

9.4.

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|---------------------------|----------------------------------|------------------------|
| 1. $\frac{2}{3}$ | 7. $\frac{\pi}{3}$ | 13. $-\log 2$ |
| 2. $e - \frac{1}{e}$ | 8. $-\frac{7}{6}$ | 14. $\log \frac{4}{3}$ |
| 3. $\frac{1}{2}(e^2 - 1)$ | 9. $\frac{1}{8} - \frac{\pi}{6}$ | 15. $\log \frac{4}{3}$ |
| 4. 1 | 10. $2(\sqrt{3} - 1)$ | 16. $\frac{3}{8}$ |
| 5. $\frac{\pi}{4}$ | 11. $\frac{4}{7}$ | 17. Non esiste |
| 6. $\lg 1$ | 12. $\log \frac{4}{3}$ | 18. $\frac{26}{81}$ |

9.9.

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|---------------------------|---------------------------------|---------------------|
| 1. $\log \frac{3}{2}$ | 6. $\log \frac{\cos e}{\cos 1}$ | 10. $\frac{3}{2}$ |
| 2. $\frac{1}{2} \log^2 3$ | 7. $\log 2$ | 11. $\frac{5}{6}$ |
| 3. $\frac{\pi}{8}$ | 8. $\frac{\pi}{12}$ | 12. $1 - \log 2$ |
| 4. 1 | 9. $\frac{1}{3} \sin 1$ | 13. $\frac{\pi}{8}$ |
| 5. $\sin 1$ | | |

9.10.

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|---|---|--|
| 1. $t - \log 1 + t $ | 4. $\frac{1}{2} \arcsin 2x$ | 7. $\frac{1}{2} e^{2 \sin x}$ |
| 2. $\frac{1}{2} \log(1 + x^2)$ | 5. $\frac{1}{6} \sin^3 x^2$ | 8. $\frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2} e^x)$ |
| 3. $x + \frac{1}{2} \log(1 + x^2) - \operatorname{arctg} x$ | 6. $\frac{1}{2} \log^2 x - \frac{1}{x}$ | 9. $\log(x^2 + x + 3)$ |
| 10. $\sqrt{1 + x^2}$ | 12. $-\frac{1}{2} e^{-x^2}$ | |
| 11. $\frac{1}{3} \cos^3 x - \cos x$ | 13. $-\sqrt{1 - x^2} + 2 \arcsin x$ | |

9.11.

- | | |
|---|--|
| 1. $\frac{x^2}{2} \left(\log x - \frac{1}{2} \right)$ | 5. $(x^2 + 1) \operatorname{arctg} x - x$ |
| 2. $-(x^2 + 2x + 2) e^{-x}$ | 6. $\frac{1}{2} e^x (\sin x + \cos x)$ |
| 3. $\sin x - x \cos x$ | 7. $(x^2 - 1) \log(x + 1) - \frac{x^2}{2} + x$ |
| 4. $\left(\frac{x^4}{4} + x \right) \log x - \frac{x^4}{16} - x$ | 8. $\left(x^2 - \frac{1}{2} \right) \arcsin x + \frac{1}{2} x \sqrt{1 - x^2}$ |

9.12.

- | | |
|---|---|
| 1. $\frac{1}{3} (\log x - 2 - \log x + 1)$ | 4. $\frac{3}{4} \log x - 3 + \frac{1}{4} \log x + 1 $ |
| 2. $\frac{1}{2} (\log x - \log x + 2)$ | 5. $\log x - \frac{1}{3} \log x + 1 + \frac{1}{3} \log x - 2 $ |
| 3. $\log x - 1 - \log x $ | 6. $3 \log x - \frac{8}{3} \log 3x + 1 $ |
| 7. $\frac{1}{4} (\log x - 2 - \log x + 2)$ | |
| 8. $-\log 1 + x + \frac{2}{3} \log 1 + 3x $ | |
| 9. $-\frac{1}{2} \log x + \frac{3}{4} \log x - 2 + \frac{3}{4} \log x + 2 $ | |
| 10. $-\log x + 2 \log 2x + 1 $ | |
| 11. $\frac{1}{6} \log x + 1 - \frac{1}{6} \log x - 1 + \frac{1}{12} \log x - 2 - \frac{1}{12} \log x + 2 $ | |

9.13.

1. $\log|x| - \log|x+1| + \frac{1}{x+1}$
2. $\log|x| - \frac{1}{4}\log|x+1| - \frac{3}{4}\log|x-1| - \frac{1}{2(x-1)}$
3. $\log|x| - \frac{1}{2}\log(x^2+1)$
4. $-3\log|x| + 3\log|x-1| + \frac{2}{x-1}$
5. $-\frac{1}{2}\log|1-x| - \frac{1}{2}\log|x+1| - \frac{1}{x+1}$
6. $\frac{3}{4}\log|x| - \frac{1}{2x} - \log|x-1| + \frac{1}{4}\log|x-2|$
7. $\frac{1}{4}\log|x-1| - \frac{1}{4}\log|x+1| - \frac{1}{2}\operatorname{arctg} x$
8. $-4\log|x| + \frac{4}{x} + \frac{3}{2x^2} + 4\log|x-1|$
9. $-\frac{1}{2}\log|x| + \frac{1}{4}\log(x^2+4) + \frac{1}{2}\operatorname{arctg} \frac{x}{2}$

9.14.

1. $\log(x + \sqrt{1+x^2})$
2. $\frac{1}{2}(x\sqrt{1+x^2} + \log(x + \sqrt{1+x^2}))$
3. $-\frac{1}{2}x\sqrt{1+x^2} + \frac{3}{2}\log(\sqrt{1+x^2} + x)$
4. $\frac{4}{3}(\sqrt{x}-2)\sqrt{1+\sqrt{x}}$
5. $\frac{x}{\sqrt{1+x^2}}$
6. $-x + \frac{2}{1 - \operatorname{tg} \frac{x}{2}}$
7. $\log \left| \operatorname{tg} \frac{x}{2} \right|$
8. $\frac{1}{2}\log \left| \frac{\operatorname{sen}^2 x}{\cos x} \right|$

9.15.

1. $-\frac{\pi}{2} + \frac{2\pi}{3\sqrt{3}} + \frac{1}{2}\log 3$
2. $1 - \frac{2}{e} + \log(e+1) - \log 2$
3. $\frac{15}{8} - \frac{9}{2}\log 2 + \frac{1}{4}\log 3$
4. $\frac{1}{2}\log 2 + \frac{\pi}{4}$

$$1. \frac{1}{2}\ln|x+1| - \ln|x+2| + \frac{1}{2}\ln|x+3|$$

$$2. \ln|x| - \frac{1}{4}\ln|2x-1| - \frac{3}{4}\ln|2x+1|$$

$$3. \frac{1}{3}x^3 + \frac{5}{2}x^2 + 19x + 90\ln|x-3| - 25\ln|x-2|$$

$$4. -\frac{3}{5}\ln|x| + \frac{2}{3}\ln|x-1| - \frac{1}{15}\ln|2x-5|$$

$$5. \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2+x+1) - \frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)$$

$$6. \ln|x| - \frac{1}{2}\ln(x^2+1)$$

$$7. \frac{1}{4}x^4 - x^3 + \frac{7}{2}x^2 - 15x + 3\ln|x+1| + 28\ln|x+2|$$

$$8. -\frac{1}{4}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{3}{16}\ln|3x-2|$$

$$9. \ln|x| - \frac{1}{3}\ln|x+1| - \frac{1}{3}\ln(x^2-x+1)$$

$$10. \ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{3}{4}\ln(x^2+1) + \frac{1}{2}\operatorname{arctg} x$$

$$11. \ln|x+1| - \ln|x| - \frac{1}{x}$$

$$12. \frac{3}{x} + \ln|x| - \frac{1}{2}\ln(x^2+1) + 3\operatorname{arctg} x$$

$$13. \frac{2}{3}\ln|t+2| + \frac{1}{3}\ln|t-1|$$

$$14. -\frac{1}{3}\ln|t| + \frac{1}{4}\ln|t+1| + \frac{1}{12}\ln|t-3|$$

$$15. \frac{1}{2}x^2 + x + \frac{1}{3}\ln|x+1| + \frac{8}{3}\ln|x-2|$$

$$16. -x + \ln|x-1| - 4\ln|x-2|$$

$$17. t + \ln|t+1| - \ln(t^2+1) + \operatorname{arctg} t$$

$$18. \frac{1}{x} + 2\operatorname{arctg} x$$

$$19. -2\ln t + 2\ln|t+1| - \frac{2t+1}{t(t+1)}$$

$$20. \frac{1}{3}\ln|t-2| - \frac{1}{6}\ln(t^2+2) + \frac{1}{3\sqrt{2}}\operatorname{arctg} \frac{t}{\sqrt{2}}$$

$$21. \frac{1}{3}\ln|t| - \frac{1}{6}\ln(t^2+2t+3) - \frac{1}{3\sqrt{2}}\operatorname{arctg} \frac{t+1}{\sqrt{2}}$$

$$22. \frac{1}{4}\ln|t+1| - \frac{1}{8}\ln(t^2+1) + \frac{1}{2}\operatorname{arctg} t + \frac{1}{4}\frac{t+1}{t^2+1}$$

$$23. \frac{5}{32}\ln|x-2| - \frac{1}{32}\ln|x+2| - \frac{1}{16}\ln(x^2+4) - \frac{3}{16}\operatorname{arctg} \frac{x}{2}$$

$$24. -\frac{1}{5} \ln |t-2| + \frac{1}{5} \ln |2t+1| = \frac{1}{5} \ln \left| \frac{2-2\cos x + \sin x}{1-\cos x - 2\sin x} \right|$$

$$25. -\frac{2}{t^2-1} - \ln |t-1| + \ln |t+1| = \frac{1}{\cos x} + 1 + \ln \left| \frac{\cos x}{1-\sin x} \right|$$

$$26. -x + \frac{2}{1 - \operatorname{tg} \frac{x}{2}} \quad 27. \ln \left| \operatorname{tg} \frac{x}{2} \right|$$

$$28. -\frac{1}{25} \ln |t+3| + \frac{9}{25} \ln |3t-1| - \frac{4}{25} \ln(t^2+1) - \frac{6}{25} \operatorname{arctg} t; \quad \left[t = \operatorname{tg} \frac{x}{2} \right] \quad 29. \operatorname{tg} \frac{x}{2}$$

$$30. \frac{1}{2} \ln \left| \frac{\sin^2 x}{\cos x} \right| \quad 31. \ln |s-1| + 2 \frac{s+1}{s^2+1} - 2 \operatorname{arctg} s \quad [s = \operatorname{tg} x]$$

$$32. -\frac{2}{t-1} + 2 \ln |t-1| = \frac{2 \sin x}{\cos x + \sin x - 1} + \ln 2 \left| \frac{1-\sin x}{1+\cos x} \right| \quad \left[t = \operatorname{tg} \frac{x}{2} \right]$$

$$33. \ln |\cos x| + \frac{3}{2 \cos^2 x}$$

$$34. -\frac{1}{t} + 2 \ln |t| - 2 \ln |t-1| = -\operatorname{cotg} \frac{x}{2} + \ln \left| \frac{1-2\cos x}{2(1-\sin x)} \right| \quad \left[t = \operatorname{tg} \frac{x}{2} \right]$$

$$35. -x + \frac{4}{\sqrt{3}} \operatorname{arctg}(\sqrt{3} \operatorname{tg} x)$$

$$36. -\frac{3}{25} \ln |t+2| - \frac{3}{25} \ln |2t-1| + \frac{3}{25} \ln(t^2+1) + \frac{8}{25} \operatorname{arctg} t \quad \left[t = \operatorname{tg} \frac{x}{2} \right]$$

$$37. \frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2} \operatorname{tg} x) \quad 38. \frac{1}{2} \ln |1+2 \operatorname{tg} x|$$

$$42. \sqrt{x^2-4x+5} - 2 \ln(x-2+\sqrt{(x-2)^2+1})$$

$$43. -\frac{1}{2} \ln |t| + 2 \operatorname{arctg} t - \sqrt{3} \operatorname{arctg} \frac{2t+1}{\sqrt{3}} \quad [t = \sqrt{x^2-1} - x]$$

$$44. \left(\frac{1}{3} x^2 - \frac{5}{12} x + \frac{15}{24} \right) \sqrt{x^2+x} - \frac{5}{16} \ln \left(x + \frac{1}{2} + \sqrt{x^2+x} \right)$$

$$45. \frac{12}{7} t^7 - 3t^4 \quad [t = \sqrt[3]{1+\sqrt[4]{x}}]$$

$$46. \frac{1}{t-1} + \frac{1}{t+1} + \ln |t-1| - \ln |t+1| \quad \left[t = \sqrt{\frac{1}{\sqrt{x}}+1} \right]$$

$$47. -4t + 4 \ln |t+2| - 4 \ln |t-2| \quad \left[t = \sqrt{4+\frac{3}{\sqrt{x}}} \right]$$

$$48. \frac{x\sqrt{2-x^2}}{2} + 2 \arcsin \frac{x}{\sqrt{2}}$$

$$49. \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x\sqrt{2}}{\sqrt{1-x^2}}$$

$$50. -\frac{1}{2} \sqrt{1+x^2} + \frac{3}{2} \ln(\sqrt{1+x^2}+x)$$

$$51. \frac{1}{8(t+1)} + \frac{3}{8} \ln |t+1| + \frac{1}{8(t-1)} - \frac{3}{8} \ln |t-1| + \frac{t}{4(t^2+1)} - \frac{3}{4} \operatorname{arctg} t \quad \left[t = \sqrt[4]{\frac{1}{x^2}+1} \right]$$

$$52. \frac{2}{\sqrt{\frac{1-x}{1+x}}-1} + 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}}$$

$$53. \frac{4}{3} (\sqrt{x}-2) \sqrt{1+\sqrt{x}}$$

$$54. -\frac{4}{3(t+1)} - \frac{4}{3} \ln |t+1| + \frac{4}{3} \frac{t+1}{t^2-t+1} + \frac{2}{3} \ln(t^2-t+1) - \frac{4}{\sqrt{3}} \operatorname{arctg} \frac{2t-1}{\sqrt{3}} \quad \left[t = \sqrt[3]{\frac{4-x}{x}} \right]$$

$$55. \frac{x}{\sqrt{1+x^2}} \quad 56. 6\sqrt{1+\sqrt[3]{x}} - 3 \ln \frac{\sqrt{1+\sqrt[3]{x}}+1}{\sqrt{1+\sqrt[3]{x}}-1}$$

$$57. \frac{4}{5} (1+\sqrt{x})^{5/2} - \frac{4}{3} (1+\sqrt{x})^{3/2}$$

$$58. -\frac{1}{4} (1+2x-2\sqrt{x^2+x}) - \frac{1}{16} \ln |1+2x-2\sqrt{x^2+x}| - \frac{15}{16} \ln |2x-2\sqrt{x^2+x}-3|$$

$$59. \frac{1}{3} u^3 - 9u + \frac{80}{3} \operatorname{arctg} u \quad [u = \sqrt{x+1} - \sqrt{x}] \quad 60. \frac{1}{2} \ln \left| \frac{2\sqrt{x^2+3}+x-3}{1+x} \right|$$

$$61. 2 \ln |t-2| - 2 \ln |t+2| + \operatorname{arctg} t \quad \left[t = \sqrt{\frac{4+e^x}{1-e^x}} \right]$$

$$62. \sqrt{2} \ln \left| \frac{1+t+\sqrt{2(t^2+1)}}{1-t} \right| = \sqrt{2} \ln \left| \frac{1+\sin x + \sqrt{1+\cos x} + \sqrt{1-\cos x}}{\cos x} \right|$$

$$63. 2u - \ln|u+1| + \ln|u-1| = 2\sqrt{1+e^x} + \ln \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} \quad [u^2 = 1+e^x]$$

$$64. \frac{1}{\sqrt{2}} \ln \frac{u^2-2u+2}{u^2+2u+2} + \sqrt{2} \operatorname{arctg}(u+1) + \sqrt{2} \operatorname{arctg}(u-1) \quad \left[u = \sqrt{\frac{2 \cos x}{1-\sin x}} \right]$$

$$65. -t + \frac{1}{2} \ln|t+1| - \frac{1}{2} \ln|t-1| \quad [t = \sqrt{1+e^{-2x}}]$$

$$66. \frac{1}{3} \ln \frac{\sqrt{1+e^{3x}}-1}{\sqrt{1+e^{3x}}+1} \quad 67. \sqrt{2} \ln(t-2+\sqrt{t^2-4t+1}) \quad \left[t = \operatorname{tg} \frac{x}{2} \right]$$

$$68. \frac{u}{2} + \ln|u-1| + \ln|u+1| - \frac{5}{4(2u+3)} + \frac{3}{2} \ln(2u+3) \quad [u = \sqrt{e^{2x}-3e^x+1} - e^x]$$

$$69. -u + \frac{1}{4} \ln|u+1| - \frac{1}{4} \ln|u-1| + \frac{1}{2} \operatorname{arctg} u \quad \left[u = (e^{-4x}+1)^{1/4} \right]$$

$$70. \sqrt{2}s - \frac{1}{2\sqrt{2}} \ln|s+1| + \frac{1}{2\sqrt{2}} \ln|s-1| \quad \left[s = \sqrt{\frac{1+\sqrt{4e^x+1}}{2}} \right]$$

$$71. -\frac{2}{\sqrt{e^{2x}+2e^x}-e^x} \quad 72. \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$73. x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) \quad 74. 2\sqrt{1+x} \arcsin x + 4\sqrt{1-x}$$

$$75. \left(\frac{x^2}{2} - \frac{1}{x} + \frac{1}{2} \right) \operatorname{arctg} x - \frac{x}{2} + \ln|x| - \frac{1}{2} \ln(x^2+1)$$

$$76. x \arcsin x + \sqrt{1-x^2} \quad 77. \frac{x^2}{2} \arccos x - \frac{1}{4} x \sqrt{1+x^2} + \frac{1}{4} \arcsin x$$

$$78. \frac{1}{2} \ln^2 x - \frac{1}{x} \ln x - \frac{1}{x}$$

$$79. \frac{x^2}{2} \left(\ln(x^3-1) - \frac{3}{2} \right) - \frac{1}{2} \ln|x-1| + \frac{1}{4} \ln(x^2+x+1) + \frac{\sqrt{3}}{2} \operatorname{arctg} \frac{2x+1}{\sqrt{3}}$$

$$80. \frac{1}{2} (x^2-2x) \ln(x+\sqrt{x^2+2}) - \frac{1}{4} \frac{(x-4)}{\sqrt{x^2+2}} \quad 81. -\cos x \ln \operatorname{tg} x + \ln \operatorname{tg} \frac{x}{2}$$

$$82. \frac{e^{2x}}{2} \ln(1-e^{-x}) - \frac{1}{2} (e^x + \ln(e^x-1)) - \frac{x}{2}$$

$$83. \left(\frac{x^2}{4} - \frac{1}{8} \right) \ln(1+2x\sqrt{1-x^2}) - \frac{x^2}{4} - \frac{1}{4} \arccos x$$

$$84. \frac{1}{2} (x^2+1) (\operatorname{arctg} x)^2 - x \operatorname{arctg} x + \frac{1}{2} \ln(1+x^2)$$

$$85. \frac{x^3}{3} \left(\ln^3 x - \ln^2 x + \frac{2}{3} \ln x - \frac{2}{9} \right) \quad 86. \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2)$$

$$87. \frac{1}{2} x^2 e^{x^2} \ln x - \frac{1}{4} e^{x^2} \quad 88. \frac{x^3}{3} \arcsin x - \frac{1}{9} \sqrt{1-x^2} (2+x^2)$$

$$89. \frac{1}{5} e^x (\sin 2x - 2 \cos 2x) \quad 90. \frac{e^{\beta x}}{\alpha^2 + \beta^2} (\beta \cos \alpha x + \alpha \sin \alpha x)$$

$$91. \frac{e^x}{2} \left\{ 1 + \frac{1}{5} (\cos 2x + 2 \sin 2x) \right\}$$

$$92. \frac{1}{4} (x^4-1) \operatorname{arctg}^2 x - \frac{1}{2} \left(\frac{x^3}{3} + x \right) \operatorname{arctg} x + \frac{1}{12} x^2 - \frac{1}{3} \ln(1+x^2)$$

$$93. -\frac{\operatorname{arctg} x}{1+x} + \frac{1}{2} \ln(1+x) - \frac{1}{4} \ln(1+x^2) + \frac{1}{2} \operatorname{arctg} x$$

$$94. -\cos x \ln \sin x + \cos x - \frac{1}{2} \ln(1+\cos x) + \frac{1}{2} \ln(1-\cos x)$$

$$95. \frac{1}{\cos t} \ln \sin t - \ln \left| \operatorname{tg} \frac{t}{2} \right| \quad 96. (\sin x - \cos x) \ln(\sin x - \cos x) + \cos x - \sin x$$

$$97. \frac{e^{\beta x}}{\alpha^2 + \beta^2} (\beta \sin \alpha x - \alpha \cos \alpha x)$$

$$98. x \arcsin \sqrt{x} - \frac{1}{2} \operatorname{arctg} \sqrt{\frac{x}{1-x}} + \frac{1}{2} \sqrt{x(1-x)}$$

$$99. (2\sqrt{x}+1) \operatorname{arctg} \left(\frac{1}{2} - \sqrt{x} \right) + \ln \left(x - \sqrt{x} + \frac{5}{2} \right)$$

100. Dalla formula di integrazione per parti si ottiene (vedi *Lezioni*, cap. 5, Esempio 5.4)

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

e dunque

$$I_n = \frac{n-1}{n} I_{n-2},$$

dove si è posto $I_n = \int_0^{\pi/2} \sin^n x \, dx$.

9.6.

1. $\frac{\pi^2}{2} + 2\pi - 2$ 2. $e - \frac{7}{3}$ 3. $\frac{3}{2} - \log 2$ 4. $\frac{16}{3}$ 5. $\frac{3}{2} - \frac{\pi}{4}$ 6. $\frac{3}{4}$

9.7. $2\sqrt{2}$

9.8. $\frac{1}{3}$