



$$1. \int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx \\ = x (\ln x - 1) + C$$

$$\int \ln^2 x \, dx = x \ln^2 x - \int x \cdot 2 \ln x \cdot \frac{1}{x} \, dx \\ = x \ln^2 x - 2 \int \ln x \, dx \\ = x (\ln^2 x - 2 \ln x + 2) + C$$

$y' = e^y \ln x$ è a variabili separabili.

$y \equiv 1$ è una soluzione costante ma non risolve il problema di Cauchy.

Soluzioni non costanti:

$$\int y'(x) e^{-y(x)} \, dx = \int \ln^2 x \, dx = x (\ln^2 x - 2 \ln x + 2) + C \\ = -e^{-y(x)}$$

$$y(x) = -\ln(-x(\ln^2 x - 2 \ln x + 2) - C)$$

è l'integrale generale. Le soluzioni del problema di Cauchy risolte

$$0 = y(1) = -\ln(-2 - C) \Rightarrow C = -3.$$

$$2. \quad f(x) = \frac{x^2}{\lg|x|-1}$$

f is peri. $D = \mathbb{R} \setminus \{0, \pm e\}$. Study for $x > 0$

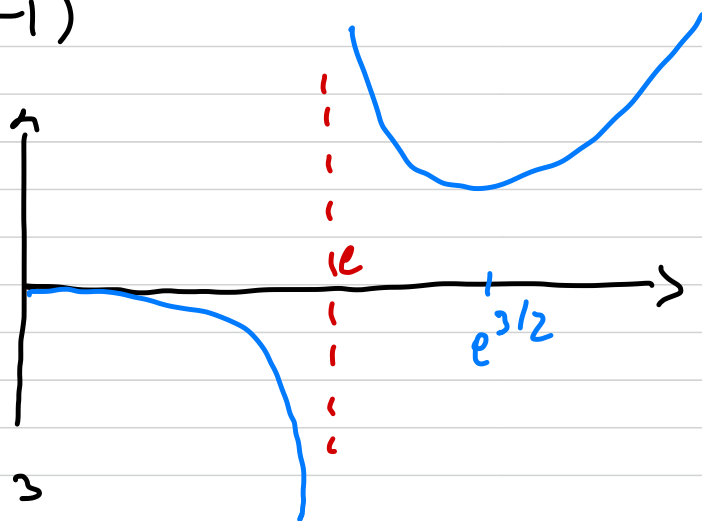
$$\lim_{x \rightarrow +\infty} \frac{x^2}{\lg|x|-1} = +\infty \quad ; \quad \lim_{x \rightarrow 0^+} \frac{x^2}{\lg|x|-1} = 0$$

$$\lim_{x \rightarrow e^\pm} \frac{x^2}{\lg|x|-1} = \pm \infty$$

$$f'(x) = \frac{2x(\lg|x|-1) - x^2 \frac{1}{x}}{(\lg|x|-1)^2} = \frac{x(2\lg|x|-3)}{(\lg|x|-1)^2}$$

sign f' : $\overset{0}{\text{---}} \overset{e^{3/2}}{\text{---}} \text{min} \text{---}$

$$\lim_{x \rightarrow 0^+} \frac{2x \lg x - 3x}{(\lg x - 1)^2} = 0$$



$$f(e^{3/2}) = 2e^3$$

$$f(D) =]-\infty, 0[\cup [2e^3, +\infty[$$

$$3. \bullet e^{\sin^3 x} - 1 = \sin^3 x + \frac{\sin^6 x}{2} + o(x^6), x \rightarrow 0$$

$$= \sin^3 x + o(x^5)$$

$$\sin^3 x = \left(x - \frac{x^3}{6} + \frac{x^5}{5!} + o(x^5) \right)^3 =$$

$$= x^3 + 3x^2 \left(-\frac{x^3}{6} \right) + o(x^5) = x^3 - \frac{x^5}{2} + o(x^5)$$

$$\Rightarrow e^{\sin^3 x} - 1 = x^3 - \frac{x^5}{2} + o(x^5), x \rightarrow 0.$$

$$\bullet \log(1 - 2x^3) = -2x^3 - \frac{(-2x^3)^2}{2} + o(x^6)$$

$$= -2x^3 + o(x^5), x \rightarrow 0$$

$$\bullet \lim_{x \rightarrow 0} \frac{e^{\sin^3 x} - 1}{\log(1 - 2x^3)} = \lim_{x \rightarrow 0} \frac{x^3}{-2x^3} = -\frac{1}{2}.$$

$$4. \left(\frac{z}{i} + 1 \right)^4 = 16$$

$$(1 - iz)^4 = 16$$

$$(1 - iz)^2 = \pm 4$$

$$-z^2 - 2iz + 1 = \pm 4$$

$$(i) \quad z^2 + 2iz + 3 = 0$$

$$z = -i \pm \sqrt{-1-3} = -i \pm 2i = \begin{array}{l} i \\ -3i \end{array}$$

$$(ii) \quad z^2 + 2iz - 5 = 0$$

$$z = -i \pm \sqrt{-1+5} = -i \pm 2 = \begin{array}{l} 2-i \\ -2-i \end{array}$$