

5.1. Dimostrare che se $a < 0$ si ha $\lim_{x \rightarrow a} \text{segno}(x) = -1$.

5.2. Calcolare i seguenti limiti:

1. $\lim_{x \rightarrow 0} x^3$
2. $\lim_{x \rightarrow 1} (1 - x(1 + x))$
3. $\lim_{x \rightarrow 0} (3x^2 - x^3)$
4. $\lim_{x \rightarrow 0} \frac{1+x}{1-x}$
5. $\lim_{x \rightarrow -1} (x^2 - x^5)$
6. $\lim_{x \rightarrow 0} x 2^x$
7. $\lim_{x \rightarrow \sqrt{2}} (2x - x^3)$
8. $\lim_{x \rightarrow 0} \sqrt{1+x}$
9. $\lim_{x \rightarrow a} x^2$
10. $\lim_{x \rightarrow 0} x \text{segno}(x)$

5.3. Dimostrare che se $\lim_{x \rightarrow x_0} f(x) = L > 0$, allora $\lim_{x \rightarrow x_0} \sqrt{f(x)} = \sqrt{L}$.

5.4. Calcolare i seguenti limiti:

1. $\lim_{x \rightarrow 0} \frac{1+x^2}{1-x}$
2. $\lim_{x \rightarrow 2} x(x+2)(x-3)$
3. $\lim_{x \rightarrow 0} \frac{2+\sqrt{x}}{1+x}$
4. $\lim_{x \rightarrow 1} \sqrt{x} + 3$
5. $\lim_{x \rightarrow 0} x \text{sen } x$
6. $\lim_{x \rightarrow 0} \frac{x+1}{x^2+1}$
7. $\lim_{x \rightarrow 0} \sqrt{2+3x}$
8. $\lim_{x \rightarrow 4} (x\sqrt{x} + 1)$
9. $\lim_{x \rightarrow 0} (x-2)\sqrt{1+x}$
10. $\lim_{x \rightarrow -1} \frac{\sqrt{|x|}}{1-x}$

5.5. Dimostrare che $\lim_{x \rightarrow x_0} f(x) = 0$ se e solo se $\lim_{x \rightarrow x_0} |f(x)| = 0$.

5.6. Dimostrare che la funzione $\text{segno}(x) = \begin{cases} 1 & \text{se } x > 0 \\ 0 & \text{se } x = 0 \\ -1 & \text{se } x < 0 \end{cases}$ non ha limite per $x \rightarrow 0$.

5.7. Calcolare i limiti seguenti:

1. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$
2. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$
3. $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{3x-2}{2x+3} - \frac{3x+2}{2x-3} \right)$
4. $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$
- 5.* $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{x}$
11. $\lim_{x \rightarrow 0} \frac{1 - \cos(x^2)}{x^3}$
12. $\lim_{x \rightarrow 0} \frac{\text{sen } x}{\sqrt{x^2}}$
13. $\lim_{x \rightarrow 0} \frac{1}{1 + \text{tg } x}$
14. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \text{sen } x}$
15. $\lim_{x \rightarrow 0} \left(\frac{1}{\text{sen } x} - \frac{1}{\text{tg } x} \right)$
16. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$
17. $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \text{sen } x}$
18. $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\pi - x}$
6. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$
7. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x^2 - x}$
8. $\lim_{x \rightarrow 0} (1 - \text{sen}(x^2))$
9. $\lim_{x \rightarrow 0} \frac{\text{sen } 2x}{3x}$
10. $\lim_{x \rightarrow 0} \frac{\text{sen } x^2}{x^2}$
19. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \text{tg } x}{\text{sen } x - \cos x}$
20. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\text{sen } x - \cos x}$
21. $\lim_{x \rightarrow 0} \frac{\text{sen}(x+x^2)}{x}$
22. $\lim_{x \rightarrow 0} \frac{1 - \cos \sqrt{x}}{\frac{1}{1+x} - \cos x}$
23. $\lim_{x \rightarrow 0} \frac{x}{\frac{1}{1+x} - \cos x}$
24. $\lim_{x \rightarrow 0} \frac{\text{tg } x - \text{sen } x}{x^3}$
25. $\lim_{x \rightarrow 0} \frac{x^x - x}{1 + x^2}$
26. $\lim_{x \rightarrow 0} \text{sen } x \log x$

5.8. Calcolare i limiti seguenti:

1. $\lim_{x \rightarrow 0} \frac{2}{|x|}$
2. $\lim_{x \rightarrow 0} \frac{\text{sen}(\text{sen } x)}{x}$
3. $\lim_{x \rightarrow 0} \frac{\text{sen}(\cos x)}{x}$
4. $\lim_{x \rightarrow 0} \frac{\text{sen}(\cos x)}{x^2}$
5. $\lim_{x \rightarrow 0} 2^x \log |x|$
6. $\lim_{x \rightarrow 0} \sqrt{1 + \frac{1}{x^2}}$
7. $\lim_{x \rightarrow 0} \sqrt[3]{1 + \frac{1}{x}}$
8. $\lim_{x \rightarrow 1} \frac{x}{|x-1|}$
9. $\lim_{x \rightarrow 0} \frac{1}{1 - \cos x}$
10. $\lim_{x \rightarrow \frac{\pi}{2}} \text{tg } x$

5.9. Calcolare i seguenti limiti:

1. $\lim_{x \rightarrow +\infty} \frac{x^2 + 3}{4x^2 + x}$

2. $\lim_{x \rightarrow +\infty} \frac{3 - x^3 - x}{1 - 2x^2}$

3. $\lim_{x \rightarrow +\infty} \frac{\operatorname{sen} x}{x}$

4. $\lim_{x \rightarrow -\infty} \frac{(1-x)(1+x^2)}{1+x^4}$

5. $\lim_{x \rightarrow +\infty} \frac{x^2 + \operatorname{sen} x}{2x + x^2 + 3}$

6. $\lim_{x \rightarrow +\infty} \frac{x^2 + 1}{3^x}$

7. $\lim_{x \rightarrow +\infty} (x^3 + 3x)2^{-x}$

8. $\lim_{x \rightarrow +\infty} (x^2 + 3^x - \log_3 x)3^{-x}$

9. $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{2x}$

10. $\lim_{x \rightarrow +\infty} \left(\frac{x^2}{x+1} - x\right)$

11. $\lim_{x \rightarrow +\infty} \frac{\sqrt{1+x^2} + \sqrt{x}}{\sqrt{x} - x}$

12. $\lim_{x \rightarrow -\infty} x(\sqrt{1+x^4} - x^2)$

5.10. Calcolare i seguenti limiti:

1. $\lim_{x \rightarrow 0^+} [x]$

2. $\lim_{x \rightarrow 0^-} [x]$

3. $\lim_{x \rightarrow 0^-} \{x\}$

4. $\lim_{x \rightarrow 1^+} \{x\}$

5. $\lim_{x \rightarrow 0^-} \frac{x + |x|}{x^2}$

6. $\lim_{x \rightarrow 0^+} \frac{x + |x|}{x^2}$

7. $\lim_{x \rightarrow 2^+} (x + [x])$

8. $\lim_{x \rightarrow 1^-} (x + [x])$

9. $\lim_{x \rightarrow 0^+} \frac{1}{\operatorname{sen} x}$

10. $\lim_{x \rightarrow 0^-} 2^{\frac{1}{\operatorname{sen} x}}$

11. $\lim_{x \rightarrow -1^-} x^2 \operatorname{segno}(1+x)$

12. $\lim_{x \rightarrow 1^+} (\log x)^{\operatorname{segno}(1-x)}$