TENA A

$$\textcircled{1} \quad \int t \sqrt{2-t} dt = \int (2-u^2) \cdot u (-2u) du$$

$$u = \sqrt{2-t}$$

$$u^2 = 2-t$$

$$t = 2-u^2$$

$$dt = -2u du$$

$$= 2 \int (u^2 - 2) u^2 du$$

$$= \frac{2}{5} u^5 - \frac{2}{3} u^3 + C$$

$$= \frac{2}{5} (2-t)^{5/2} - \frac{2}{3} (2-t)^{3/2} + C$$

$$u'(t) + \frac{u(t)}{t} = 0$$

$$\lim \quad 1^{\text{ord}} \quad a(t) = -\frac{1}{t}$$

$$A(t) = -\log t$$

$$u(t) = C e^{A(t)} = \frac{C}{t}$$

$$\text{cruo sol. part.} \quad u_p(t) = (t)/t$$

$$\Rightarrow c'(t) = t \sqrt{2-t}$$

$$c(t) = \int t \sqrt{2-t} dt$$

$$u_p(t) = \frac{2}{5t} (2-t)^{5/2} - \frac{2}{3t} (2-t)^{3/2}$$

$$\text{sol. gen:} \quad u(t) = \frac{1}{t} \left(C + \frac{2}{5} (2-t)^{5/2} - \frac{2}{3} (2-t)^{3/2} \right)$$

$$u(1) = C + \frac{2}{5} - \frac{2}{3} \Rightarrow C + \frac{6-10}{15} = C - \frac{4}{15}$$

$$(2) \quad f(x) = \log \left(\frac{x - |x^2 - 2|}{x} \right)$$

si ha $x - |x^2 - 2| > 0$ se $x \in]1, 2[$

dunque $x - |x^2 - 2| > 0$

	0	1	2
	-	-	+
	-	+	+
	⊕		⊕

$$D =]-\infty, 0[\cup]1, 2[.$$

$$\lim_{x \rightarrow -\infty} \frac{x - |x^2 - 2|}{x} = +\infty \Rightarrow \lim_{x \rightarrow -\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow 0^-} \frac{x - |x^2 - 2|}{x} = +\infty \Rightarrow \lim_{x \rightarrow 0^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 1^+} \frac{x - |x^2 - 2|}{x} = 0^+ \Rightarrow \lim_{x \rightarrow 1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} \frac{x - |x^2 - 2|}{x} = 0^+ \Rightarrow \lim_{x \rightarrow 2^-} f(x) = -\infty.$$

no asintoti orizzontali né obliqui

Il dominio è diviso dai punti $\pm\sqrt{2}$
dove $|x^2 - 2|$ si annulla

① dove $|x^2-2| > 0$ allora

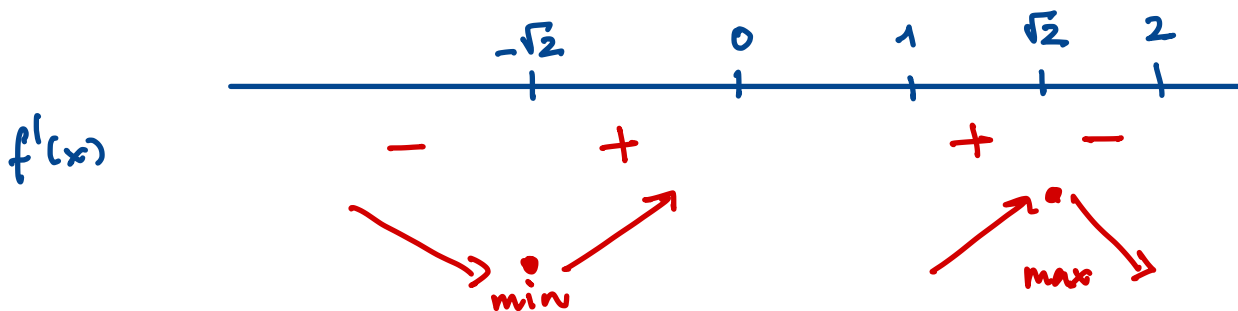
$$f(x) = \log\left(\frac{x-x^2+2}{x}\right)$$

$$\begin{aligned} f'(x) &= \frac{x}{x-x^2+2} \cdot \frac{(-2x+1)x - (x-x^2+2)}{x^2} \\ &= \frac{-x^2-2}{-(x-2)(x+1)x} = \frac{x^2+2}{x(x-2)(x+1)} \end{aligned}$$

② dove $|x^2-2| < 0$

$$f(x) = \log\left(\frac{x+x^2-2}{x}\right)$$

$$\begin{aligned} f'(x) &= \frac{x}{x^2+x-2} \cdot \frac{(2x+1)x - (x+x^2-2)}{x^2} \\ &= \frac{x^2+2}{x(x+2)(x-1)} \end{aligned}$$

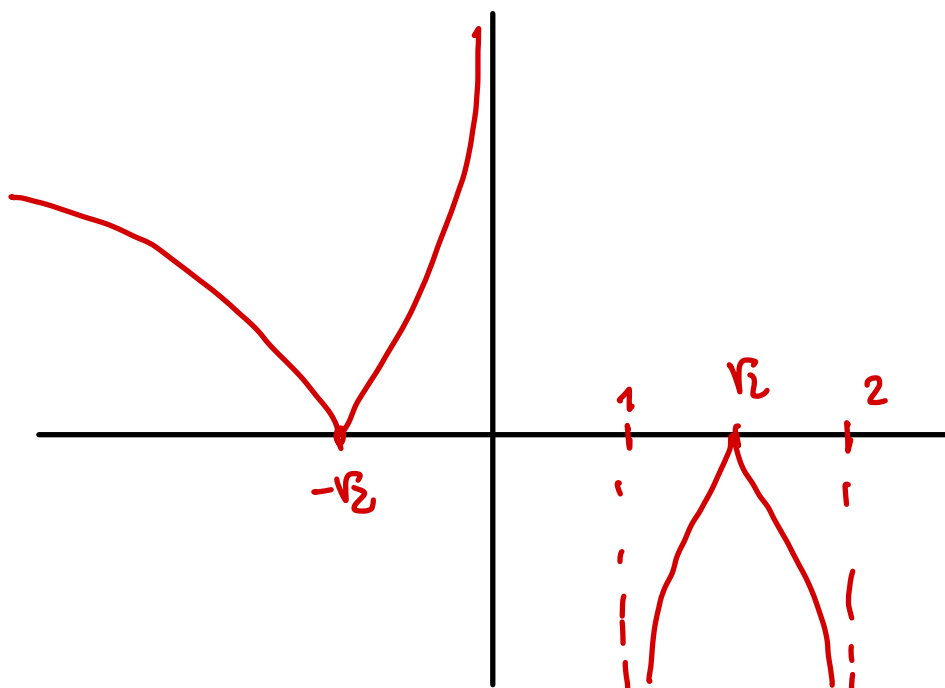


sono punti di non derivabilità!

$$\begin{aligned} \lim_{x \rightarrow \sqrt{2}^+} f'(x) &= \lim_{x \rightarrow \sqrt{2}^+} \frac{x^2+2}{x(x-2)(x+1)} = \frac{4}{\sqrt{2}(\sqrt{2}-2)(\sqrt{2}+1)} \\ \lim_{x \rightarrow \sqrt{2}^-} f'(x) &= \lim_{x \rightarrow \sqrt{2}^-} \frac{x^2+2}{x(x+2)(x-1)} = \frac{4}{\sqrt{2}(\sqrt{2}+2)(\sqrt{2}-1)} \end{aligned} \quad \left| \begin{array}{l} \text{sono} \\ \text{diversi!} \end{array} \right.$$

Calcoli simili si possono fare per
l'altro punto critico.

Nota che $f(-\sqrt{2}) = \log(1) = 0$
 $f(\sqrt{2}) = \log(1) = 0.$



③ Taylor

$$\begin{aligned} f(x) &= 1 - e^{2x^2} = 1 - \left(1 + 2x^2 + \frac{4x^4}{2} + \frac{8x^6}{3!} + o(x^6) \right) \\ &= -2x^2 - 2x^4 - \frac{4}{3}x^6 + o(x^6) \end{aligned}$$

$$g(x) = \sin(y)$$

$$= y - \frac{y^3}{6} + o(y^3)$$

$$y = \log(1+x^2)$$

$$= x^2 - \frac{x^4}{2} + \frac{x^6}{3} + o(x^6)$$

$$= \left(x^2 - \frac{x^4}{2} + \frac{x^6}{3} \right) - \frac{x^6}{6} + o(x^6)$$

$$= x^2 - \frac{x^4}{2} + \frac{x^6}{6} + o(x^6).$$

$$\lim_{x \rightarrow 0^+} \frac{f(x) + 2g(x) + 3x^4}{x^\alpha \cos(x)}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x^6 + o(x^6)}{x^\alpha + o(x^\alpha)}$$

$$\text{sc} \quad \alpha = 6 \Rightarrow L = -1$$

$$\alpha < 6 \Rightarrow L = -\infty$$

$$\alpha > 6 \Rightarrow L = 0.$$

$$z = e^{-i\frac{\pi}{6}} + e^{-i\frac{\pi}{2}}$$

$$\Rightarrow z = \cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) + \cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)$$

$$= \frac{\sqrt{3}}{2} - i \frac{1}{2} + -i$$

$$= \frac{\sqrt{3}}{2} - i \frac{3}{2} = \sqrt{3} \left(\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

$$w^3 = z \quad \text{allora} \quad w = r e^{i\theta}$$

$$\begin{cases} r^3 = \sqrt{3} \\ 3\theta = -\frac{\pi}{3} + 2k\pi \end{cases} \Rightarrow \begin{cases} r = \sqrt[6]{3} \\ \theta = -\frac{\pi}{9} + \frac{2\pi}{3}k \end{cases}$$

$$w_1 = \sqrt[6]{3} e^{-i\frac{\pi}{9}}$$

$$w_2 = \sqrt[6]{3} e^{-i\left(\frac{\pi}{9} + \frac{2\pi}{3}\right)}$$

$$w_3 = \sqrt[6]{3} e^{-i\left(\frac{\pi}{9} - \frac{2\pi}{3}\right)}.$$

Teoria

- ① la successione $a_n = \frac{n \cos(n)}{n+1}$
è limitata in quanto

$$|a_n| = \left| \frac{n \cos(n)}{n+1} \right| \leq \left| \frac{n}{n+1} \right| \leq 1.$$

dunque ammette una sottosucc. convergente
per il teorema di Bolzano

- ② Vedi lezioni

- ③ Per il teorema di Lagrange

$$\begin{aligned} f(1) - f(-1) &= f'(c) (1 - (-1)) \\ &= f'(c) \cdot 2 \end{aligned}$$

allora $f(1) = f(-1) + f'(c) \cdot 2$

per un certo $c \in]-1, 1[$.

D'altra parte $f(c) \leq 3$ allora

$$\begin{aligned} f(1) &\leq f(-1) + 3 \cdot 2 \\ &\leq -1 + 6 = 5 \end{aligned}$$

④ Essendo f continua allora

$$F(x) = \int_a^x f(t) dt \quad \text{è derivabile}$$

$$\text{con } F'(x) = f(x) < 0$$

Dunque f strettamente decrescente

$$\text{Allora } F(b) - F(a) = \int_a^b f(x) dx < 0$$