

9.4. Calcolare i seguenti integrali:

1. $\int_0^1 \sqrt{x} dx$
2. $\int_{-1}^1 e^x dx$
3. $\int_0^1 e^{2x} dx$
4. $\int_0^{\frac{\pi}{2}} \cos x dx$
5. $\int_0^1 \frac{dx}{1+x^2}$
6. $\int_0^1 \frac{dx}{\cos^2 x}$
7. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$
8. $\int_0^1 (x^2 - 3x) dx$
9. $\int_0^{\frac{1}{2}} \left(x - \frac{1}{\sqrt{1-x^2}} \right) dx$
10. $\int_0^2 \frac{dx}{\sqrt{1+x}}$
11. $\int_0^1 \sqrt[4]{x^3} dx$
12. $\int_0^1 \frac{dx}{x+3}$
13. $\int_{-2}^{-1} \frac{dx}{x}$
14. $\int_0^1 \frac{dx}{(x+1)(x+2)}$
15. $\int_0^1 \left(\frac{1}{x+1} - \frac{1}{x+2} \right) dx$
16. $\int_1^2 \frac{dx}{x^3}$
17. $\int_0^2 \frac{dx}{(x-1)^4}$
18. $\int_0^2 \frac{dx}{(x+1)^4}$

9.9. Calcolare i seguenti integrali:

1. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2 + \cos x} dx$
2. $\int_1^3 \frac{\log x}{x} dx$
3. $\int_0^1 \frac{x dx}{1+x^4}$
4. $\int_{-1}^1 \frac{e^x dx}{1+e^x}$
5. $\int_1^e \frac{\cos \log x}{x} dx$
6. $\int_0^1 e^x \operatorname{tg} e^x dx$
7. $\int_e^{e^2} \frac{dx}{x \log x}$
8. $\int_0^{\frac{1}{\sqrt{2}}} \frac{x dx}{\sqrt{1-x^4}}$
9. $\int_0^1 x^2 \cos x^3 dx$
10. $\int_0^{\frac{\pi}{2}} \sin 2x (1 + \cos^2 x) dx$
11. $\int_1^e \frac{\log x + \log^2 x}{x} dx$
12. $\int_1^e \frac{\log x}{x(1 + \log x)} dx$
13. $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 + \cos^4 x} dx$

9.10. Calcolare i seguenti integrali indefiniti:

1. $\int \frac{t}{1+t} dt$
2. $\int \frac{x dx}{1+x^2}$
3. $\int \frac{x^2+x}{1+x^2} dx$
4. $\int \frac{dx}{\sqrt{1-4x^2}}$
5. $\int x \cos x^2 \sin^2 x^2 dx$
6. $\int \frac{x \log x + 1}{x^2} dx$
7. $\int \cos x e^{2 \sin x} dx$
8. $\int \frac{e^x dx}{1+2e^{2x}}$
9. $\int \frac{2x+1}{x^2+x+3} dx$
10. $\int \frac{x dx}{\sqrt{1+x^2}}$
11. $\int \sin^3 x dx$
12. $\int x e^{-x^2} dx$
13. $\int \frac{x+2}{\sqrt{1-x^2}} dx$

9.11. Calcolare i seguenti integrali, e verificare il risultato derivando:

1. $\int x \log x dx$
2. $\int x^2 e^{-x} dx$
3. $\int x \sin x dx$
4. $\int (x^3 + 1) \log x dx$
5. $\int 2x \operatorname{arctg} x dx$
6. $\int e^x \cos x dx$
7. $\int 2x \log(x+1) dx$
8. $\int 2x \operatorname{arcsen} x dx$

9.12. Calcolare i seguenti integrali e verificare i risultati:

1. $\int \frac{dx}{(x+1)(x-2)}$
2. $\int \frac{dx}{x(x+2)}$
3. $\int \frac{dx}{x^2-x}$
4. $\int \frac{x dx}{(x-3)(x+1)}$
5. $\int \frac{x^2-2}{(x^2+x)(x-2)} dx$
6. $\int \frac{x+3}{x+3x^2} dx$
7. $\int \frac{dx}{x^2-4}$
8. $\int \frac{1-x}{(1+x)(1+3x)} dx$
9. $\int \frac{x^2+2}{x^3-4x} dx$
10. $\int \frac{2x-1}{2x^2+x} dx$
11. $\int \frac{dx}{(x^2-1)(x^2-4)}$

9.13. Calcolare i seguenti integrali, e verificare i risultati ottenuti:

$$\begin{array}{lll} 1. \int \frac{dx}{x(x+1)^2} & 4. \int \frac{x-3}{x(x^2-2x+1)} dx & 7. \int \frac{dx}{(x^2-1)(x^2+1)} \\ 2. \int \frac{dx}{x(x+1)(x-1)^2} & 5. \int \frac{x^2+1}{(x+1)^2(1-x)} dx & 8. \int \frac{x+3}{x^3(x-1)} dx \\ 3. \int \frac{dx}{x(x^2+1)} & 6. \int \frac{dx}{x^2(x^2-3x+2)} & 9. \int \frac{x-2}{x(x^2+4)} dx \end{array}$$

9.14. Calcolare i seguenti integrali indefiniti:

$$\begin{array}{lll} 1. \int \frac{dx}{\sqrt{1+x^2}} & 4. \int \frac{dx}{\sqrt{1+\sqrt{x}}} & 7. \int \frac{dx}{\sin x} \\ 2. \int \sqrt{1+x^2} dx & 5. \int \frac{dx}{(1+x^2)\sqrt{1+x^2}} & 8. \int \frac{2-\sin^2 x}{\sin 2x} dx \\ 3. \int \frac{1-x^2}{\sqrt{1+x^2}} dx & 6. \int \frac{\sin x}{1-\sin x} dx & \end{array}$$

9.15. Calcolare i seguenti integrali definiti:

$$1. \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^2 x} dx \quad 2. \int_0^1 \frac{e'+2}{e'(e'+1)} dt \quad 3. \int_0^{\frac{5}{4}} \frac{x+\sqrt{x+1}}{2x-3\sqrt{x+1}} dx \quad 4. \int_0^1 \frac{\sqrt{x}+2}{(\sqrt{x}+1)(x+1)} dx$$

$$\begin{array}{lll} 1. \int \frac{dx}{(x+1)(x+2)(x+3)} & 2. \int \frac{x-1}{4x^3-x} dx & 3. \int \frac{x^4+9}{x^2-5x+6} dx \\ 4. \int \frac{x-3}{2x^3-7x^2+5x} dx & 5. \int \frac{x}{x^3-1} dx & 6. \int \frac{dx}{x^3+x} \\ 7. \int \frac{x^5+4}{x^2+3x+2} dx & 8. \int \frac{dx}{3x^3+4x^2-4x} & 9. \int \frac{dx}{x(x^3+1)} \\ 10. \int \frac{x(x+3)}{x^4-1} dx & 11. \int \frac{dx}{x^2(x+1)} & 12. \int \frac{x-3}{x^4+x^2} dx \\ 13. \int \frac{t dt}{(t+2)(t-1)} & 14. \int \frac{dt}{t(t+1)(t-3)} & 15. \int \frac{x^3 dx}{(x+1)(x-2)} \\ 16. \int \frac{x^2 dx}{(1-x)(x-2)} & 17. \int \frac{t^3+3}{(t+1)(t^2+1)} dt & 18. \int \frac{x^2-1}{x^2(x^2+1)} dx \\ 19. \int \frac{dt}{t^2(t+1)^2} & 20. \int \frac{t dt}{(t^2+2)(t-2)} & 21. \int \frac{dt}{t(t^2+2t+3)} \\ 22. \int \frac{dt}{(t+1)(t^2+1)^2} & 23. \int \frac{x+3}{(x^4-16)} dx & \end{array}$$

$$\begin{array}{lll} 24. \int \frac{dx}{3 \sin x + 4 \cos x} & 25. \int \frac{1+\operatorname{tg} x}{\cos x} dx & 26. \int \frac{\sin x}{1-\sin x} dx \\ 27. \int \frac{dx}{\sin x} & 28. \int \frac{1+\cos x}{4 \sin x - 3 \cos x} dx & 29. \int \frac{dx}{1+\cos x} \\ 30. \int \frac{2-\sin^2 x}{\sin 2x} dx & 31. \int \frac{1+\sin 4x}{(\sin x - \cos x) \cos x} dx & \\ 32. \int \frac{\sin x}{(1-\sin x)(1+\cos x)} dx & 33. \int \frac{2 \sin x + \sin^3 x}{\cos^3 x} dx & \\ 34. \int \frac{\cos x}{(1-\sin x)(1-\cos x)} dx & 35. \int \frac{1+2 \cos^2 x}{1+2 \sin^2 x} dx & \\ 36. \int \frac{\cos x}{4 \sin x - 3 \cos x} dx & 37. \int \frac{dx}{1+\sin^2 x} & 38. \int \frac{dx}{\sin 2x + \cos^2 x} \\ 42. \int \frac{x-4}{\sqrt{x^2-4x+5}} dx & 43. \int \frac{\sqrt{x^2-1}}{x(2x-1)} dx & 44. \int \frac{x^3}{\sqrt{x^2+x}} dx \\ 45. \int \frac{\sqrt[3]{1+\sqrt{x}}}{\sqrt{x}} dx & 46. \int \frac{dx}{\sqrt[4]{x} \sqrt{1+\sqrt{x}}} & 47. \int \sqrt{\frac{3+4\sqrt{x}}{x}} \frac{dx}{x} \\ 48. \int \frac{3-x^2}{\sqrt{2-x^2}} dx & 49. \int \frac{dx}{(1+x^2)\sqrt{1-x^2}} & 50. \int \frac{1-x^2}{\sqrt{1+x^2}} dx \\ 51. \int \frac{(1+x^2)^{3/4}}{\sqrt{x}} dx & 52. \int \frac{\sqrt{1-x^2}+1}{x^2} dx & 53. \int \frac{dx}{\sqrt{1+\sqrt{x}}} \\ 54. \int \sqrt[3]{\frac{4-x}{x}} dx & 55. \int \frac{dx}{(1+x^2)\sqrt{1+x^2}} & 56. \int \frac{\sqrt{1+\sqrt[3]{x}}}{x} dx \\ 57. \int \sqrt{1+\sqrt{x}} dx & 58. \int \frac{\sqrt{1+x}-\sqrt{x}}{5\sqrt{x}+3\sqrt{1+x}} dx & 59. \int \frac{2x+1-2\sqrt{x^2+x}}{4\sqrt{x}+5\sqrt{1+x}} dx \\ 60. \int \frac{dx}{(x+1)\sqrt{x^2+3}} & 61. \int \sqrt{\frac{4+e^x}{1-e^x}} dx & 62. \int \sqrt{\frac{1}{1-\sin x}} dx \\ 63. \int \sqrt{1+e^x} dx & 64. \int \sqrt{\frac{\cos x}{1-\sin x}} dx & 65. \int \sqrt{1+e^{2x}} e^{-x} dx \\ 66. \int \frac{dx}{\sqrt{1+e^{3x}}} & 67. \int \frac{dx}{\sqrt{(1-2 \sin x)(1+\cos x)}} & 68. \int \sqrt{e^{2x}-3e^x+1} dx \\ 69. \int \sqrt[4]{1+e^{4x}} e^{-x} dx & 70. \int \sqrt{1+\sqrt{4e^x+1}} dx & 71. \int \frac{dx}{\sqrt{e^{2x}+2e^x}} \end{array}$$

$$72. \int x \sin^2 x \, dx$$

$$74. \int \frac{\arcsin x}{\sqrt{1+x}} \, dx$$

$$76. \int \arcsin x \, dx$$

$$78. \int \frac{x+1}{x^2} \ln x \, dx$$

$$80. \int (x-1) \ln(x + \sqrt{x^2+2}) \, dx$$

$$82. \int e^{2x} \ln(1 - e^{-x}) \, dx$$

$$84. \int x(\operatorname{arctg} x)^2 \, dx$$

$$86. \int x^2 \operatorname{arctg} x \, dx$$

$$88. \int x^2 \arcsin x \, dx$$

$$90. \int e^{\beta x} \cos \alpha x \, dx$$

$$92. \int x^3 (\operatorname{arctg} x)^2 \, dx$$

$$94. \int \sin x \ln(\sin x) \, dx$$

$$96. \int (\sin x + \cos x) \ln(\sin x - \cos x) \, dx$$

$$98. \int \arcsin \sqrt{x} \, dx$$

$$100. \int_0^{\pi/2} \sin^n x \, dx.$$

$$73. \int \operatorname{arctg} x \, dx$$

$$75. \int \left(x + \frac{1}{x^2}\right) \operatorname{arctg} x \, dx$$

$$77. \int x \arccos x \, dx$$

$$79. \int x \ln(x^3 - 1) \, dx$$

$$81. \int \sin x \ln \operatorname{tg} x \, dx$$

$$83. \int x \ln(x + \sqrt{1-x^2}) \, dx$$

$$85. \int x^2 (\ln x)^3 \, dx$$

$$87. \int x(1+x^2) e^{x^2} \ln x \, dx$$

$$89. \int e^x \sin 2x \, dx$$

$$91. \int e^x \cos^2 x \, dx$$

$$93. \int \frac{\operatorname{arctg} x}{(1+x)^2} \, dx$$

$$95. \int \frac{\sin t \ln \sin t}{1 - \sin^2 t} \, dt$$

$$97. \int e^{\beta x} \sin \alpha x \, dx$$

$$99. \int \frac{\operatorname{arctg} \left(\frac{1}{2} - \sqrt{x}\right)}{\sqrt{x}} \, dx$$

9.6. Calcolare l'area delle regioni di piano seguenti:

$$1. \{(x, y) : 0 \leq x \leq \pi; \sin x \leq y \leq x + 2\}$$

$$2. \{(x, y) : 0 \leq x \leq 1; x^2 + 1 \leq y \leq e^x\}$$

$$3. \left\{ (x, y) : 1 \leq x \leq 2; \frac{1}{x} \leq y \leq x \right\}$$

$$4. \{(x, y) : 0 \leq x \leq 2; 1 - x^2 \leq y \leq 1 + x^2\}$$

$$5. \left\{ (x, y) : 0 \leq x \leq 1; \frac{1}{1+x^2} \leq y \leq 1+x \right\}$$

$$6. \left\{ (x, y) : 0 \leq x \leq 1; \frac{1}{(1+x)^2} \leq y \leq 1+x^3 \right\}$$

9.7. I grafici delle funzioni $\sin x$ e $\cos x$, $0 < x < 2\pi$, racchiudono una regione piana. Calcolarne l'area.

9.8. Calcolare l'area della regione finita compresa tra le curve $y = \sqrt{x}$ e $y = x^2$.