

9.1.

1. $\frac{3}{2}$

2. $e - \frac{1}{e}$

3. $\frac{b^3 - a^3}{3}$

9.2. Se φ e ψ sono una maggiorante e una minorante di f , le funzioni φ^+ e ψ^+ sono una maggiorante e una minorante di f^+ , e si ha $\int \varphi^+ dx - \int \psi^+ dx \leq \int \varphi dx - \int \psi dx$. Il risultato segue applicando il teorema 9.1. Per f^- si può ripetere il ragionamento, o più semplicemente osservare che $f^- = (-f)^+$.

9.3. Si ha $|f(x)| = f^+(x) + f^-(x)$, e per l'esercizio 9.2 ambedue queste funzioni sono integrabili. Il viceversa non è vero; un controesempio è dato dalla funzione che vale 1 se x è razionale e -1 se è irrazionale, che non è integrabile, mentre il suo modulo è integrabile, dato che vale identicamente 1.

9.5. Se indichiamo con M e m l'estremo superiore e l'estremo inferiore di f in $[x_1, x_2]$, si ha $mp(t) \leq f(t)p(t) \leq Mp(t)$ in $[x_1, x_2]$, e quindi

$$m \int_{x_1}^{x_2} p(t) dt \leq \int_{x_1}^{x_2} f(t)p(t) dt \leq M \int_{x_1}^{x_2} p(t) dt.$$

La quantità

$$\frac{\int_{x_1}^{x_2} f(t)p(t) dt}{\int_{x_1}^{x_2} p(t) dt}$$

è allora compresa tra m ed M ; per la continuità di f esisterà allora un punto $\xi \in [x_1, x_2]$ tale che essa vale $f(\xi)$.

10.4.

1. $\sqrt{x^2+x}-x$

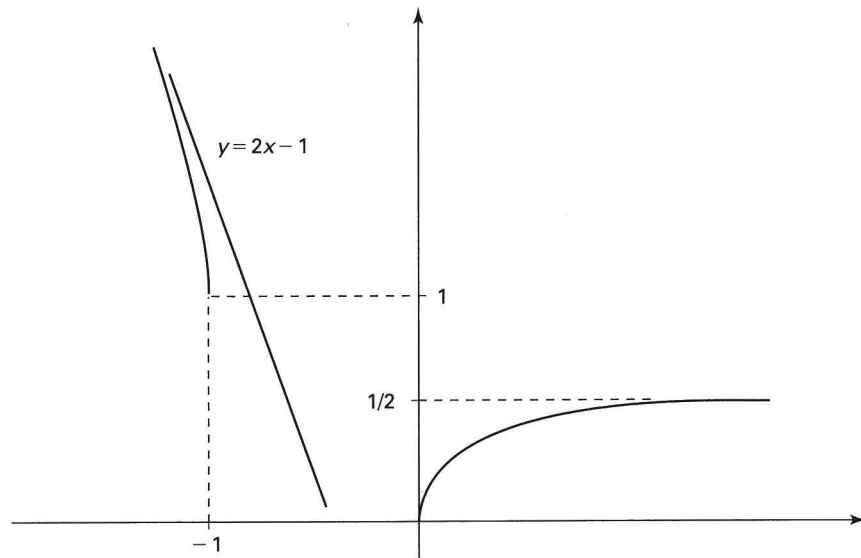


Figura 10.21

2. $\sqrt{x+1}-\sqrt{x}$

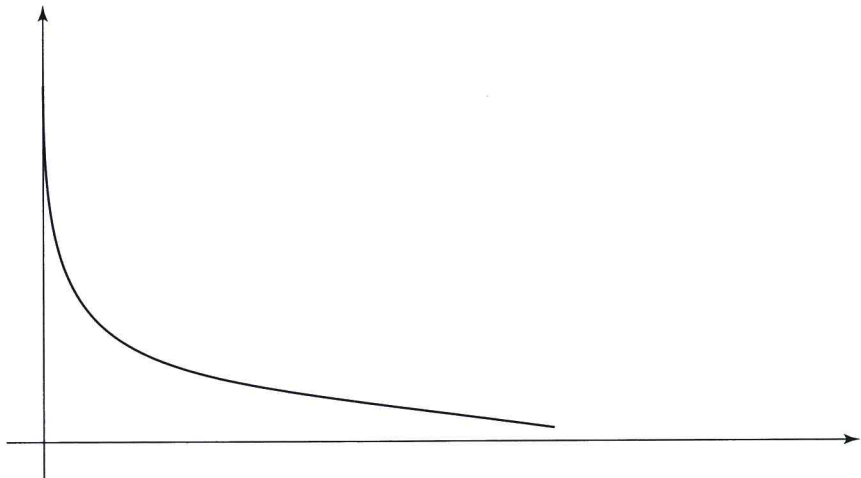


Figura 10.22

3. $(x^2+2x)e^x$

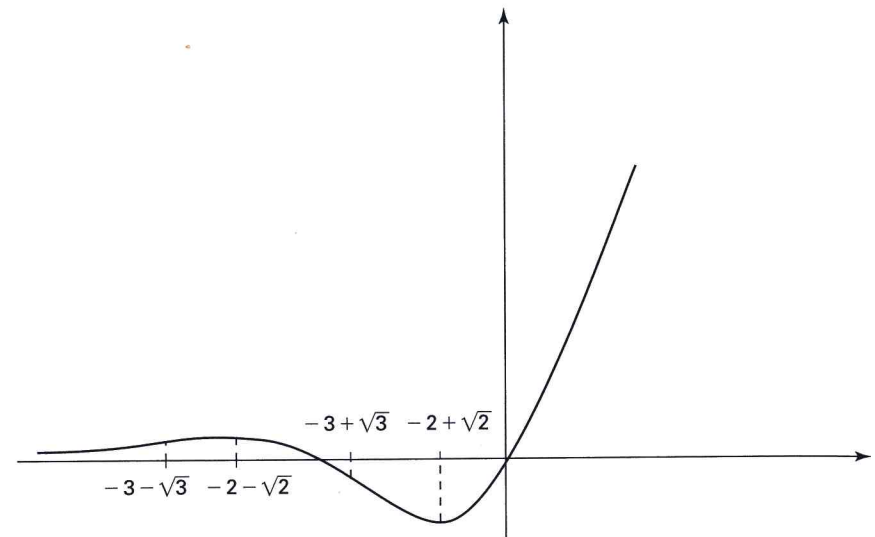


Figura 10.23

4. $-\log \operatorname{sen} x$

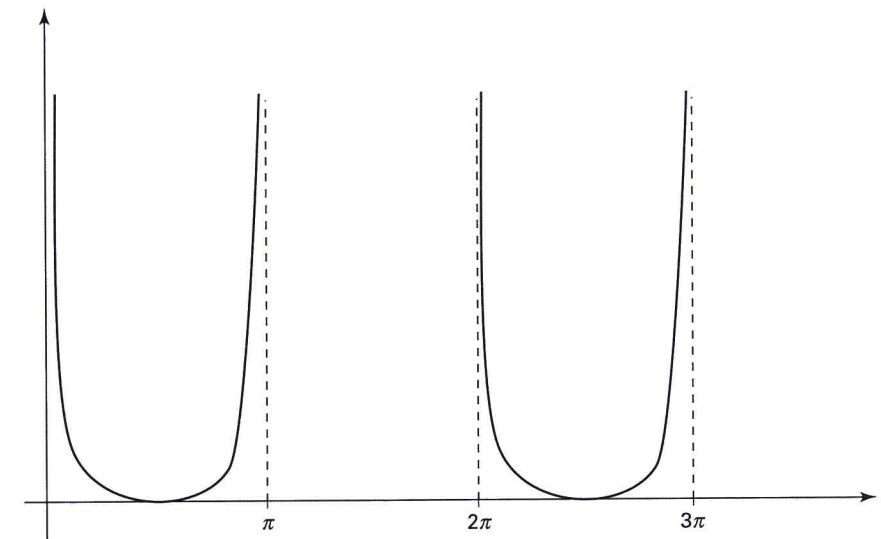


Figura 10.24

5. $\sqrt{\frac{x^3}{x+3}}$

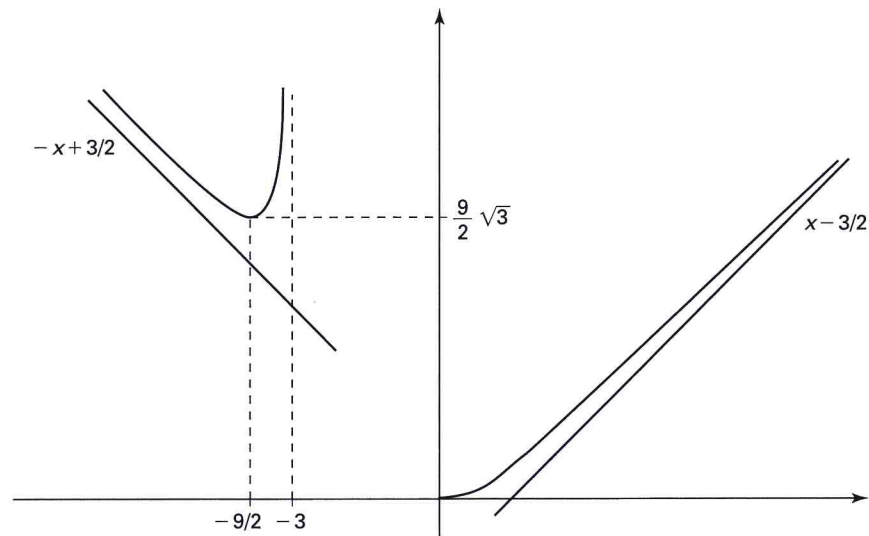


Figura 10.25

7. $\frac{e^x + 1}{e^x - 1}$

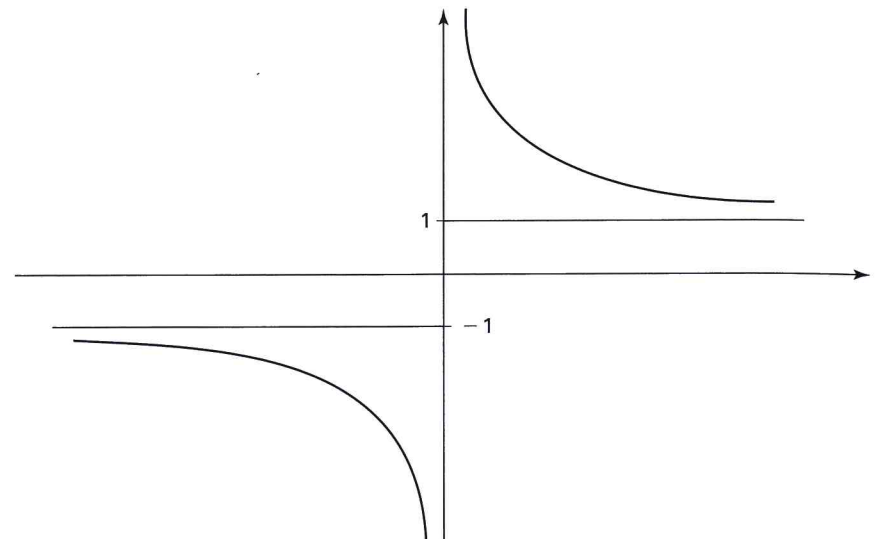


Figura 10.27

6. $\frac{1-x}{x^2+1}$

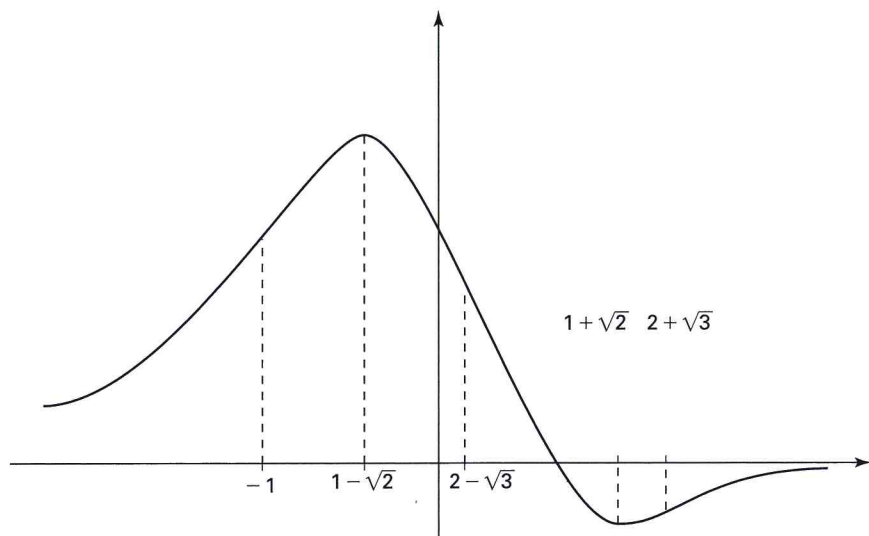


Figura 10.26

8. $\frac{x-1}{x^3}$

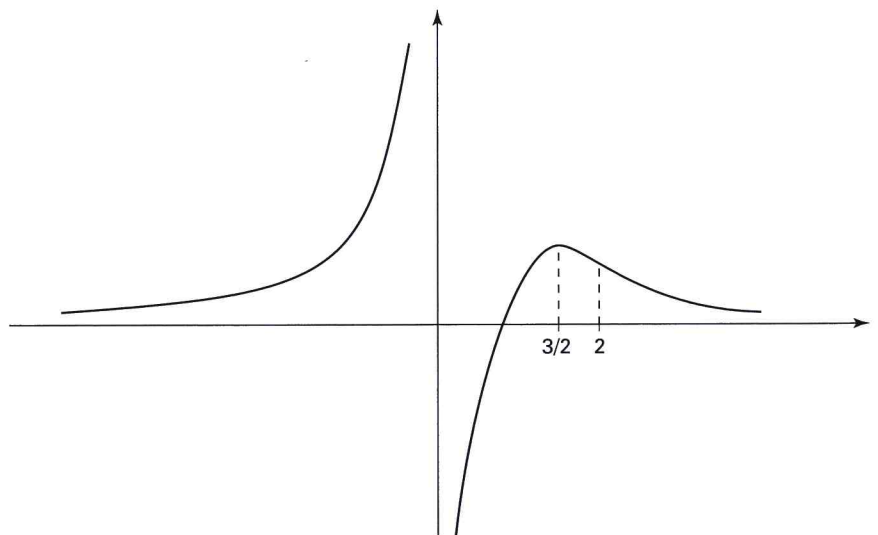


Figura 10.28

9. $x^2(\log|x| - 1)$

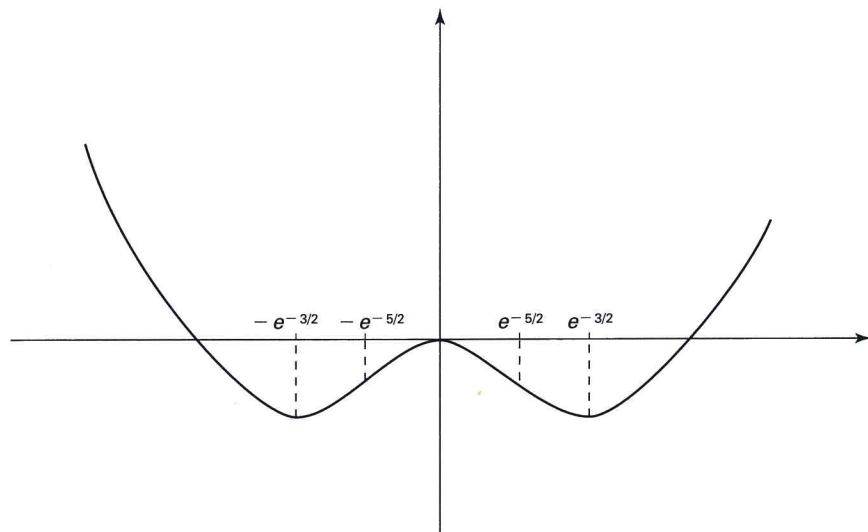


Figura 10.29

11. $\frac{x^2 - 2}{x + 1}$

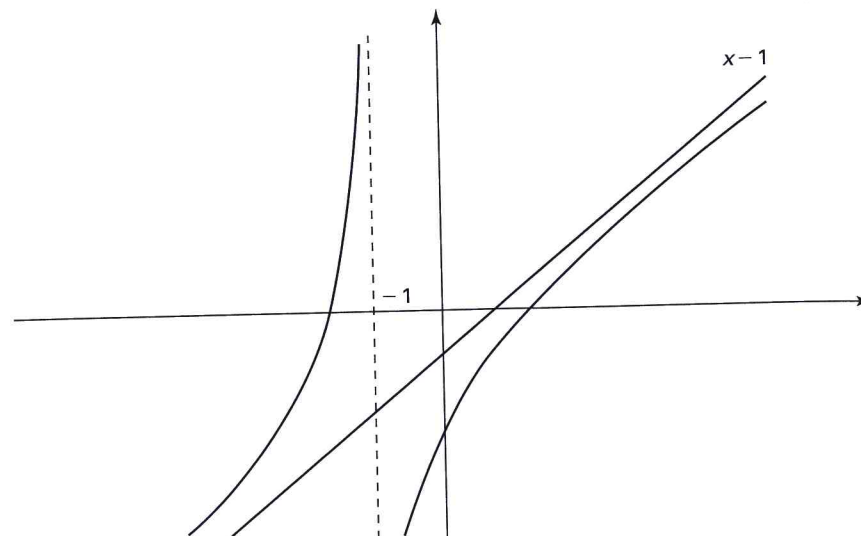


Figura 10.31

10. $\frac{x + 3}{x^2 - 5}$

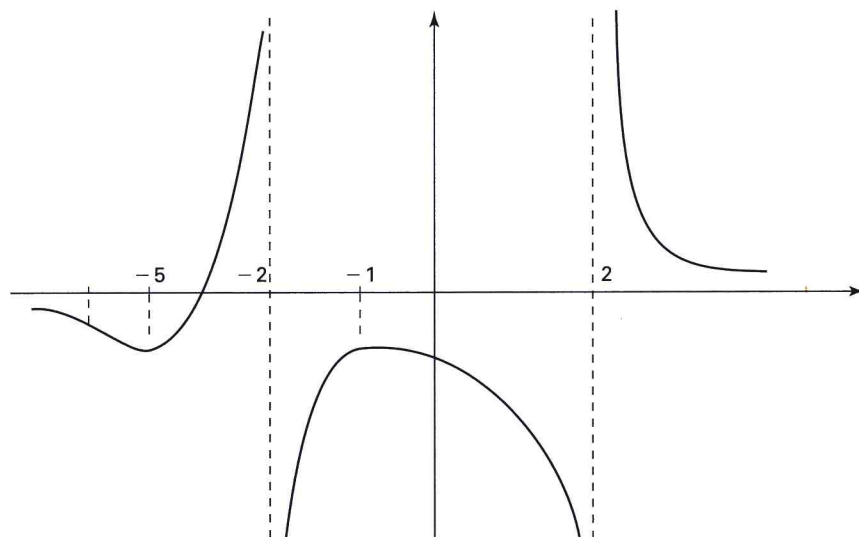


Figura 10.30

12. $\frac{xe^{-x}}{x - 1}$

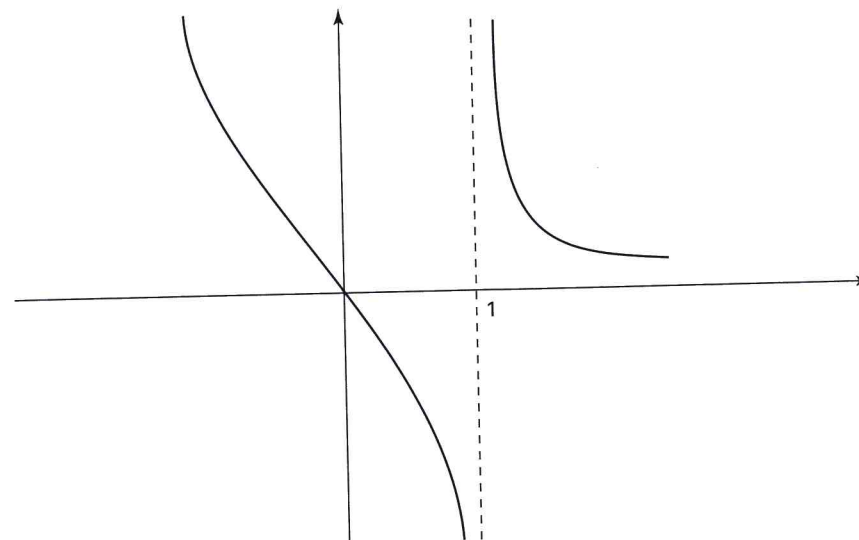


Figura 10.32

13. $\log(1 - 2\sin^2 x)$

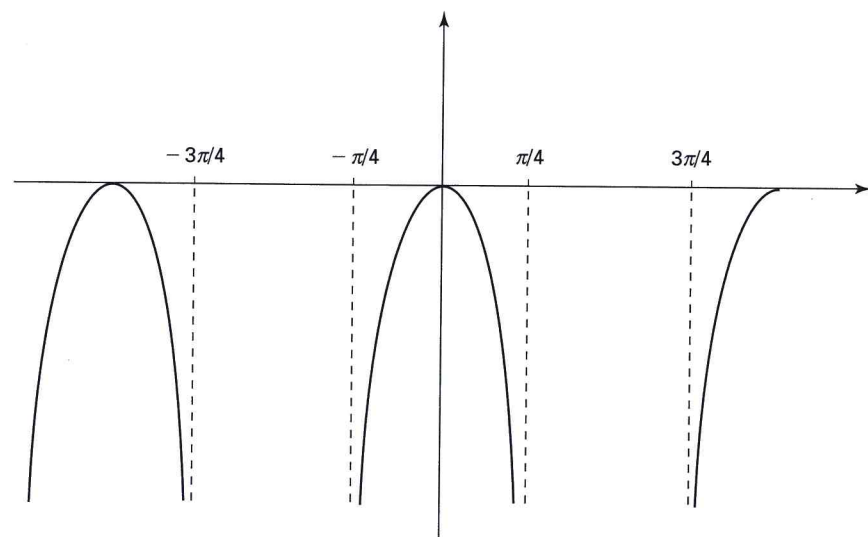


Figura 10.33

15. $2x - \arcsin x$

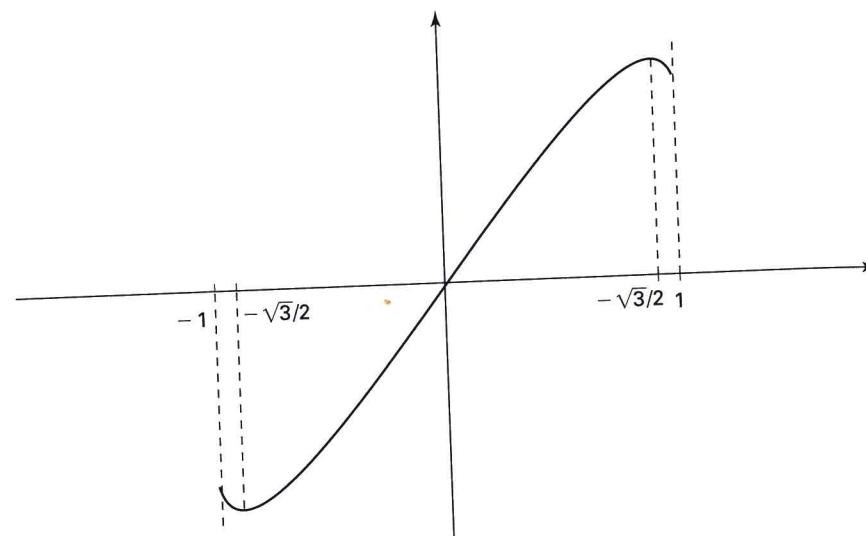


Figura 10.35

14. $\log \frac{x^2}{x-1}$

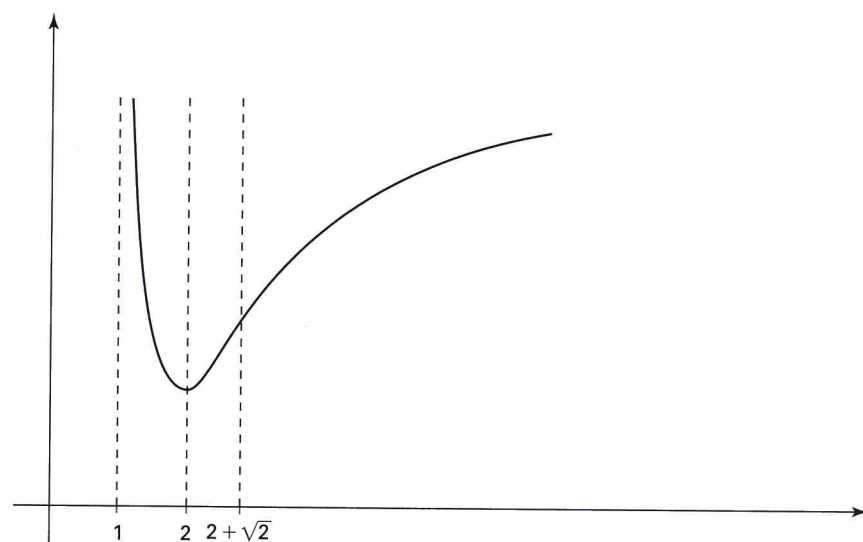


Figura 10.34

16. $\frac{\log x}{x^2}$

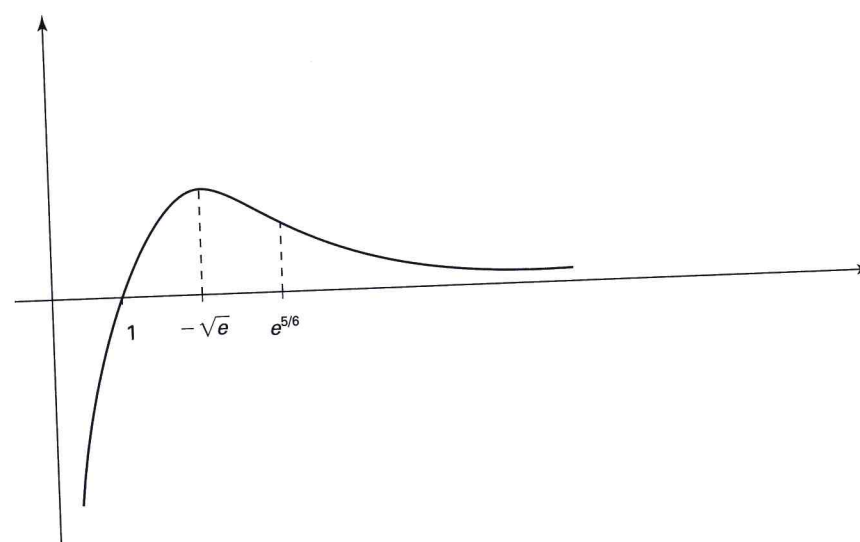


Figura 10.36

17. $\sin x(1 - \sin x)$

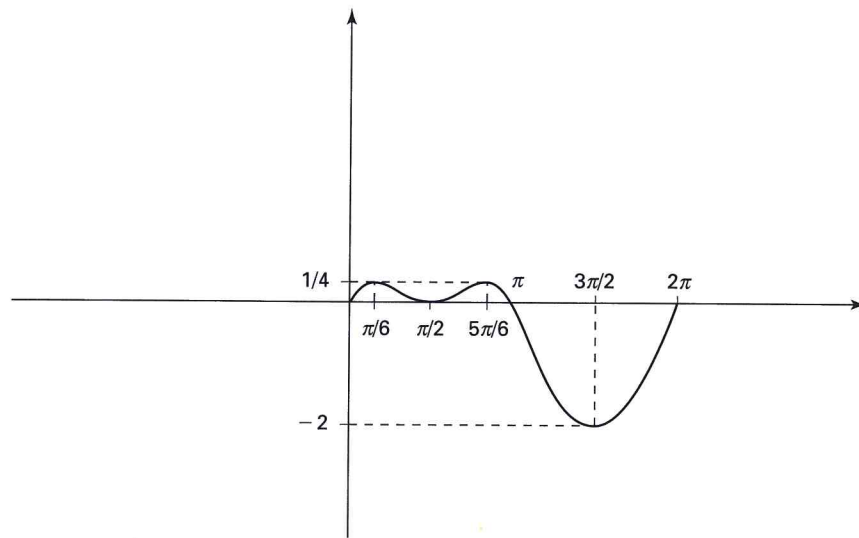


Figura 10.37

19. $\frac{|x|}{|x+1|}$

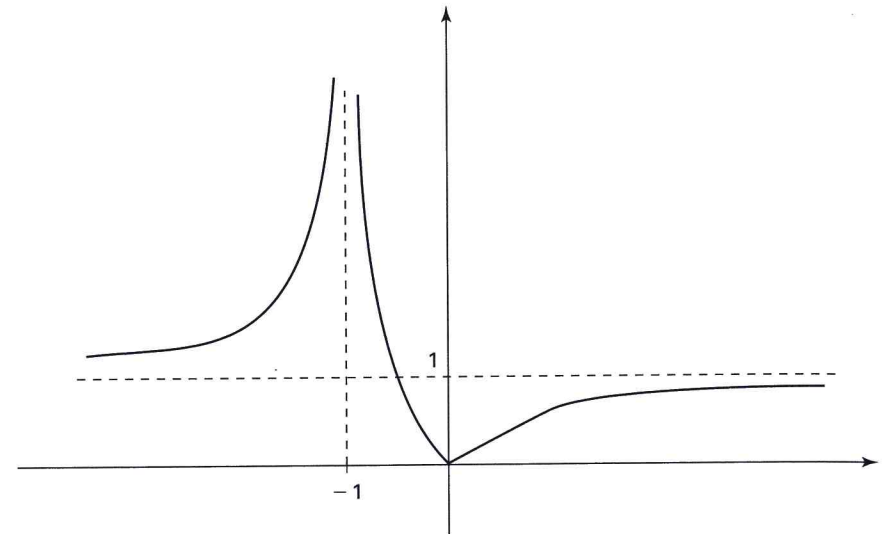


Figura 10.39

18. $x - 2 \arctg x$

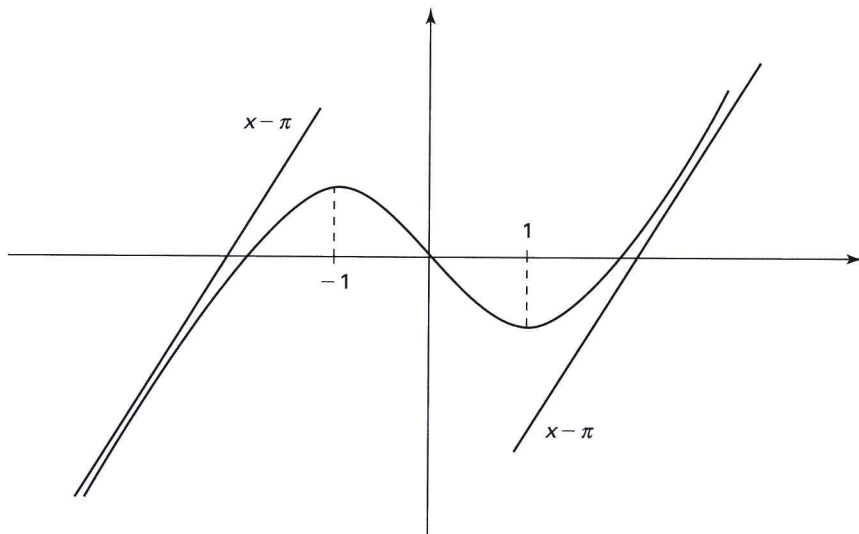
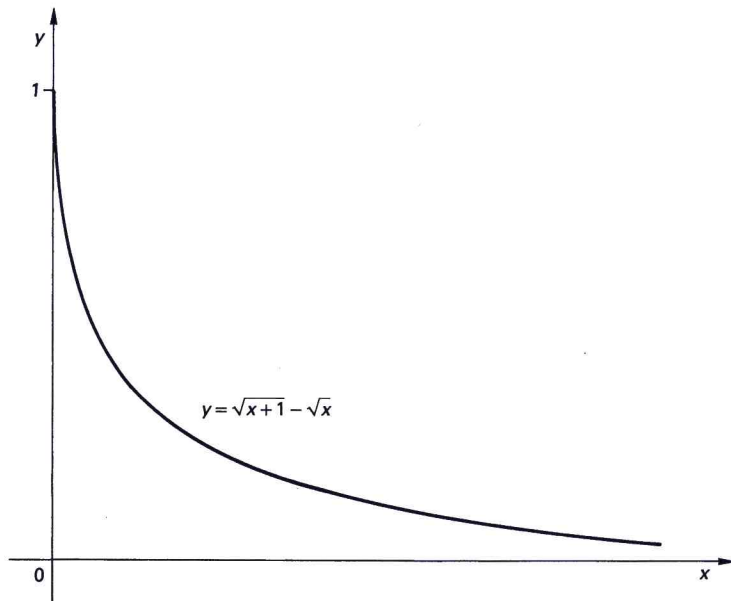


Figura 10.38

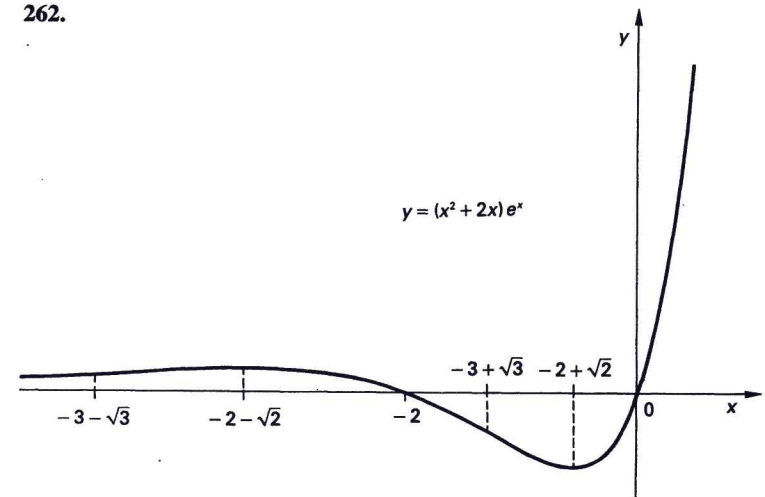
10.5. Sia $0 < a < 1$. Posto $f(x) = x - \log_a x$, risulta $f(a) = a - \log_a a = a - 1 < 0$ e $f(1) = 1 > 0$. Il punto ξ in cui la funzione $f(x)$ si annulla è dunque compreso tra a e 1 ; quando a tende a 1 anche ξ tende a 1 .

10.6. Si ha $f(0) = -\frac{\pi}{2}$ e $f(1) = 4 - \log 2 > 0$, e quindi $f(x)$ si annulla almeno una volta in $[0, 1]$. D'altra parte $f'(x) = 8x - \frac{2x}{1+x^2} + \frac{2}{1+x^2} > 0$ in $[0, 1]$, e quindi $f(x)$ è strettamente crescente, e si può annullare solo una volta.

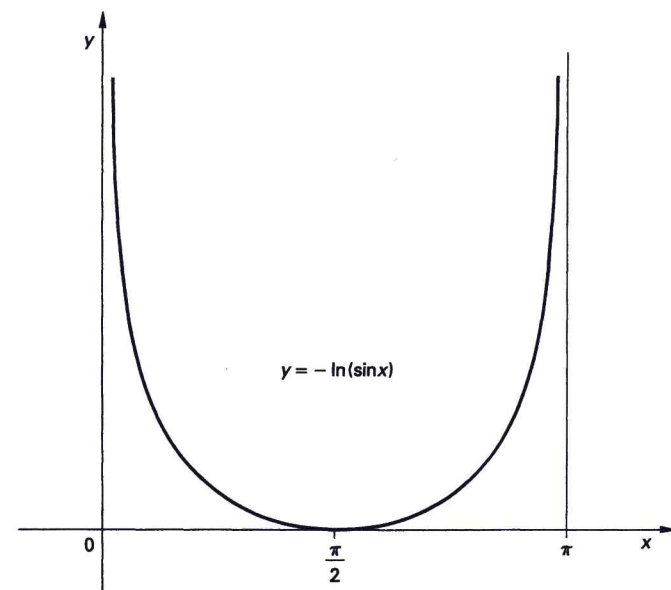
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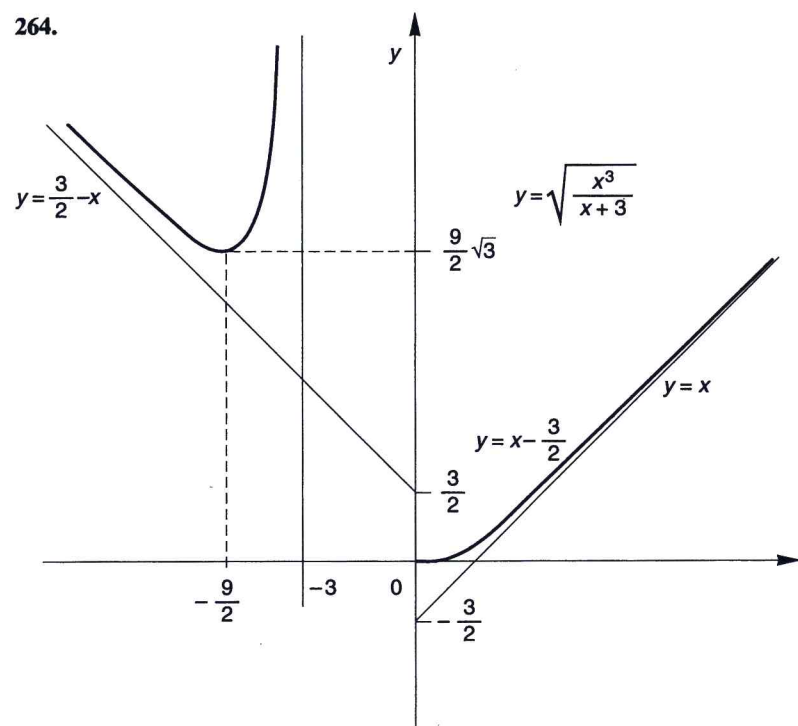
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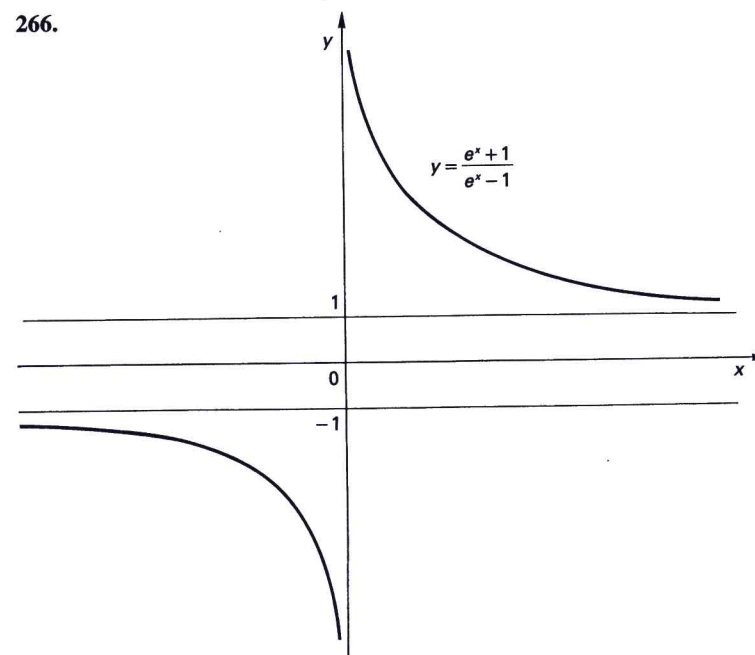
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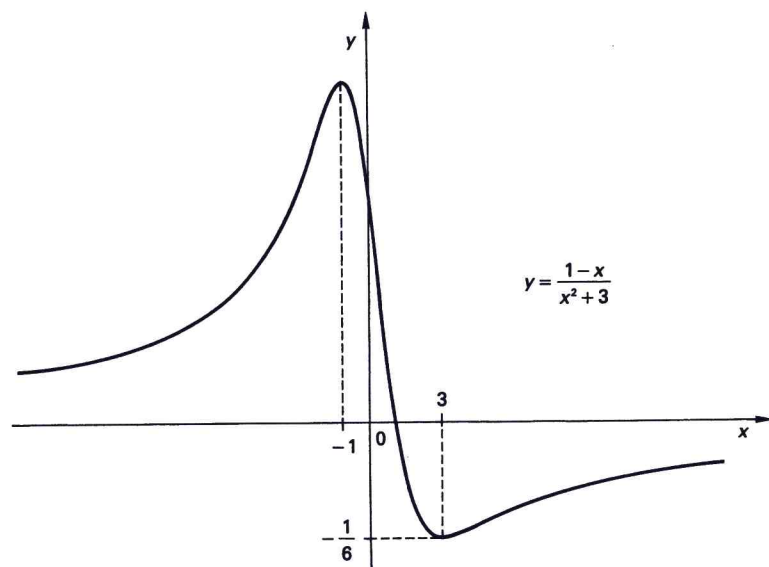
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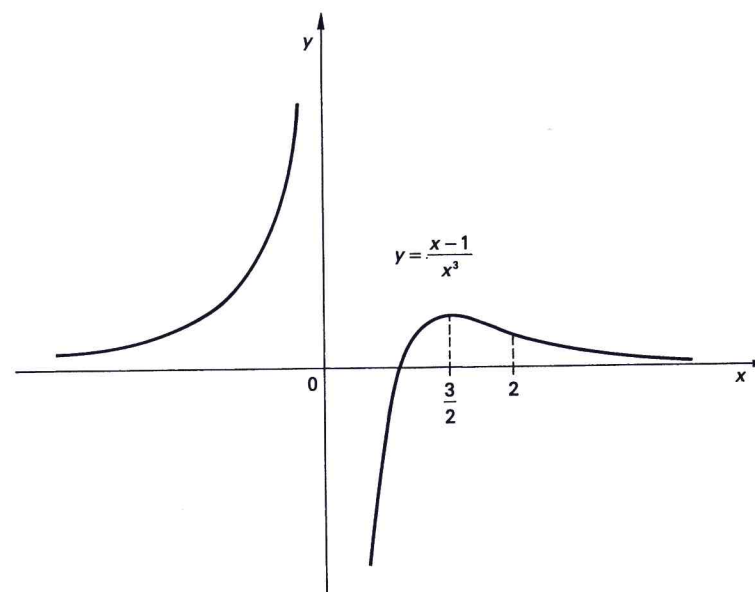
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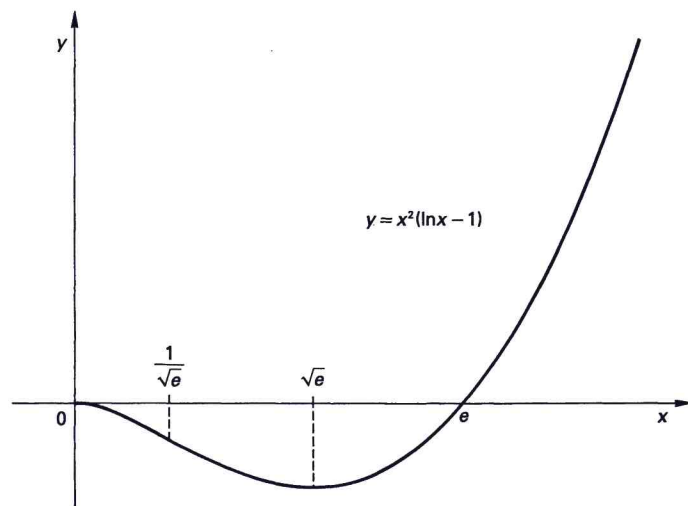
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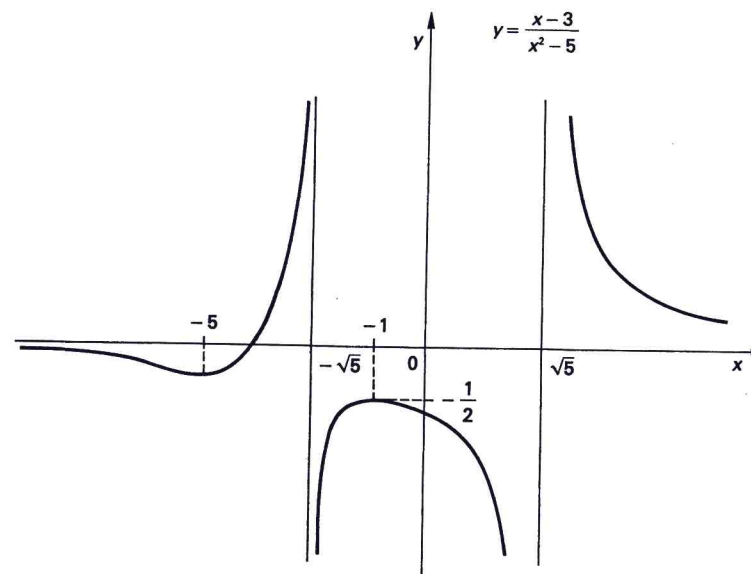
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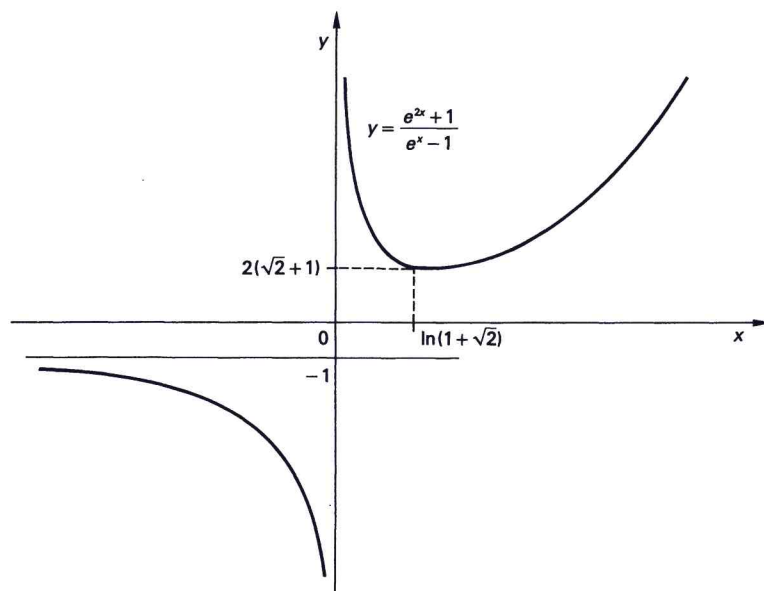
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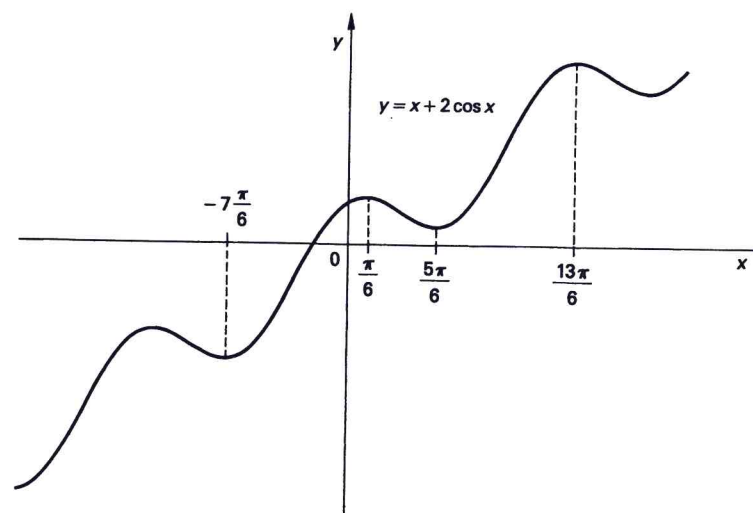
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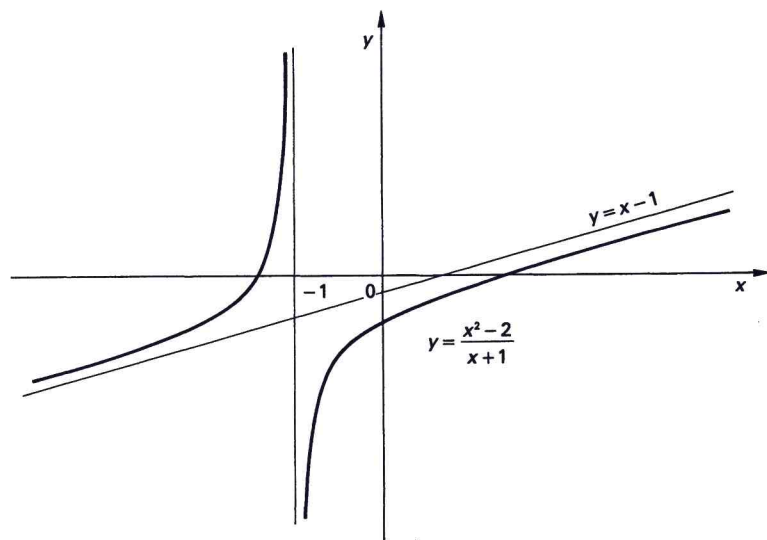
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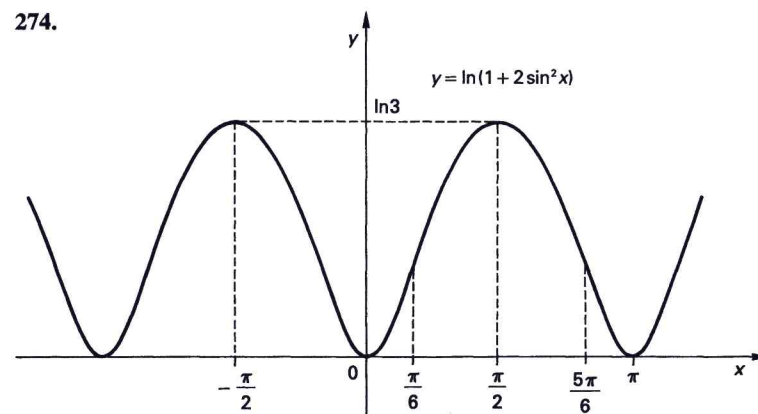
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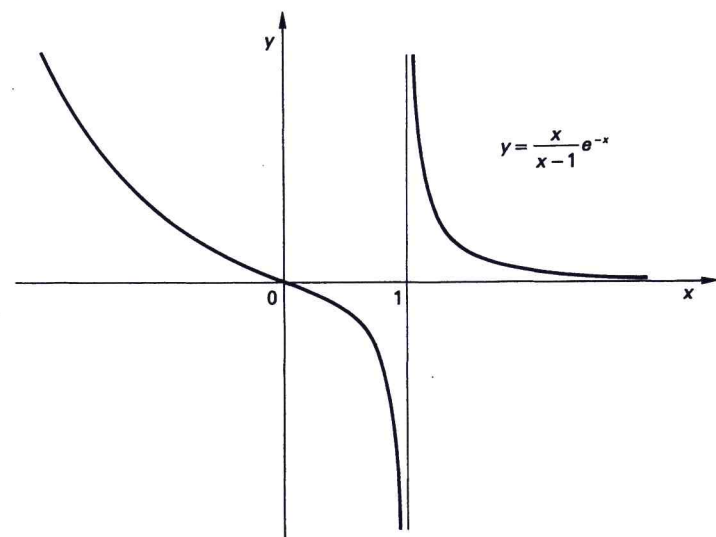
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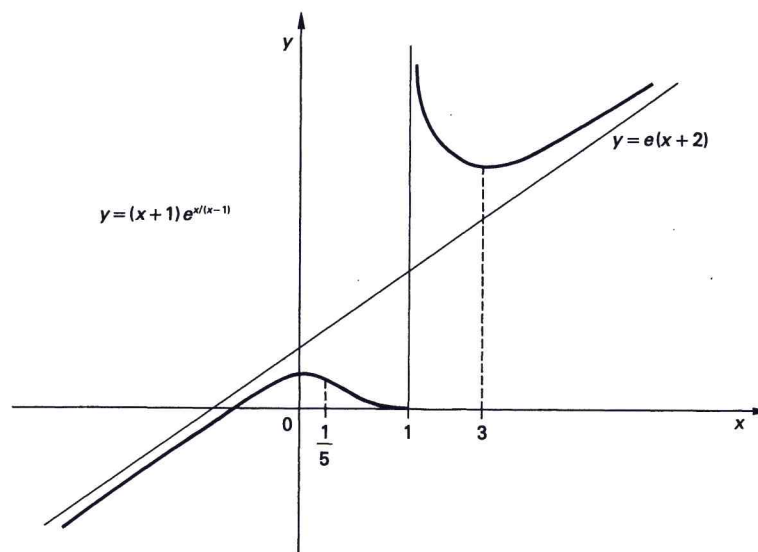
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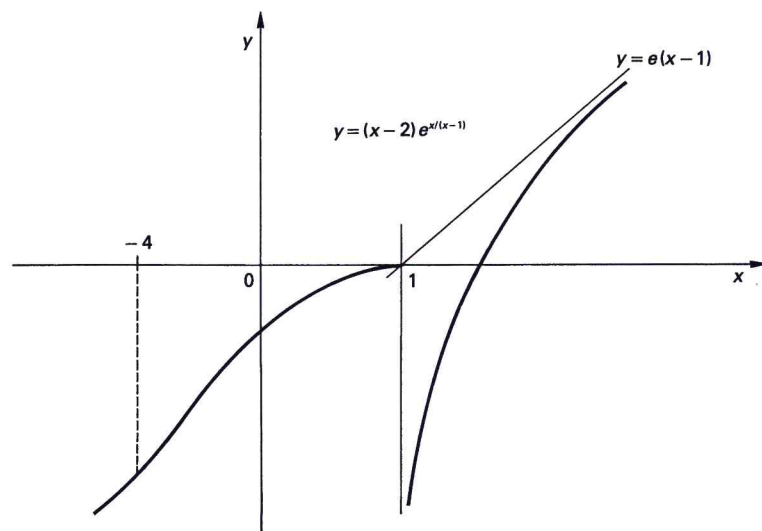
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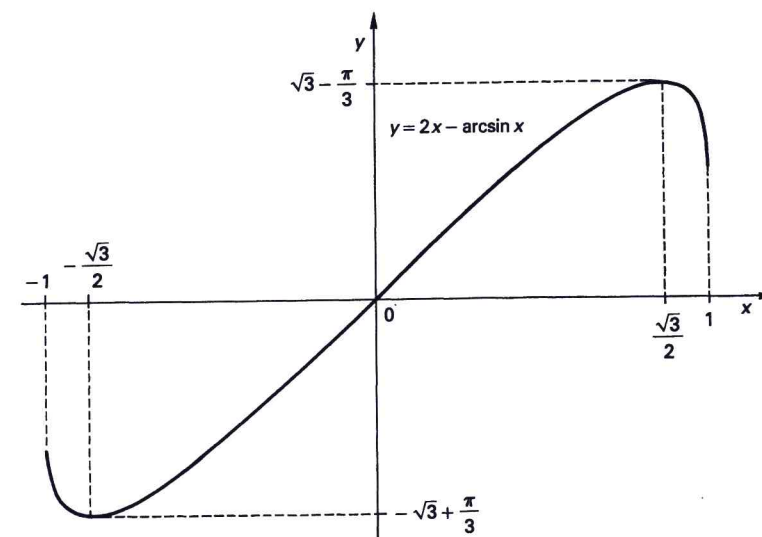
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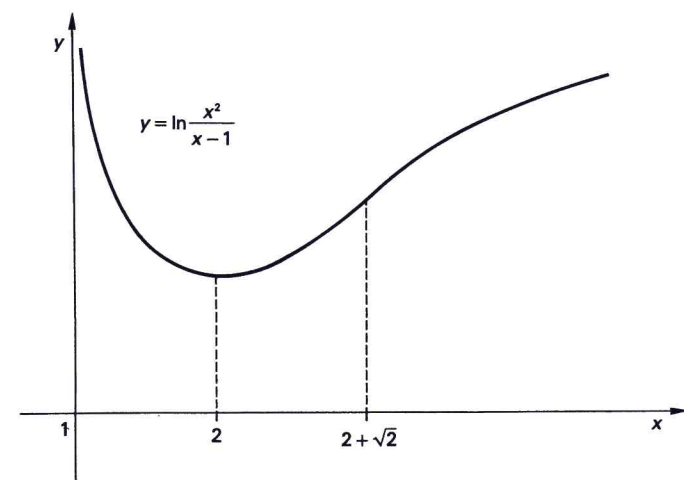
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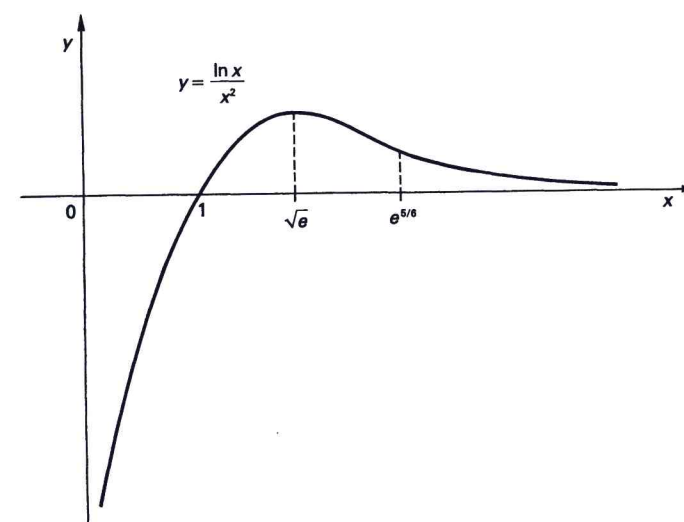
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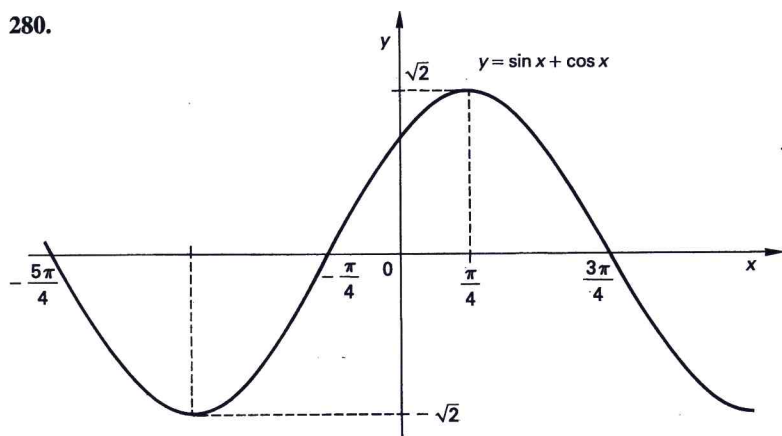
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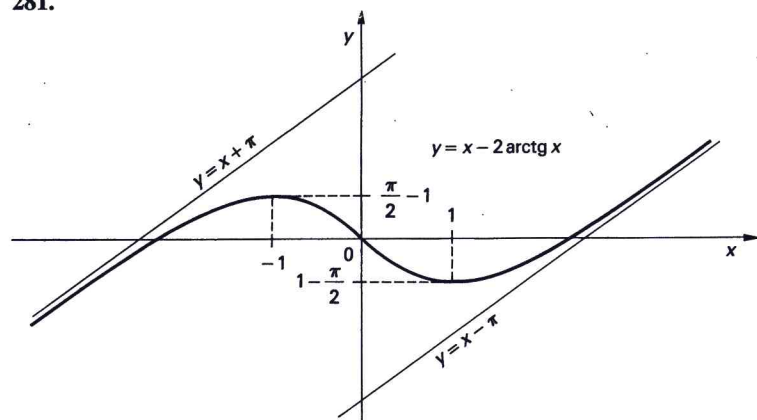
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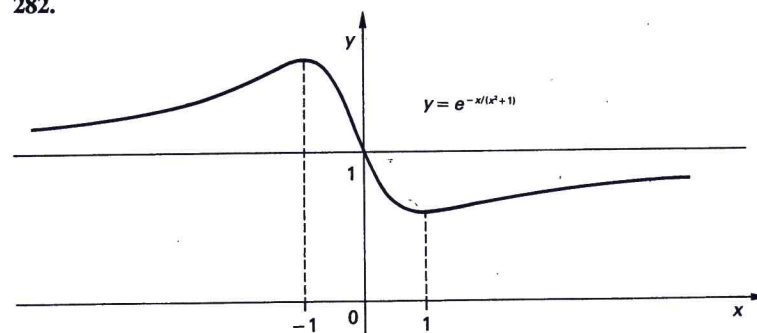
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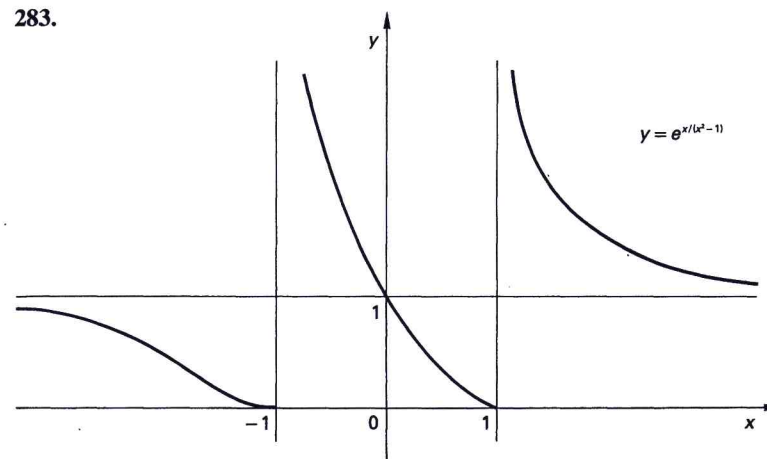
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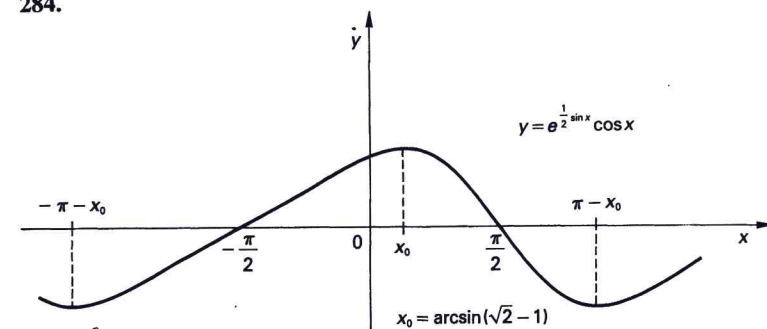
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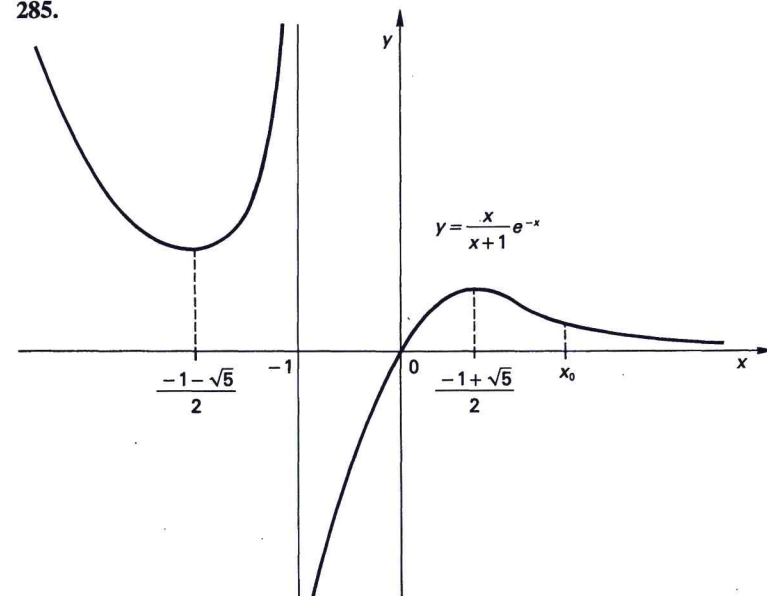
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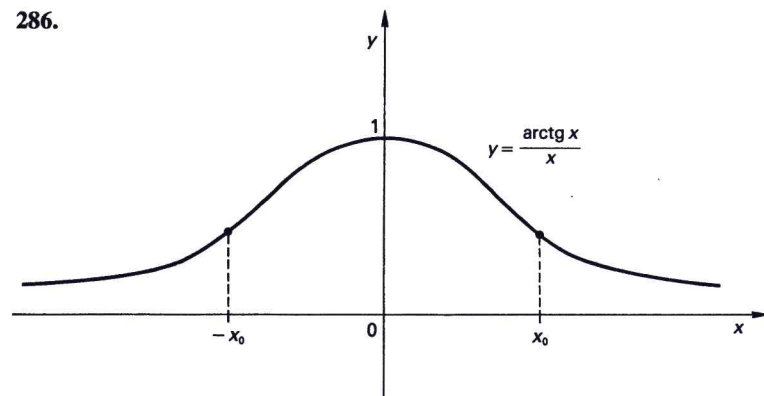
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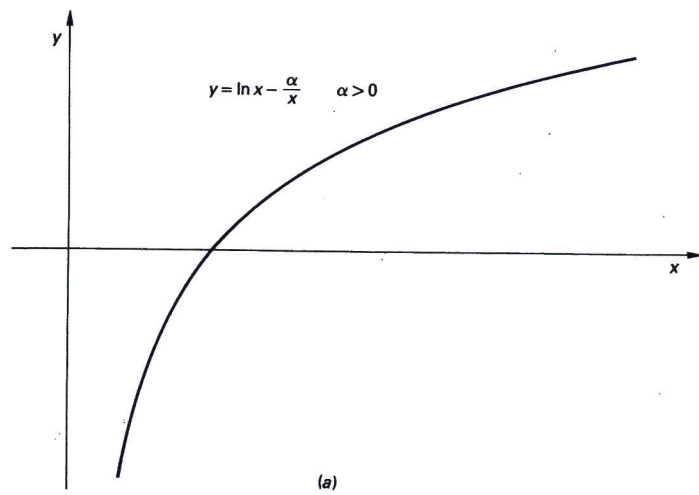
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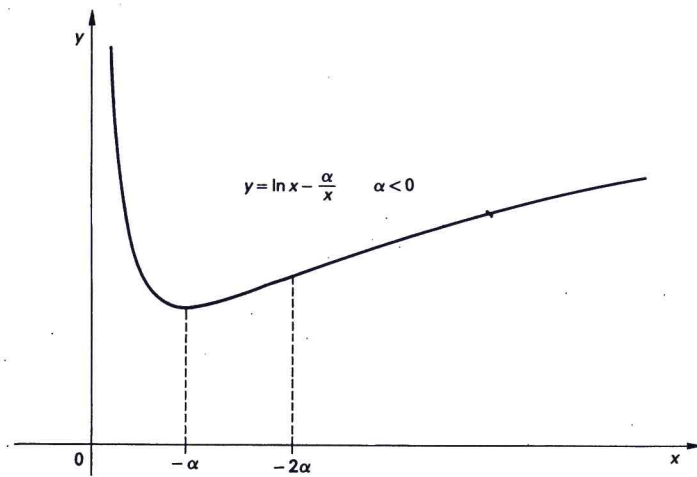
286.



287.



(a)



(b)