



$$1. \int_0^{\pi/4} \tan(x) dx = \int_0^{\pi/4} \frac{\sin x}{\cos x} dx = -\log(\cos x) \Big|_0^{\pi/4} = \log \sqrt{2}$$

oppure $y = \tan(x)$, $x = \arctan y$ "dx = $\frac{1}{1+y^2} dy$ "

$$= \frac{1}{2} \int_0^1 \frac{2y}{1+y^2} dy = \frac{1}{2} \log(1+y^2) \Big|_0^1 = \frac{1}{2} \log 2.$$

$y' = \tan(x) y + 4$ è lineare del 1° ordine

$y' = \tan(x) y$ omogenea esplicita

$$\int \tan(x) dx = -\log \cos x$$

$y(x) = k e^{-\log \cos x}$ $k \in \mathbb{R}$

integrali generali omogenea associata

$y(x) = k(x) e^{-\log \cos x}$ soluzioni particolari

$$y' = k' e^{-\log \cos x} + k e^{-\log \cos x} \tan(x)$$

$$\stackrel{!}{=} \tan(x) y + 4$$

$$\Rightarrow k' e^{-\log \cos x} = 4 \Rightarrow k' = 4 \cos x$$

$$\Rightarrow k(x) = 4 \sin x + c, \quad c \in \mathbb{R}$$

$$y(x) = (4 \sin x + c) e^{-\log \cos x} = \frac{4 \sin x + c}{\cos x}$$

è l'integrale generale.

$$1 = y(0) = \frac{c}{1} \Rightarrow y(x) = \frac{4 \sin x + 1}{\cos x}$$

risolvere problema di Cauchy

$$(\text{sec.}) \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{4 \sin x + c}{\cos x} = \begin{cases} +\infty & \text{se } c > -4 \\ -\infty & \text{se } c < -4 \end{cases}$$

$$\text{se } \boxed{c = -4}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{4(\sin x - 1)}{\cos x} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{4 \cos x}{-\sin x} = 0$$

de l'Hopital

$$2. f(x) = |x| e^{\arctan \frac{4}{x}}$$

$$D = \mathbb{R} \setminus \{0\}.$$

$$\lim_{x \rightarrow 0^\pm} |x| e^{\arctan \frac{4}{x}} = 0$$

$\rightarrow 0 \quad \rightarrow e^{\pm \pi/2}$

$$\lim_{x \rightarrow \pm\infty} |x| e^{\arctan \frac{4}{x}} = +\infty \cdot 1 = +\infty$$

Asintoto obliquo:

$$\lim_{x \rightarrow \pm\infty} \frac{|x| e^{\arctan \frac{4}{x}}}{x} = (\pm 1) \cdot 1 = \pm 1$$

$$\lim_{x \rightarrow +\infty} x e^{\arctan \frac{4}{x}} - x = \lim_{x \rightarrow +\infty} x (e^{\arctan \frac{4}{x}} - 1) =$$

$$= \lim_{x \rightarrow +\infty} x \arctan \frac{4}{x} = \lim_{x \rightarrow +\infty} x \cdot \frac{4}{x} = 4$$

$\Rightarrow y = x + 4$ è es. obliquo per $x \rightarrow +\infty$

$$\lim_{x \rightarrow -\infty} -x e^{\arctan \frac{4}{x}} + x = \lim_{x \rightarrow -\infty} -x (e^{\arctan \frac{4}{x}} - 1) = -4$$

$\Rightarrow y = -x - 4$ es. obliquo per $x \rightarrow -\infty$.

Derivate in $\mathbb{R} \setminus \{0\}$

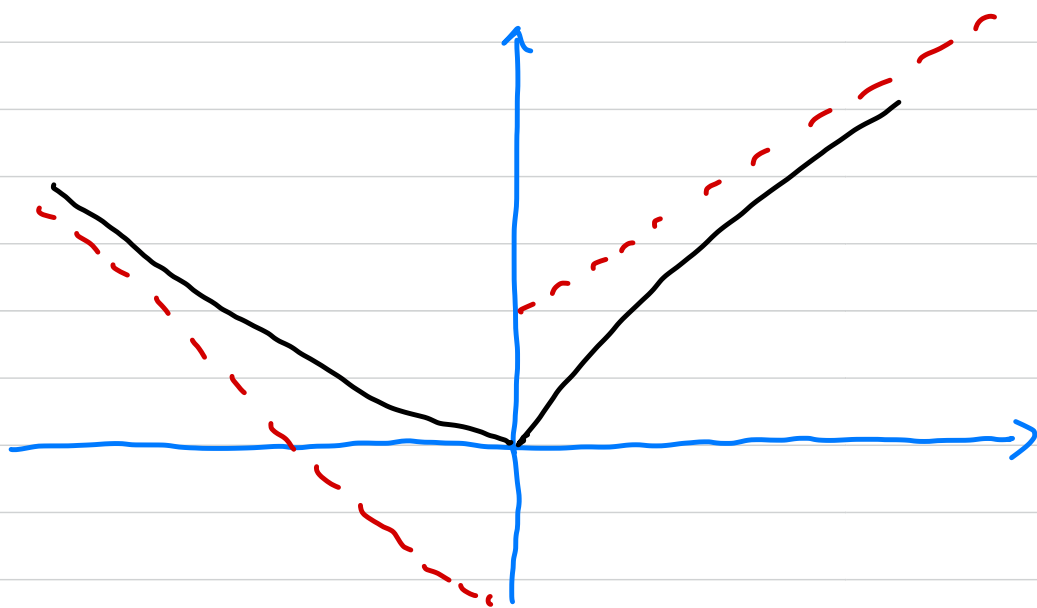
$$f'(x) = e^{\arctan \frac{4}{x}} \left(\operatorname{sgn} x + \frac{|x|}{1 + \frac{16}{x^2}} \cdot \frac{-4}{x^2} \right) =$$

$$= \underbrace{e^{\arctan \frac{4}{x}}}_{>0} \cdot \operatorname{sgn} x \cdot \underbrace{\left(\frac{x^2 - 4x + 16}{x^2 + 16} \right)}_{>0}$$

sgn f' 

decreciente in $]-\infty, 0[$, crescente in $]0, +\infty[$

non ha stami locali.



$$\lim_{x \rightarrow 0^+} f'(x) = e^{\pi/2}, \quad \lim_{x \rightarrow 0^-} f'(x) = -e^{-\pi/2}$$

3. $f(x) = e^{\sin x} - e^x \quad (x \rightarrow 0)$

$$e^y = 1 + y + \frac{y^2}{2} + \frac{y^3}{6} + o(y^3) \quad \text{as } y \rightarrow 0$$

$$e^{4x} = 1 + 4x + 8x^2 + \frac{32}{3}x^3 + o(x^3)$$

$$e^{\sin 4x} = 1 + \underbrace{\sin(4x)}_{4x - \frac{64}{6}x^3 + o(x^3)} + \frac{1}{2} \underbrace{\sin^2(4x)}_{\frac{1}{2} 16x^2 + o(x^3)} + \frac{1}{6} \underbrace{\sin^3(4x)}_{\frac{1}{6} 64x^3 + o(x^3)} + o(x^3)$$

$$\Rightarrow e^{\sin(4x)} - e^{4x} = 1 + 4x - \frac{32}{3}x^3 + 8x^2 + \frac{32}{3}x^3 - 1 - 4x - 8x^2 - \frac{32}{3}x^3 + o(x^3)$$

$$= -\frac{32}{3}x^3 + o(x^3) \quad \text{for } x \rightarrow 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos(x^2)}{e^{\sin(4x)} - e^{4x}} = \lim_{x \rightarrow 0} \frac{x^4/2}{-\frac{32}{3}x^3} = 0.$$

$$4. \int 4^x \arctan 4^x dx =$$

$$y = 4^x = e^{x \log 4} \quad ; \quad "dy = 4^x \log 4 dx"$$

$$= \int \frac{1}{\log 4} \arctan y dy \Big|_{y=4^x} = \frac{1}{\log 4} \left(y \arctan y - \int \frac{y}{1+y^2} dy \right)$$

$$= \frac{1}{\log 4} \left(y \arctan y - \frac{1}{2} \log(1+y^2) + C \right) \Big|_{y=4^x} =$$

$$= \frac{1}{\log 4} \left(4^x \arctan 4^x - \frac{1}{2} \log(1+16^x) + C \right) \quad C \in \mathbb{R}$$