



$$1. \int \frac{e^x - 1}{e^x + 1} dx = 2 \log(e^x + 1) - x + C$$

$$y = e^x \Rightarrow x = \log y \Rightarrow "dx = \frac{dy}{y}" . \text{ Per sostituzione}$$

$$I = \int \frac{y-1}{y+1} \frac{1}{y} dy = \int \left(\frac{2}{y+1} - \frac{1}{y} \right) dy =$$

$$= 2 \log(y+1) - \log y + C = 2 \log(e^x + 1) - x + C .$$

$$\bullet y' + \frac{e^x - 1}{e^x + 1} y = 0 \quad \text{è lineare del primo ordine}$$

$$A(x) = \int e(x) dx, \quad e(x) = \frac{e^x + 1}{e^x + 1}$$

$$y(x) = k e^{-A(x)}, \quad k \in \mathbb{R}$$

$$= k \frac{e^x}{(e^x + 1)^2}$$

$$\bullet y' + \frac{e^x - 1}{e^x + 1} y = \frac{1}{(e^x + 1)^2}$$

$$\text{se } y(x) = k(x) \frac{e^x}{(e^x + 1)^2} \quad \text{allora}$$

$$y' = k' \frac{e^x}{(e^x + 1)^2} - \frac{e^x - 1}{e^x + 1} y = \frac{1}{(e^x + 1)^2} - \frac{e^x - 1}{e^x + 1} y$$

$$\Rightarrow k' = e^{-x} \Rightarrow k(x) = -e^{-x} + C$$

$$\Rightarrow y(x) = C \frac{e^x}{(e^x + 1)^2} - \frac{1}{(e^x + 1)^2}$$

Per la condizione iniziale

$$-1 = y(0) = c \cdot \frac{1}{4} - \frac{1}{4} \Rightarrow \underline{c = -3}$$

2. $f(x) = |x| \sqrt{1-x^2}$

$$D: 1-x^2 \geq 0 \Rightarrow x \leq 1, D =]-\infty, 1]$$

$$f(1) = 0$$

$$\lim_{x \rightarrow -\infty} |x| \sqrt{1-x^2} = +\infty$$

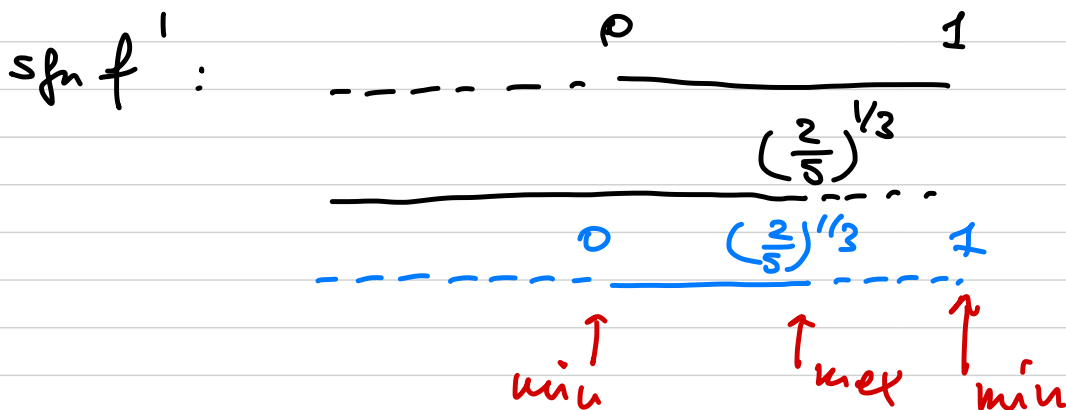
$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} -\sqrt{1-x^2} = -\infty \text{ no estremo obbligato.}$$

derivate in $]-\infty, 1[\setminus \{0\}$

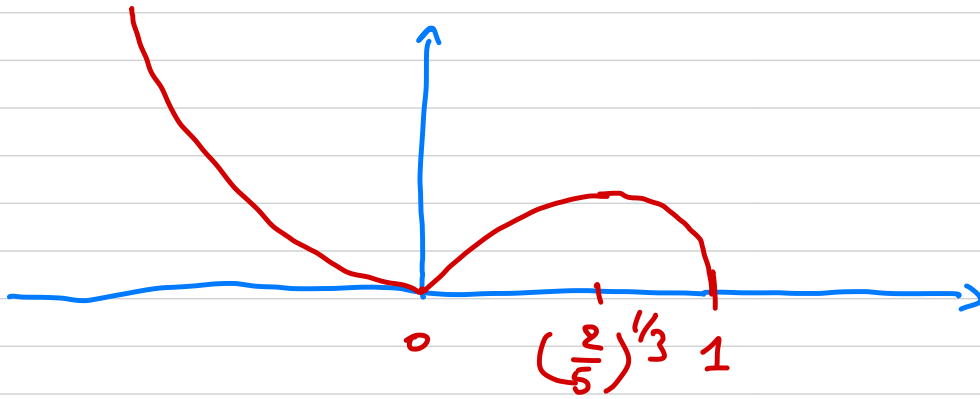
$$f'(x) = \operatorname{sgn}(x) \sqrt{1-x^2} + |x| \frac{-3x^2}{2\sqrt{1-x^2}} =$$

$$= \frac{\operatorname{sgn}(x)}{2\sqrt{1-x^2}} \cdot (2 - 2x^3 + x \cdot (-3x^2))$$

$$= \frac{\operatorname{sgn}(x)}{2\sqrt{1-x^2}} (2 - 5x^3)$$



$$f'(0-) = -1 \quad ; \quad f'(0+) = 1 \quad ; \quad f'(1-) = -\infty$$



$$f(D) = [0, +\infty[\quad , \quad 0 \text{ ist Minimum global.}$$

3. $\log(1 - \sin^2(x)) = -x^2 - \frac{x^4}{6} + o(x^5) \quad , \quad x \rightarrow 0$

$$\begin{aligned} \log(1 - \sin^2(x)) &= -\sin^2 x - \frac{\sin^4 x}{2} + o(x^5) \\ &= -\left(x - \frac{x^3}{6} + o(x^4)\right)^2 - \frac{1}{2}\left(x + o(x^2)\right)^4 + o(x^5) \\ &= -x^2 + \frac{x^4}{3} - \frac{x^4}{2} + o(x^5) = -x^2 - \frac{x^4}{6} + o(x^5). \end{aligned}$$

$$\cos(x^2) - \cosh(x^2) = x^4 + o(x^5) \quad , \quad x \rightarrow 0$$

$$\begin{aligned} \cos(x^2) - \cosh(x^2) &= \left(1 - \frac{x^4}{2} + \frac{x^6}{6} + o(x^6)\right) \\ &\quad - \left(1 + \frac{x^4}{2} + \frac{x^6}{6} + o(x^6)\right) = \\ &= -x^4 + o(x^6) \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x^2 + \log(1 - \sin^2(x))}{\cos(x^2) - \cosh(x^2)} = \lim_{x \rightarrow 0} \frac{-\frac{x^4}{6}}{-x^4} = \frac{1}{6}.$$

$$4. \quad z^3 = (1+i)^{12} = 2^6 \left(\cos\left(\frac{\pi}{4}\right) + i \sin\frac{\pi}{4} \right)^{12}$$

$$= 64 \left(\cos(3\pi) + i \sin(3\pi) \right) = -64$$

$$\Rightarrow |z| = 4$$

$$\text{Arg}(z) = \frac{3\pi + 2k\pi}{3}, \quad k = 0, 1, 2$$

$$= \pi + \frac{2}{3}k\pi, \quad k = 0, 1, 2$$

$$z = -4, \quad 4 \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right), \quad 4 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right).$$