



Tema A

$$1. \int \frac{1}{x^2 + 4x + 5} dx = \int \frac{1}{(x+2)^2 + 1} dx = \\ = \arctan(x+2) + C, \quad C \in \mathbb{R}$$

$y' = y^2 + 4y + 5$ $\bar{t}$ e variabili separabili
non ha soluzioni costanti

$$\int 1 dx = x + C \quad C \in \mathbb{R}$$

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$$\int \frac{y'}{y^2 + 4y + 5} dx = \arctan(y(x) + 2)$$

$$\Rightarrow \arctan(y(x) + 2) = x + C$$

$$\text{se } y(0) = 1 \Rightarrow$$

$$\arctan(1+2) = C$$

$$\Rightarrow y(x) = \tan(x + \arctan(3)) - 2$$

$$\text{per } -\frac{\pi}{2} - \arctan 3 < x < \frac{\pi}{2} - \arctan 3$$

2. $f(x) = e^x \left| \frac{x}{x-2} \right| \quad (x \geq 0)$

$$D = \mathbb{R} \setminus \{2\}$$

$$\lim_{x \rightarrow +\infty} f(x) = +\infty \cdot 1 = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = 0 \cdot 1 = 0$$

$$\lim_{x \rightarrow 2} f(x) = e^2 \cdot +\infty = +\infty$$

$x=2$ asintota verticale

$y=0$ asintota orizzontale per $x \rightarrow -\infty$

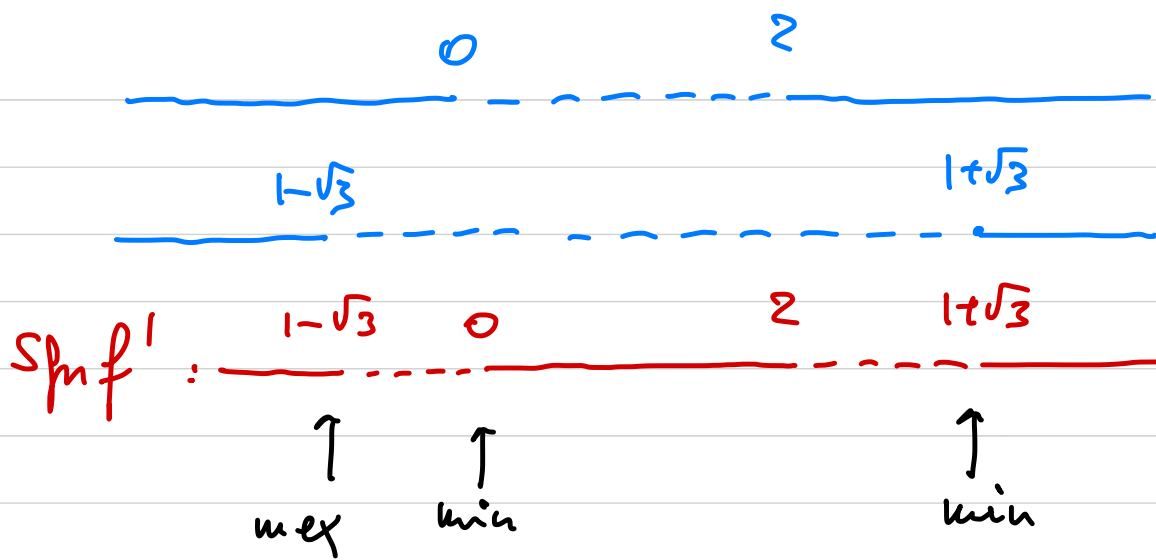
Se $x \neq 2$, $x \neq 0$

$$f'(x) = e^x \left(\left| \frac{x}{x-2} \right| + \operatorname{sgn} \left(\frac{x}{x-2} \right) \frac{x-2-x}{(x-2)^2} \right) =$$

$$= \frac{e^x}{(x-2)^2} \operatorname{sgn} \left(\frac{x}{x-2} \right) (x(x-2) - 2)$$

$|x| = x \operatorname{sgn} x$ $x^2 - 2x - 2$
 $(x = 1 \pm \sqrt{1+2})$

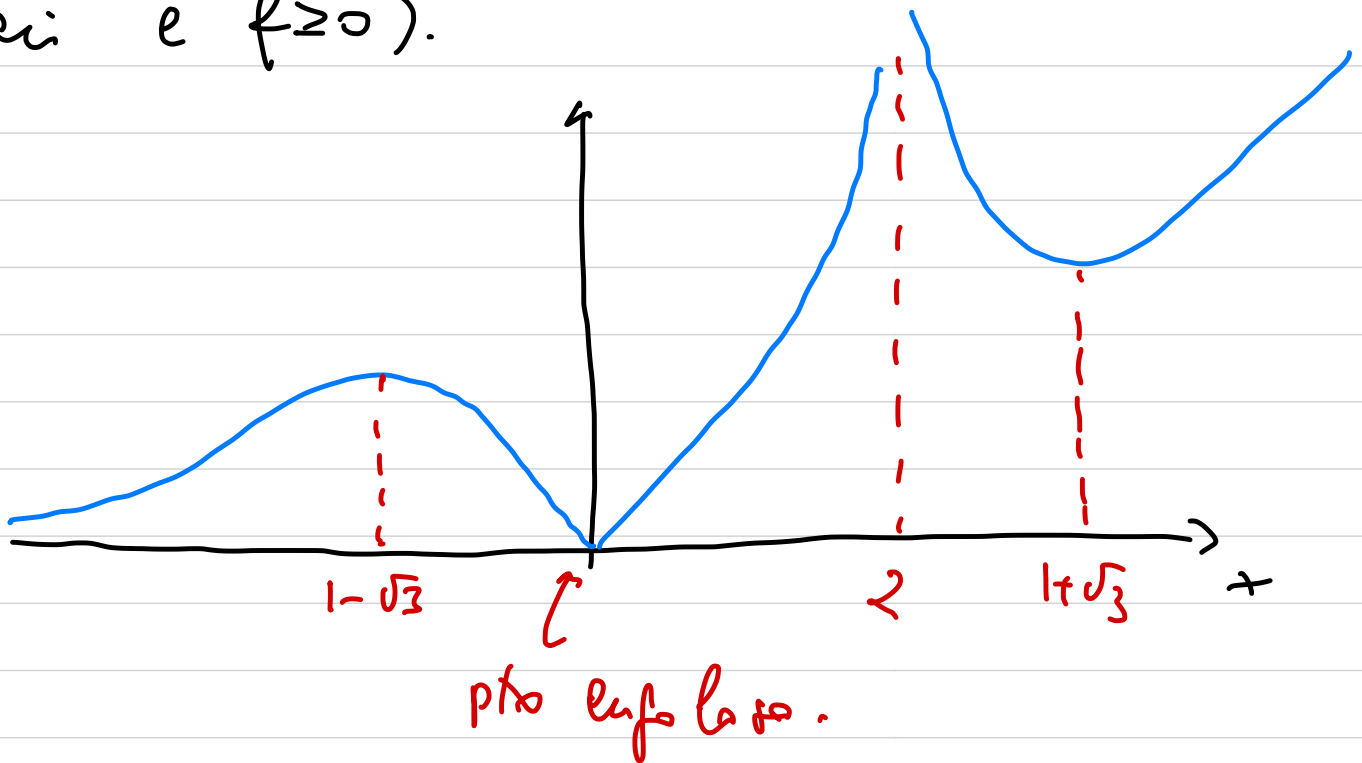
$$\operatorname{sgn} f' = \operatorname{sgn} \left(\frac{x}{x-2} \right) \cdot \operatorname{sgn} (x^2 - 2x - 2)$$



$$\lim_{x \rightarrow 0^+} f'(x) = \frac{1}{2} \quad ; \quad \lim_{x \rightarrow 0^-} f'(x) = -\frac{1}{2}$$

$$f(D) = [0, +\infty[\quad 0 \text{ è minimo assoluto}$$

(già $f([0, 2[) = [0, +\infty[$ per il teorema degli zeri e $f \geq 0$).



3. $f(x) = e^{-2x} - (1 + \sin(2x))^{-1} + 2 \log(1+x^2)$

de Teylor en $x_0 = 0$:

$$e^{-2x} = 1 - 2x + 4 \frac{x^2}{2} - \frac{8x^3}{6} + o(x^3)$$

$$2 \log(1+x^2) = 2x^2 - 2 \frac{x^4}{2} + o(x^4)$$

$$\begin{aligned} (1 + \sin(2x))^{-1} &= 1 - \sin(2x) + \sin^2(2x) - \sin^3(2x) + o(x^3) \\ &= 1 - 2x + \frac{8x^3}{6} + 4x^2 - 8x^3 + o(x^3) \end{aligned}$$

$$\Rightarrow f(x) = x^3 \left(-\frac{4}{3} - \frac{4}{3} + 8 \right) + o(x^3)$$

$$a = \frac{16}{3}, \quad b = 3$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{(\ln(x))^3} = \lim_{x \rightarrow 0} \frac{\frac{16}{3} x^3}{x^3} = \frac{16}{3}$$

4. $\frac{z^3}{z^3 + 2} = 1 - \frac{i}{\sqrt{3}}$

$$\Leftrightarrow z^3 = z^3 + 2 - \frac{i}{\sqrt{3}} (z^3 + 2), \quad z^3 \neq -2$$

$$\Leftrightarrow 2 = \frac{i}{\sqrt{3}} (z^3 + 2)$$

$$\Leftrightarrow z^3 + 2 = -2\sqrt{3}i$$

$$\Leftrightarrow z^3 = -2 - 2\sqrt{3}i = 4 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) =$$

$$= 4 \left(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi \right) \quad \text{forme trigonométrique}$$

$$\Rightarrow |z| = 4^{1/3}$$

$$\text{Arg}(z) = \frac{4}{3}\pi + k \frac{2\pi}{3}, \quad k=0, 1, 2$$

