



Chapter 2

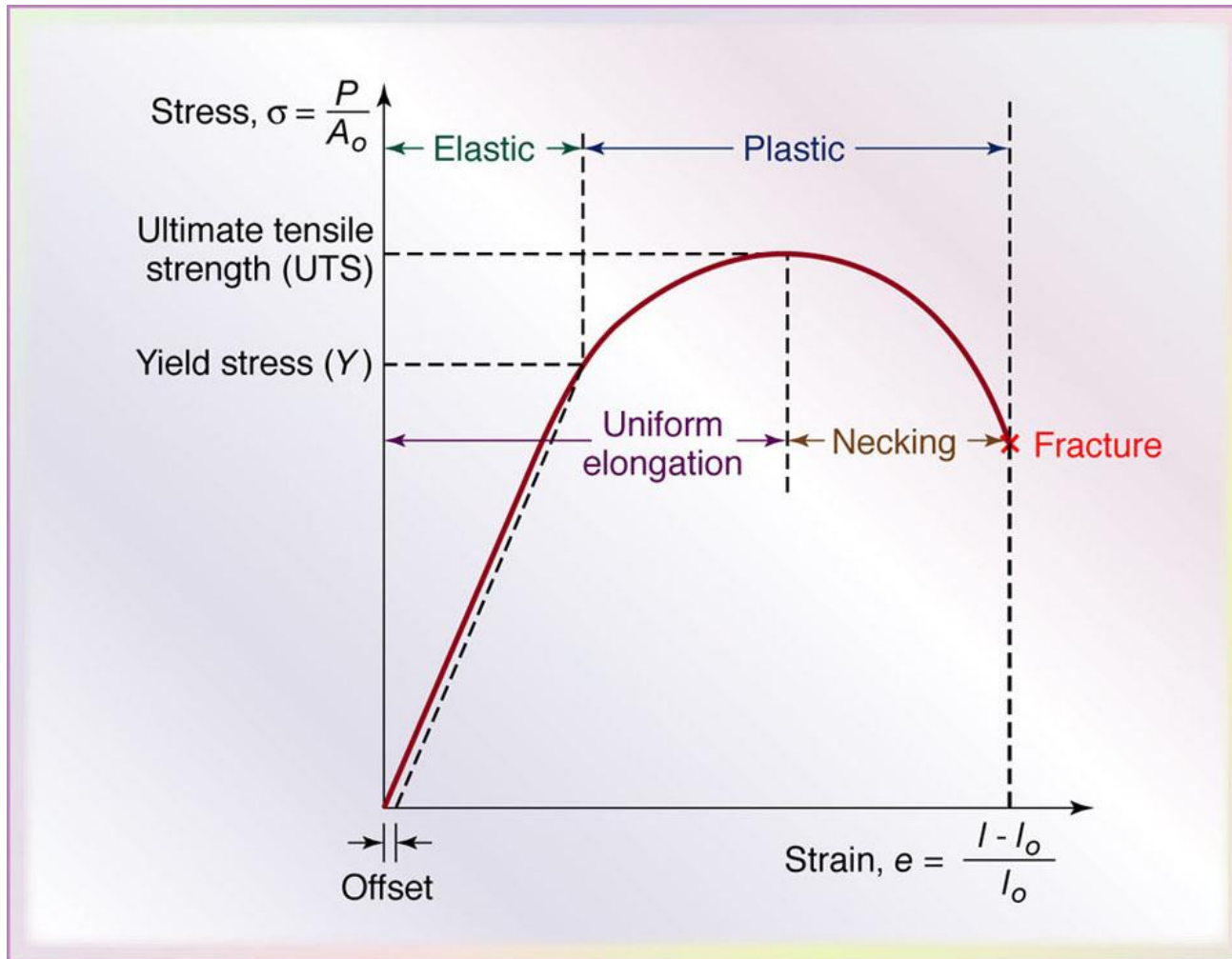
Mechanical behaviour of metallic materials

Mechanical behaviour of metallic materials at room temperature

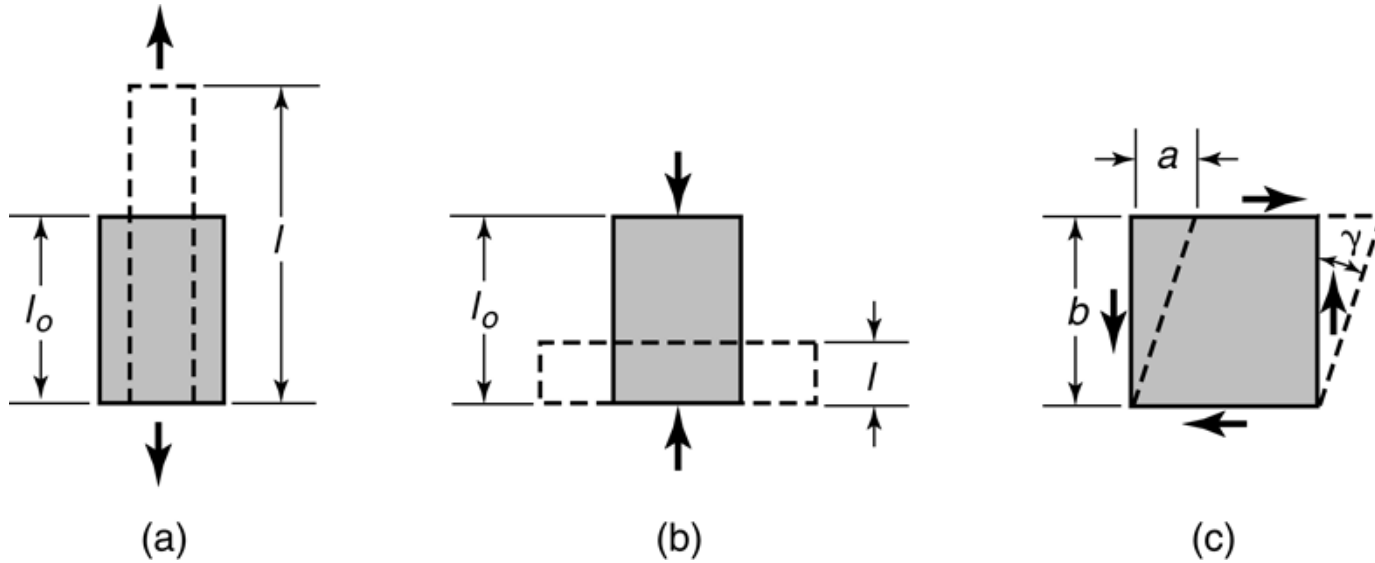
METALS (WROUGHT)	E (GPa)	Y (MPa)	UTS (MPa)	ELONGATION (%) in 50 mm	POISSON'S RATIO (n)
Aluminum and its alloys	69-79	35-550	90-600	45-5	0.31-0.34
Copper and its alloys	105-150	76-1100	140-1310	65-3	0.33-0.35
Lead and its alloys	14	14	20-55	50-9	0.43
Magnesium and its alloys	41-45	130-305	240-380	21-5	0.29-0.35
Molybdenum and its alloys	330-360	80-2070	90-2340	40-30	0.32
Nickel and its alloys	180-214	105-1200	345-1450	60-5	0.31
Steels	190-200	205-1725	415-1750	65-2	0.28-0.33
Stainless steels	190-200	240-480	480-760	60-20	0.28-0.30
Titanium and its alloys	80-130	344-1380	415-1450	25-7	0.31-0.34
Tungsten and its alloys	350-400	550-690	620-760	0	0.27
NONMETALLIC MATERIALS					
Ceramics	70-1000	-	140-2600	0	0.2
Diamond	820-1050	-	-	-	-
Glass and porcelain	70-80	-	140	0	0.24
Rubbers	0.01-0.1	-	-	-	0.5
Thermoplastics	1.4-3.4	-	7-80	1000-5	0.32-0.40
Thermoplastics, reinforced	2-50	-	20-120	10-1	-
Thermosets	3.5-17	-	35-170	0	0.34
Boron fiber	380	-	3500	0	-
Carbon fibers	275-415	-	2000-5300	1-2	-
Glass fibers (S, E)	73-85	-	3500-4600	5	-
Kevlar fibers (29, 49, 129)	70-113	-	3000-3400	3-4	-
Spectra fibers (900, 1000)	73-100	-	2400-2800	3	-

Note: In the upper table the lowest values for E , Y , and UTS and the highest values for elongation are for the pure metals. Multiply GPa by 145,000 to obtain psi, and MPa by 145 to obtain psi. For example, 100 GPa = 14,500 ksi, and 100 MPa = 14,500 psi.

Engineering stress-strain curve



Strain



Types of strain. (a) Tensile, (b) compressive, and (c) shear

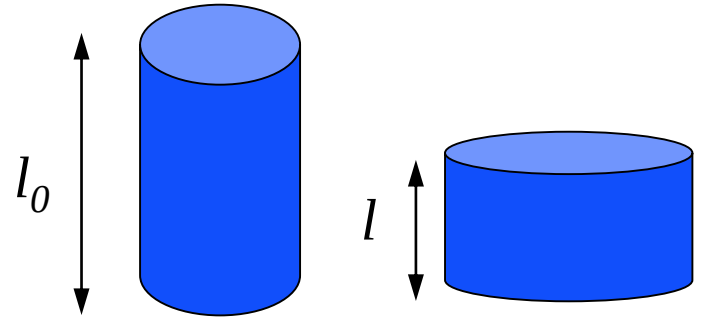
$$e = \frac{l - l_0}{l_0} \quad \text{Engineering strain}$$

$$\gamma = \frac{a}{b} \quad \text{Shear strain}$$

Strain

True

$$d\varepsilon = \frac{dl}{l} \quad \bar{\varepsilon} = \ln\left(\frac{l}{l_0}\right)$$



Engineering

$$e = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}$$



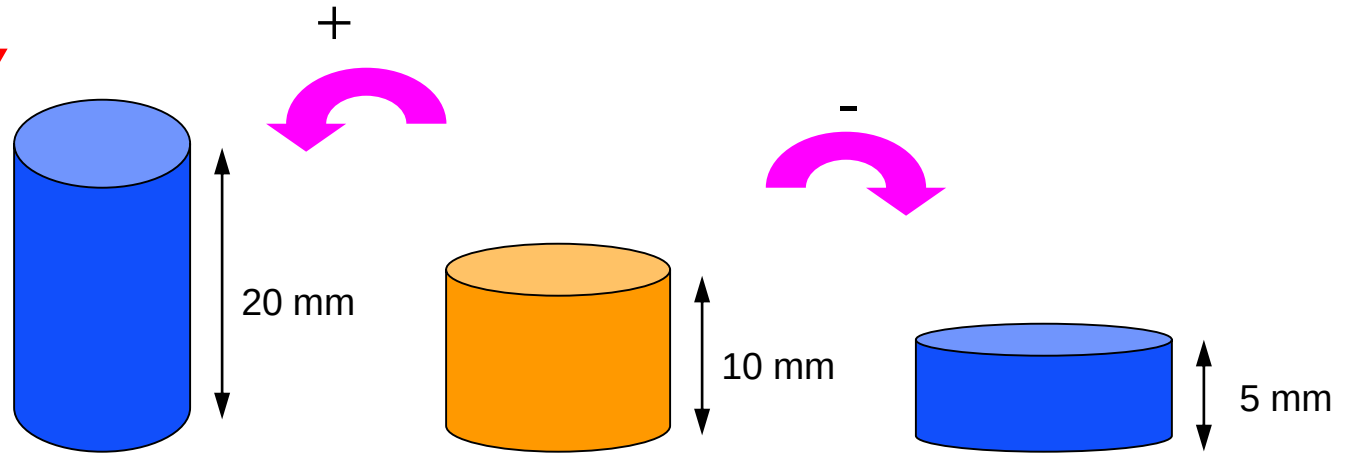
$$\bar{\varepsilon} = \ln(1 + e)$$

$$e = \frac{5 - 10}{10} = -0.5 \Rightarrow -50\%$$

$$\bar{\varepsilon} = \ln\left(\frac{5}{10}\right) = -0.693 \approx -0.7$$

Strain

Symmetry



$$e_1 = \frac{20-10}{10} = 1. \Rightarrow 100\%$$

$$e_2 = \frac{5-10}{10} = -0.5 \Rightarrow -50\%$$

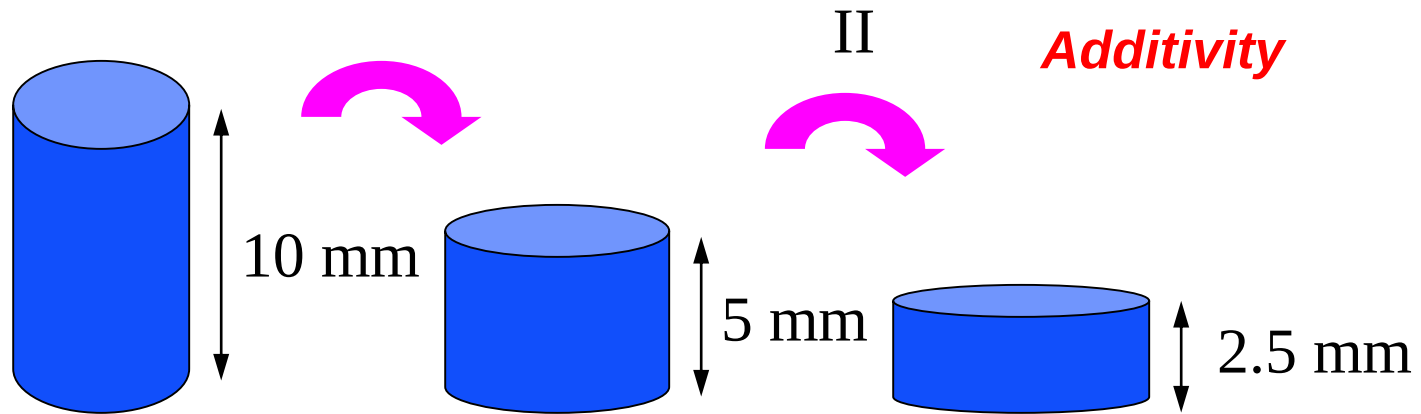
$$\bar{\varepsilon}_1 = \ln\left(\frac{20}{10}\right) = 0.693 \approx 0.7$$

$$\bar{\varepsilon}_2 = \ln\left(\frac{5}{10}\right) = -0.693 \approx -0.7$$

Strain

$$\bar{\varepsilon} = \ln\left(\frac{l}{l_0}\right)$$

$$e = \frac{\Delta l}{l_0}$$



$$e_1 = \frac{5-10}{10} = -0.5$$

$$e_2 = \frac{2.5-5}{5} = -0.5$$

$$\bar{\varepsilon}_1 = \ln\left(\frac{5}{10}\right) = -0.693 \quad \bar{\varepsilon}_2 = \ln\left(\frac{2.5}{5}\right) = -0.693$$

$$e_{tot} = \frac{2.5-10}{10} = -0.75$$

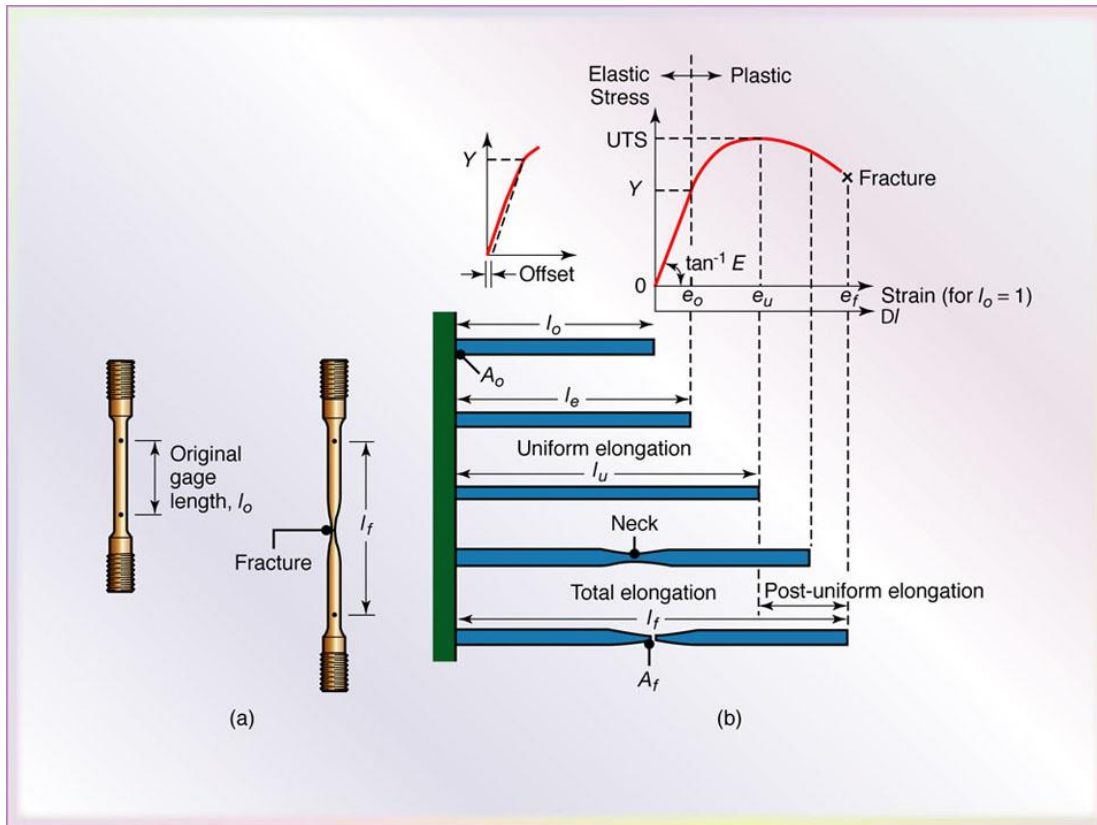
$$e_1 + e_2 = -1.0$$

$$\bar{\varepsilon}_{tot} = \ln\left(\frac{2.5}{10}\right) = -1.3863 \quad \bar{\varepsilon}_1 + \bar{\varepsilon}_2 = -1.3863$$

$$e_1 + e_2 \neq e_{tot}$$

$$\bar{\varepsilon}_1 + \bar{\varepsilon}_2 = \bar{\varepsilon}_{tot}$$

Tensile test



$$S = \frac{F}{A_0} \quad \text{Engineering stress}$$

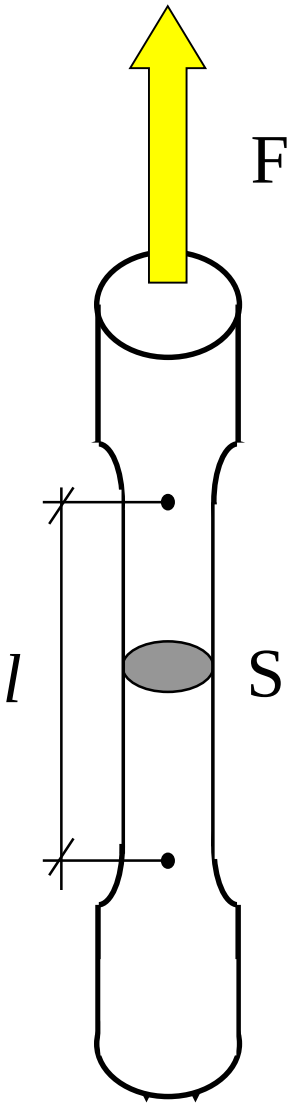
$$E = \frac{\sigma}{e} \quad \text{Young's module}$$

True stress

$$s = \frac{F}{A} \quad \text{True stress}$$

• *Hp: volume constancy*

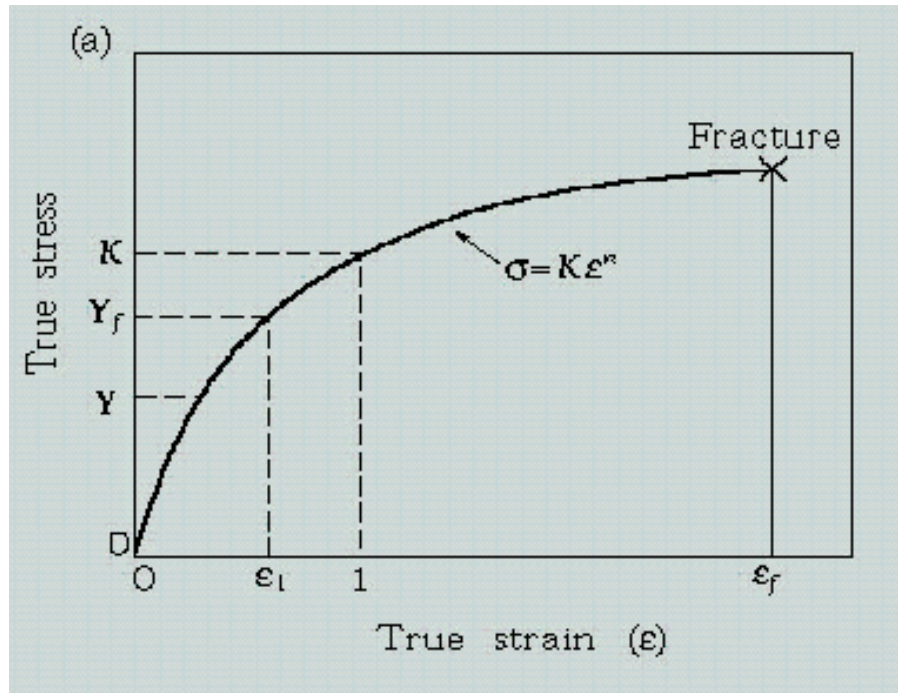
$$\varepsilon = \ln\left(\frac{l}{l_0}\right) = \ln\left(\frac{A_0}{A}\right) = \ln\left(\frac{D_0}{D}\right)^2 = 2 \ln\left(\frac{D_0}{D}\right)$$



Homologous temperature

Process	T/T_m
Cold working	< 0.3
Warm working	0.3 to 0.5
Hot working	> 0.6

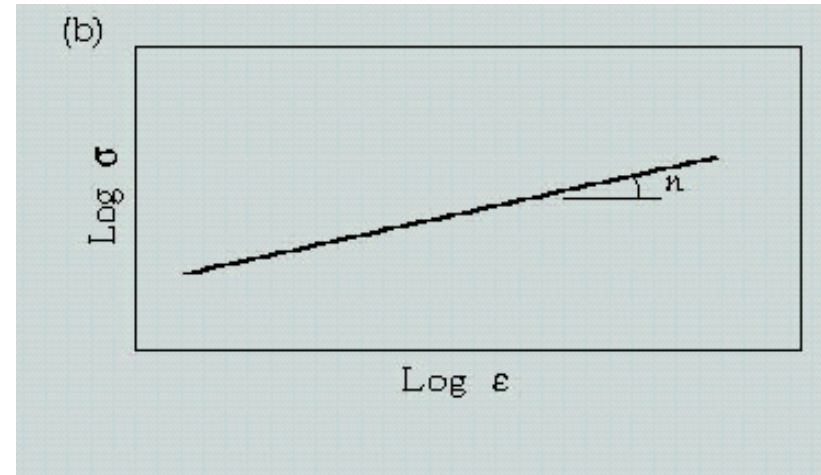
True stress modelling in cold conditions



$$\sigma = K \cdot \epsilon^n$$

Hollomon's equation

$$\text{toughness} = \int_0^{\epsilon_f} \sigma d\epsilon$$



$$\ln \sigma = \ln K + n \ln \epsilon$$

n strain hardening exponent

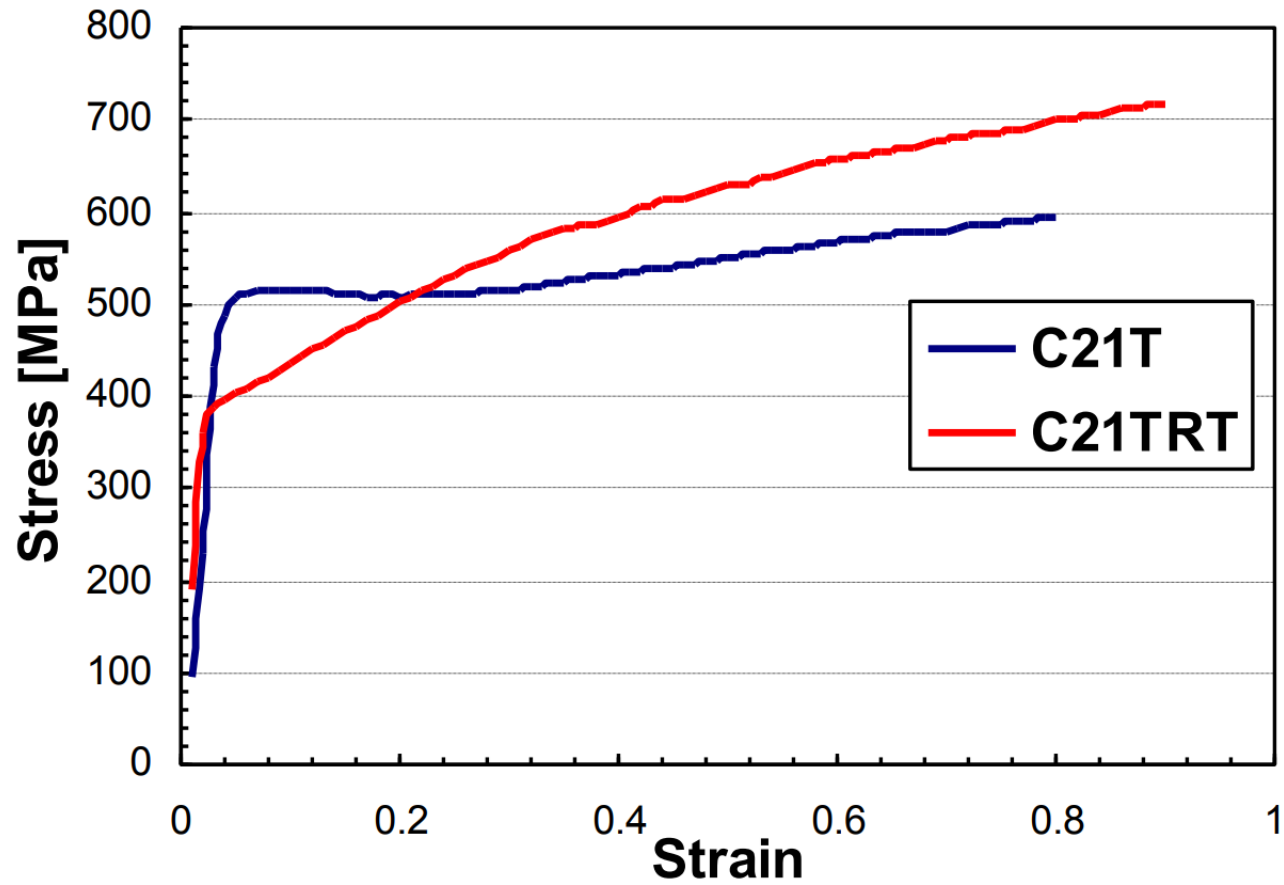
K strength coefficient

Hollomon's equation

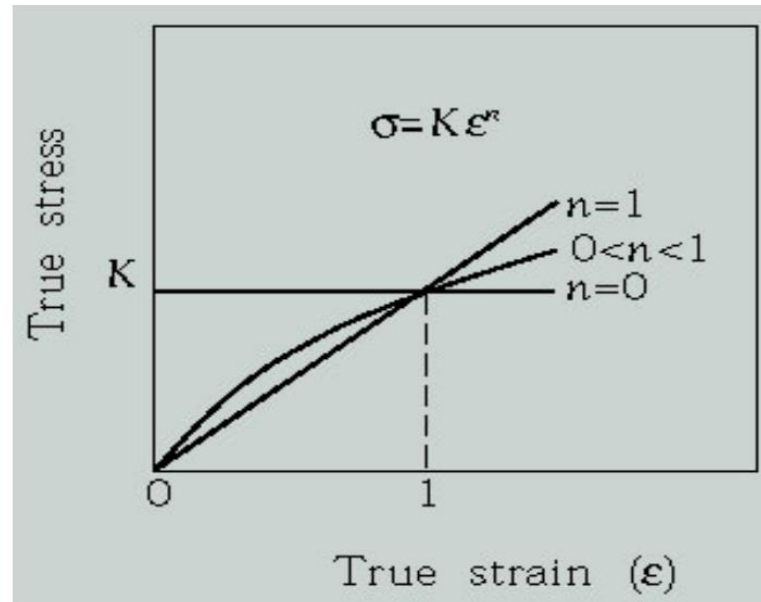
MATERIAL	K (MPa)	n
Aluminum, 1100-O	180	0.20
2024-T4	690	0.16
5052-O	210	0.13
6061-O	205	0.20
6061-T6	410	0.05
7075-O	400	0.17
Brass, 70-30, annealed	895	0.49
85-15, cold-rolled	580	0.34
Bronze (phosphor), annealed	720	0.46
Cobalt-base alloy, heat treated	2070	0.50
Copper, annealed	315	0.54
Molybdenum, annealed	725	0.13
Steel, low-carbon, annealed	530	0.26
1045 hot-rolled	965	0.14
1112 annealed	760	0.19
1112 cold-rolled	760	0.08
4135 annealed	1015	0.17
4135 cold-rolled	1100	0.14
4340 annealed	640	0.15
17-4 P-H annealed	1200	0.05
52100 annealed	1450	0.07
304 stainless, annealed	1275	0.45
410 stainless, annealed	960	0.10

K e n typical values in cold conditions

Example

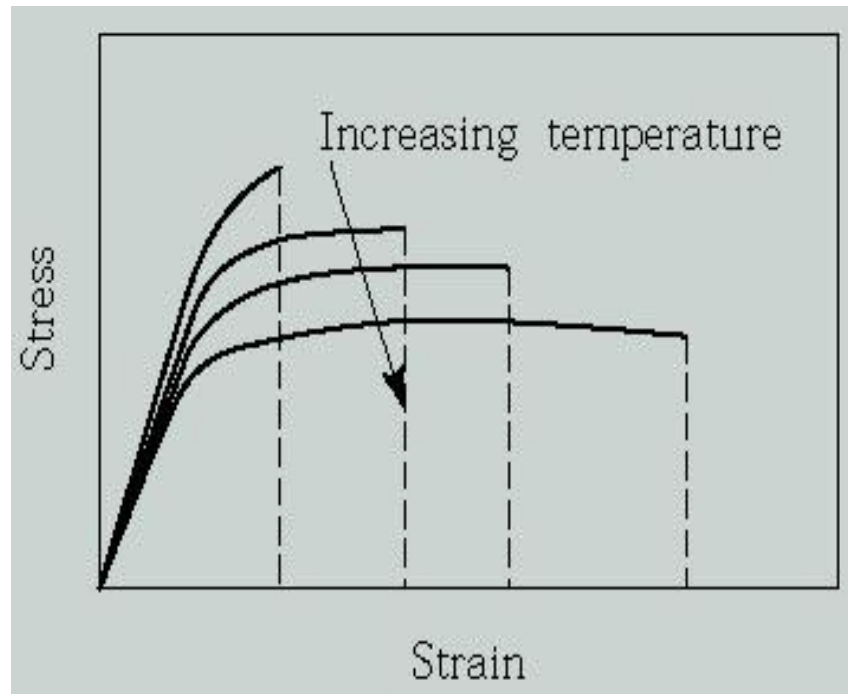


Example



The effect of the strain -hardening exponent, n , on the shape of true-stress-true-strain curves. When $n = 1$, the material is elastic, and when $n = 0$, it is rigid and perfectly plastic.

Temperature effects



Typical effects of temperature on engineering stress-strain curves. Temperature affects the modulus of elasticity, yield stress, ultimate tensile strength, and toughness of materials.

Strain rate

Engineering strain rate $\dot{\epsilon} = \frac{de}{dt} = \frac{d\left(\frac{l-l_0}{l_0}\right)}{dt} = \frac{1}{l_0} \frac{dl}{dt} = \frac{v}{l_0}$

True strain rate $\dot{\epsilon} = \frac{d\epsilon}{dt} = \frac{d\left[\ln\left(\frac{l}{l_0}\right)\right]}{dt} = \frac{1}{l} \frac{dl}{dt} = \frac{v}{l}$

Strain rate

PROCESS	TRUE STRAIN	DEFORMATION SPEED (m/s)	STRAIN RATE (s^{-1})
Cold working			
Forging, rolling	0.1-0.5	0.1-100	$1-10^3$
Wire and tube drawing	0.05-0.5	0.1-100	$1-10^4$
Explosive forming	0.05-0.2	10-100	$10-10^5$
Hot working and warm working			
Forging, rolling	0.1-0.5	0.1-30	$1-10^3$
Extrusion	2-5	0.1-1	$10^{-1}-10^2$
Machining	1-10	0.1-100	10^3-10^6
Sheet-metal forming	0.1-0.5	0.05-2	$1-10^2$
Superplastic forming	0.2-3	$10^{-4}-10^{-2}$	$10^{-4}-10^{-2}$

Typical ranges of strain, deformation speed, and strain rates in metalworking processes.

True stress modelling in hot conditions

$$\sigma = C \dot{\epsilon}^m$$

m = strain-rate exponent coefficient

C = strength coefficient

MATERIAL	TEMPERATURE, °C	C		m
		psi x 10 ³	MPa	
Aluminum	200-500	12-2	82-14	0.07-0.23
Aluminum alloys	200-500	45-5	310-35	0-0.20
Copper	300-900	35-3	240-20	0.06-0.17
Copper alloys (brasses)	200-800	60-2	415-14	0.02-0.3
Lead	100-300	1.6-0.3	11-2	0.1-0.2
Magnesium	200-400	20-2	140-14	0.07-0.43
Steel				
Low-carbon	900-1200	24-7	165-48	0.08-0.22
Medium-carbon	900-1200	23-7	160-48	0.07-0.24
Stainless	600-1200	60-5	415-35	0.02-0.4
Titanium	200-1000	135-2	930-14	0.04-0.3
Titanium alloys	200-1000	130-5	900-35	0.02-0.3
Ti-6Al-4V*	815-930	9.5-1.6	65-11	0.50-0.80
Zirconium	200-1000	120-4	830-27	0.04-0.4

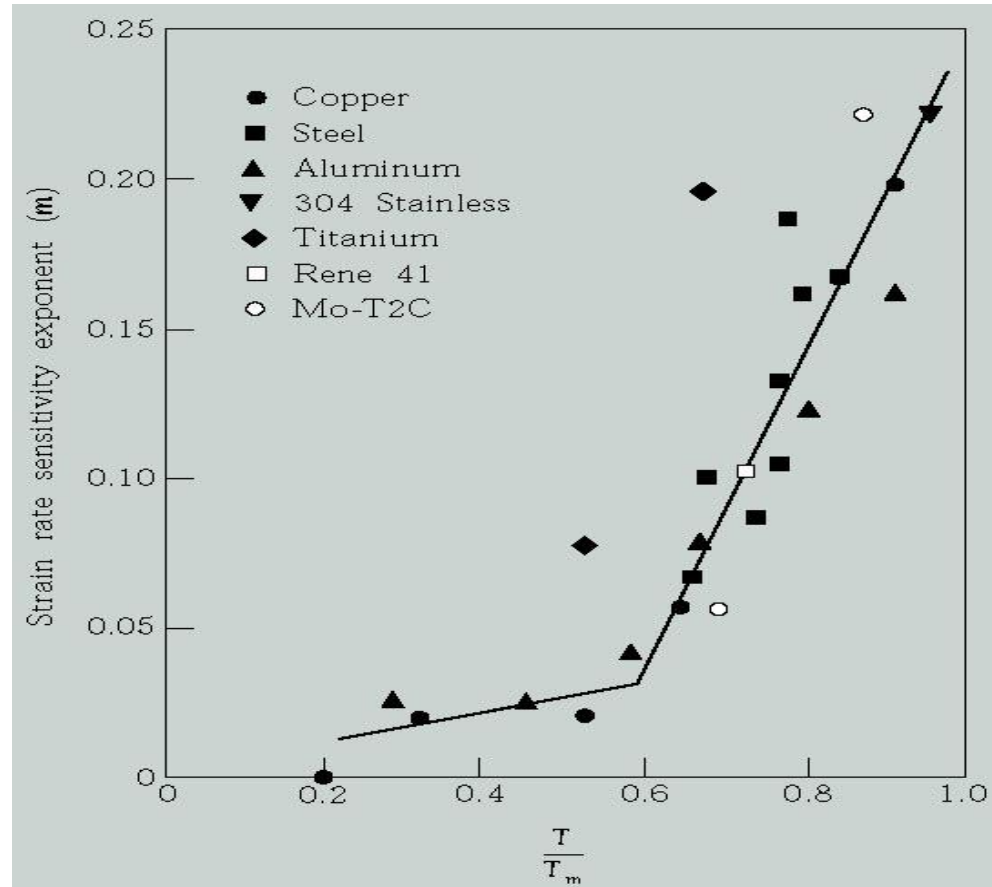
* At a strain rate of $2 \times 10^{-4} \text{ s}^{-1}$.

Note: As temperature increases, *C* decreases and *m* increases. As strain increases, *C* increases and *m* may increase or decrease, or it may become negative within certain ranges of temperature and strain.

Source: After T. Altan and F.W. Boulger.

Approximate range of values for *C* and *m* for various annealed materials at true strains ranging from 0.2 to 1.0.

Strain rate sensitivity



Dependence of the strain-rate sensitivity exponent m on the homologous temperature T/T_m for various materials. T is the testing temperature, and T_m is the melting point of the metal, both on the absolute scale. The transition in the slopes of the curve occurs at about the recrystallization temperature of the metals.

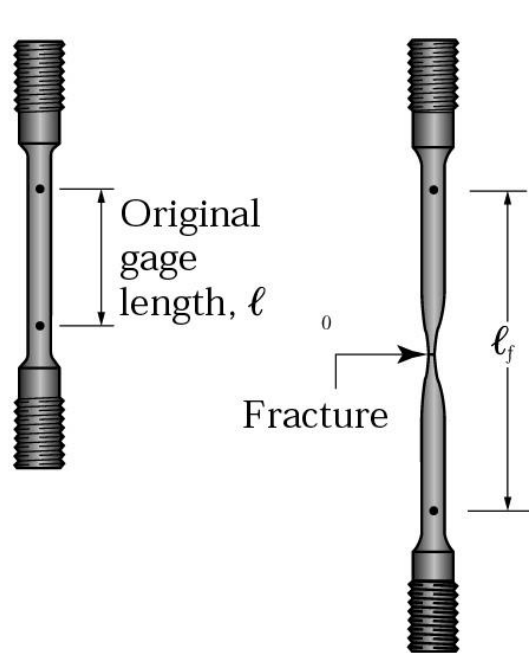
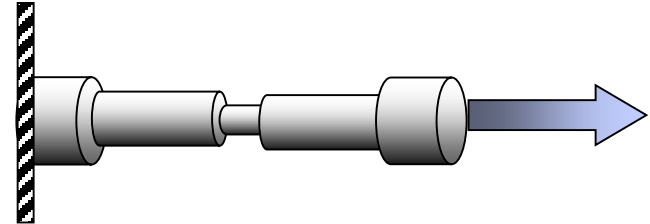
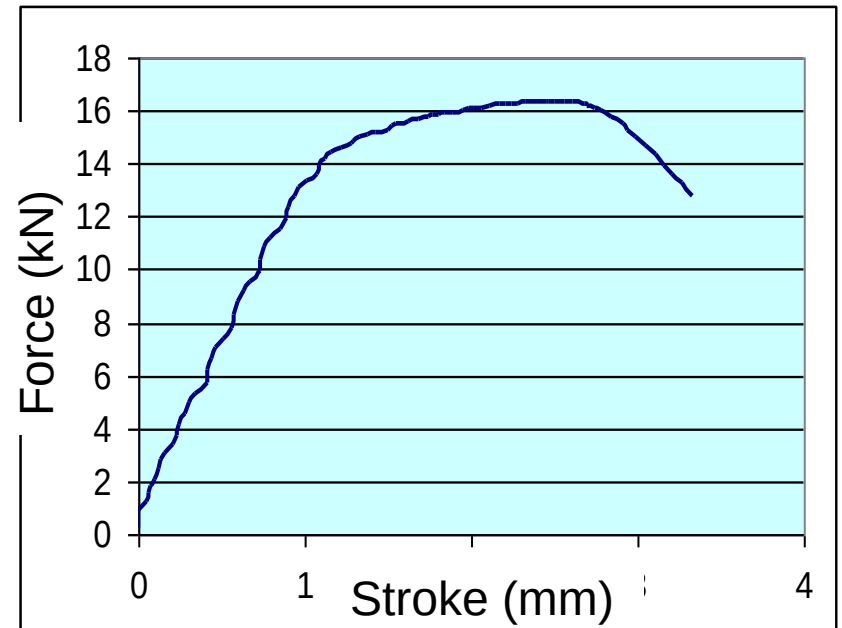
Mechanical tests

➤ Tensile

➤ Compression

➤ Torsion

Tensile test

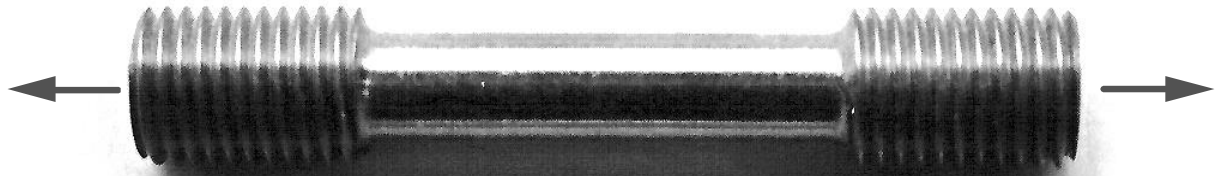


No friction

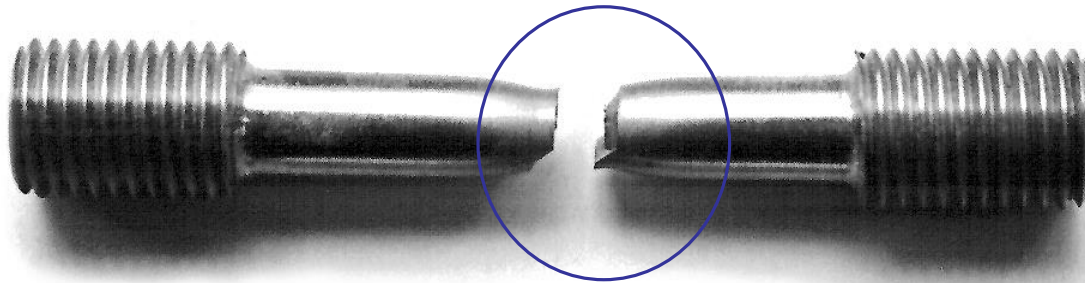
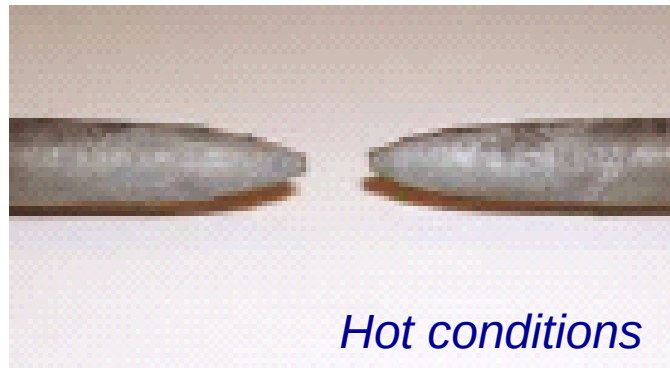
Small amount of strain

Necking

Necking and fracture

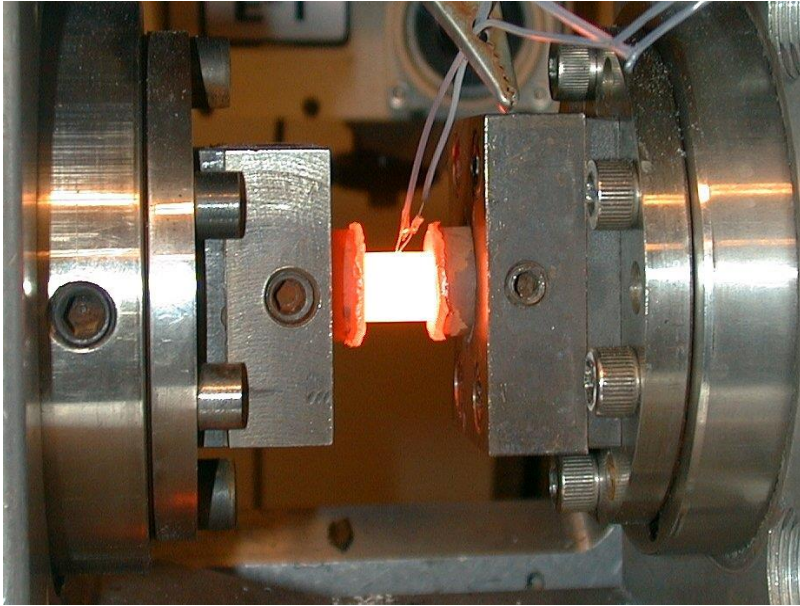
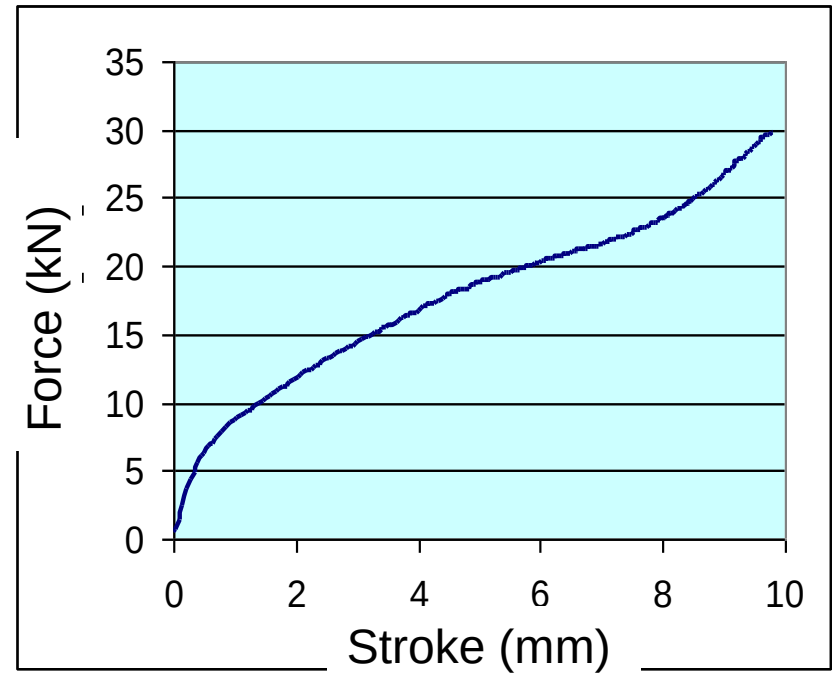


Cold conditions



Hot conditions

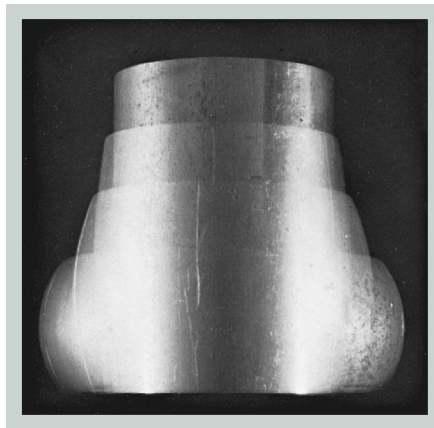
Compression test



Friction

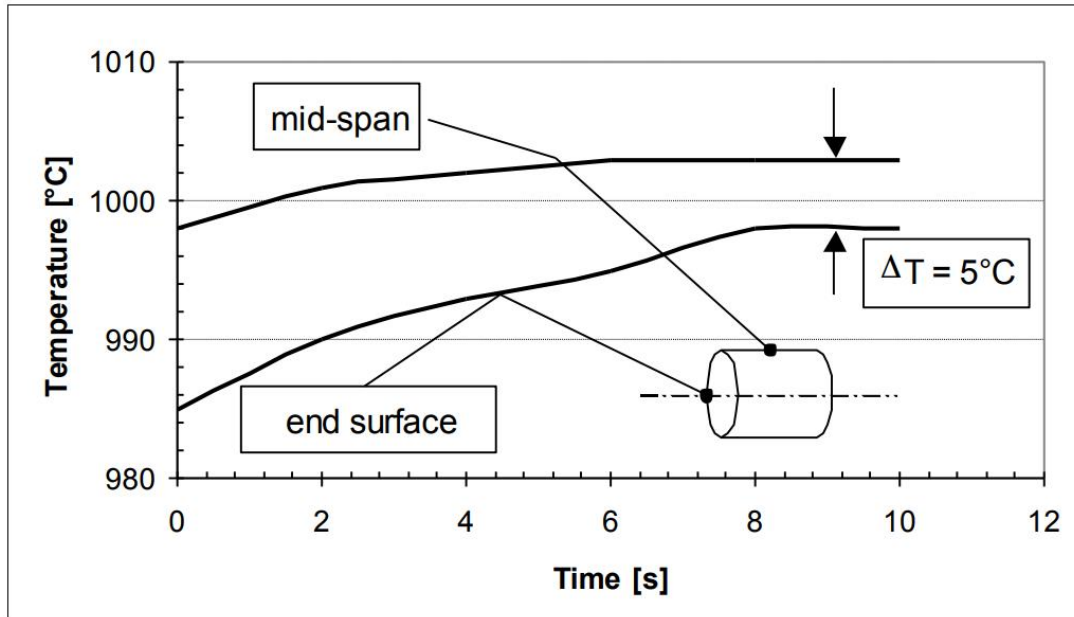
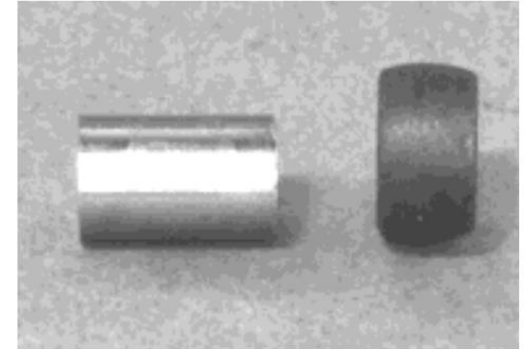
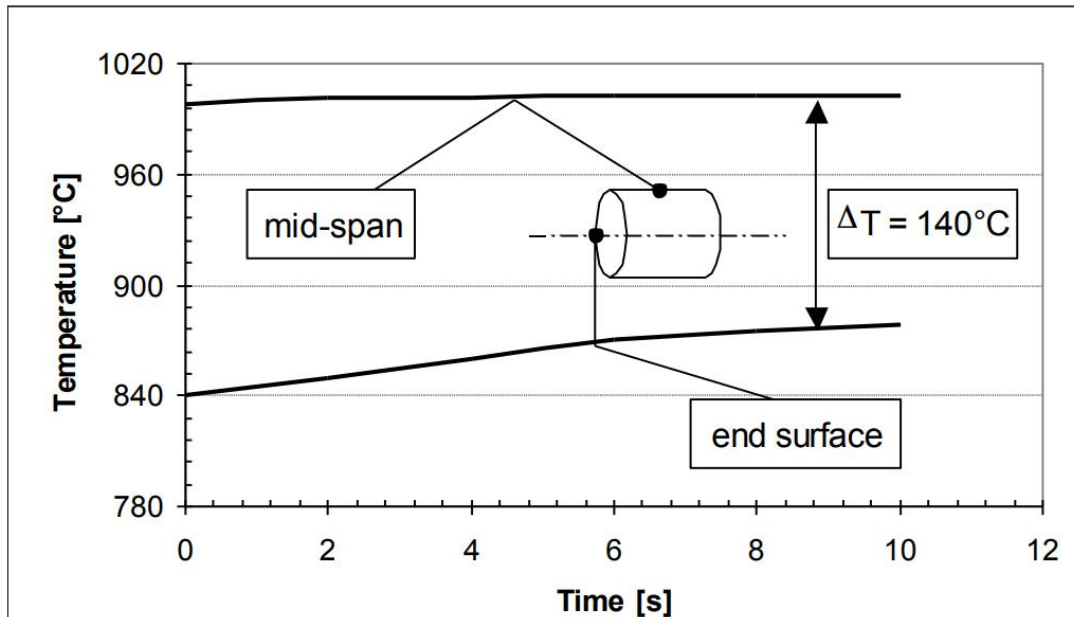
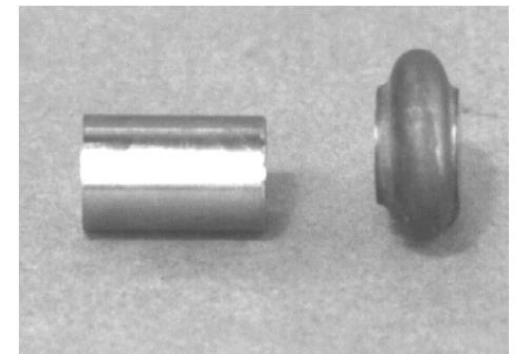
No necking

Barreling

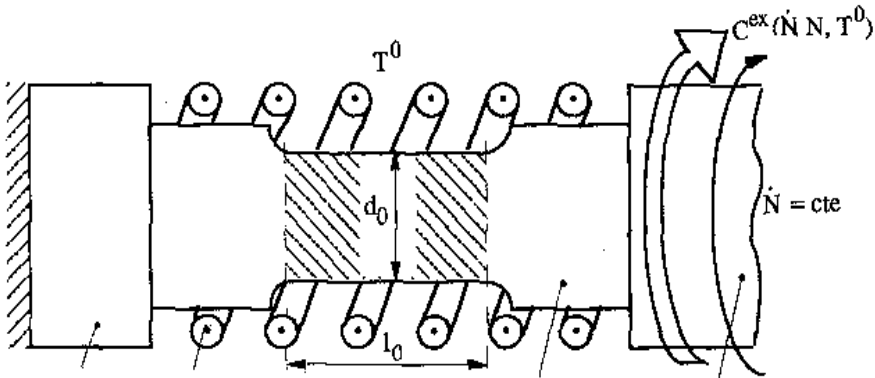
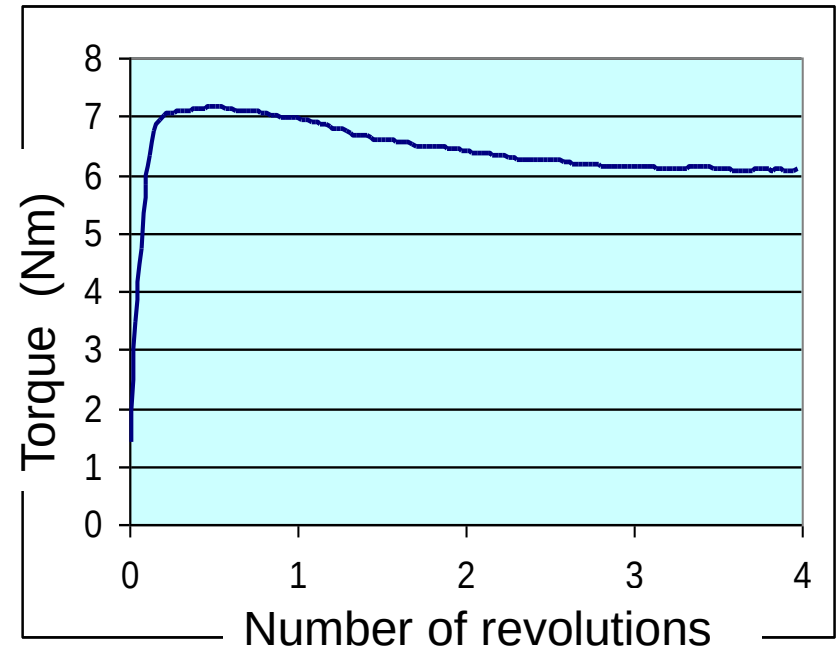


NB: State of stress similar to those of most of bulk forming processes

Compression test in hot conditions



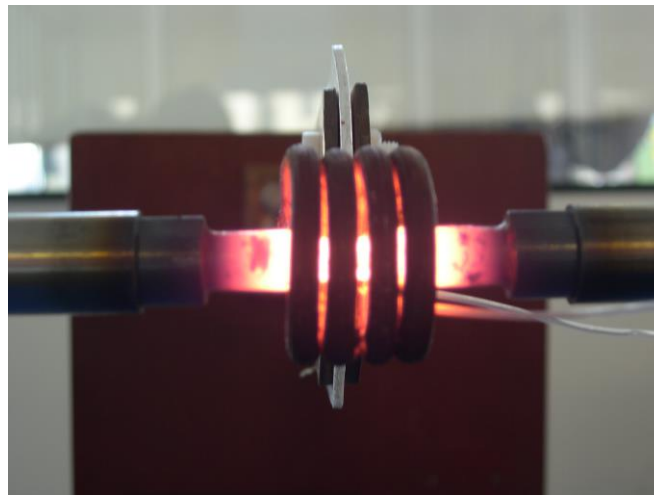
Torsion test



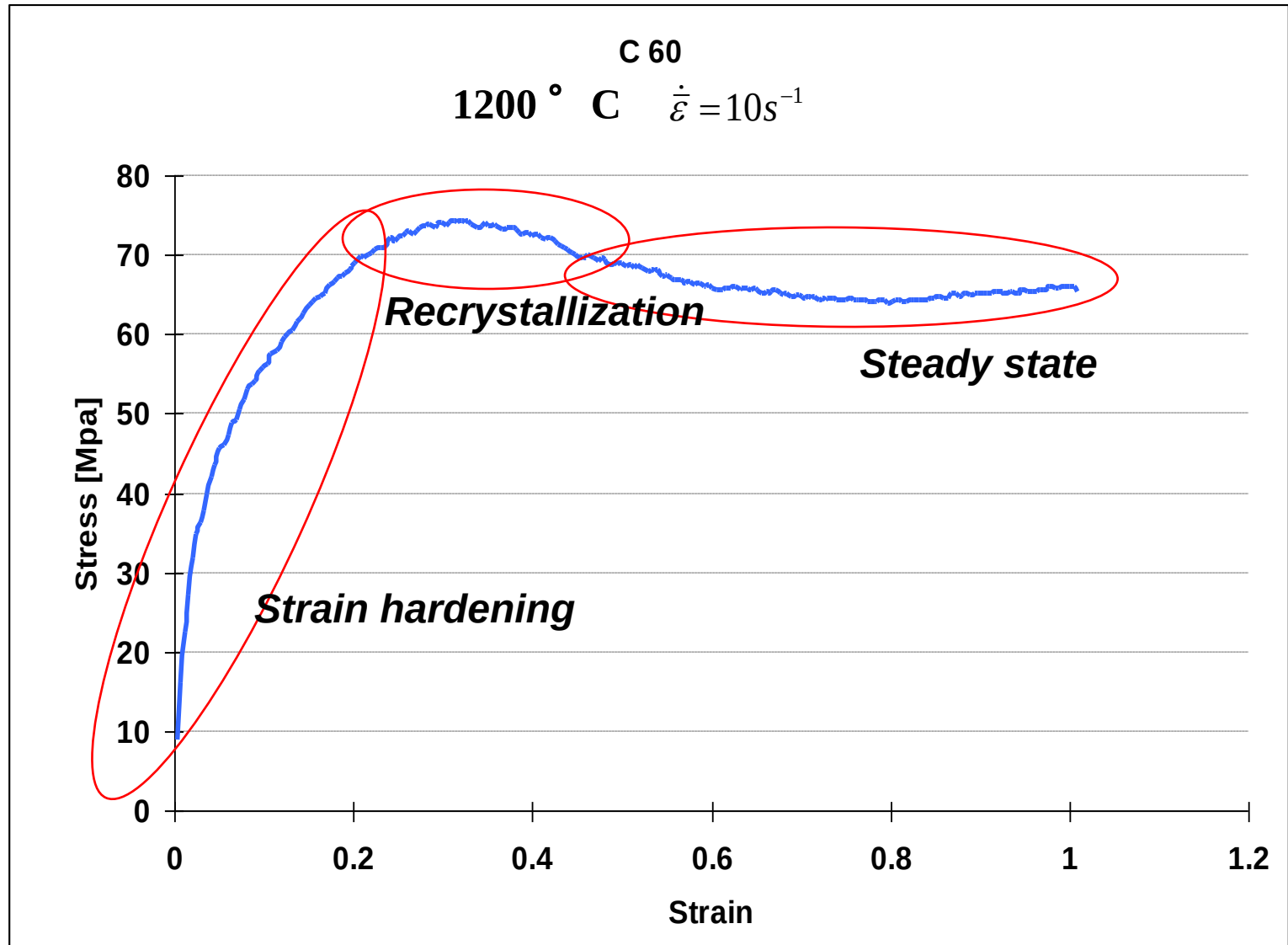
No friction

High amount of strain

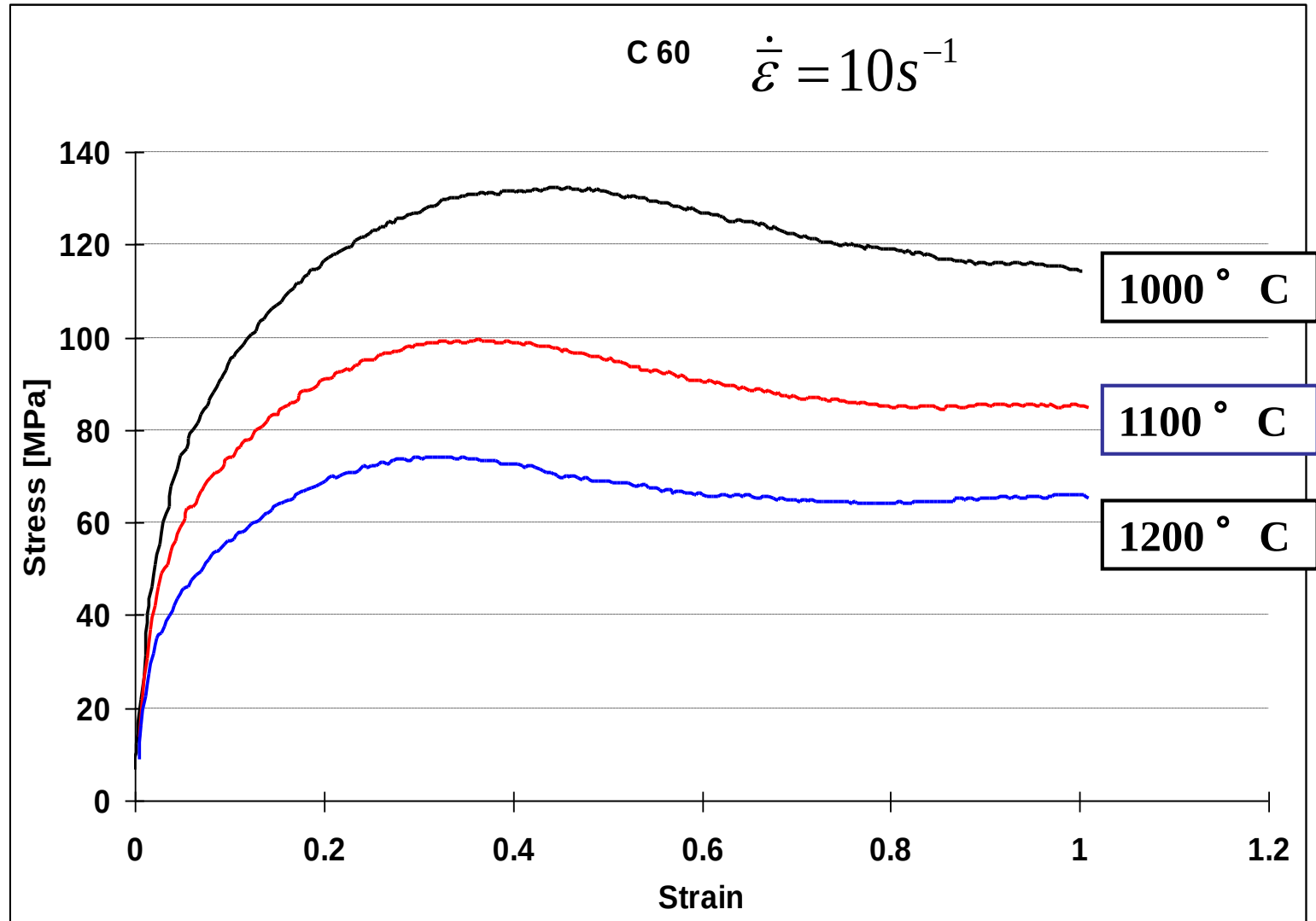
Strain gradient



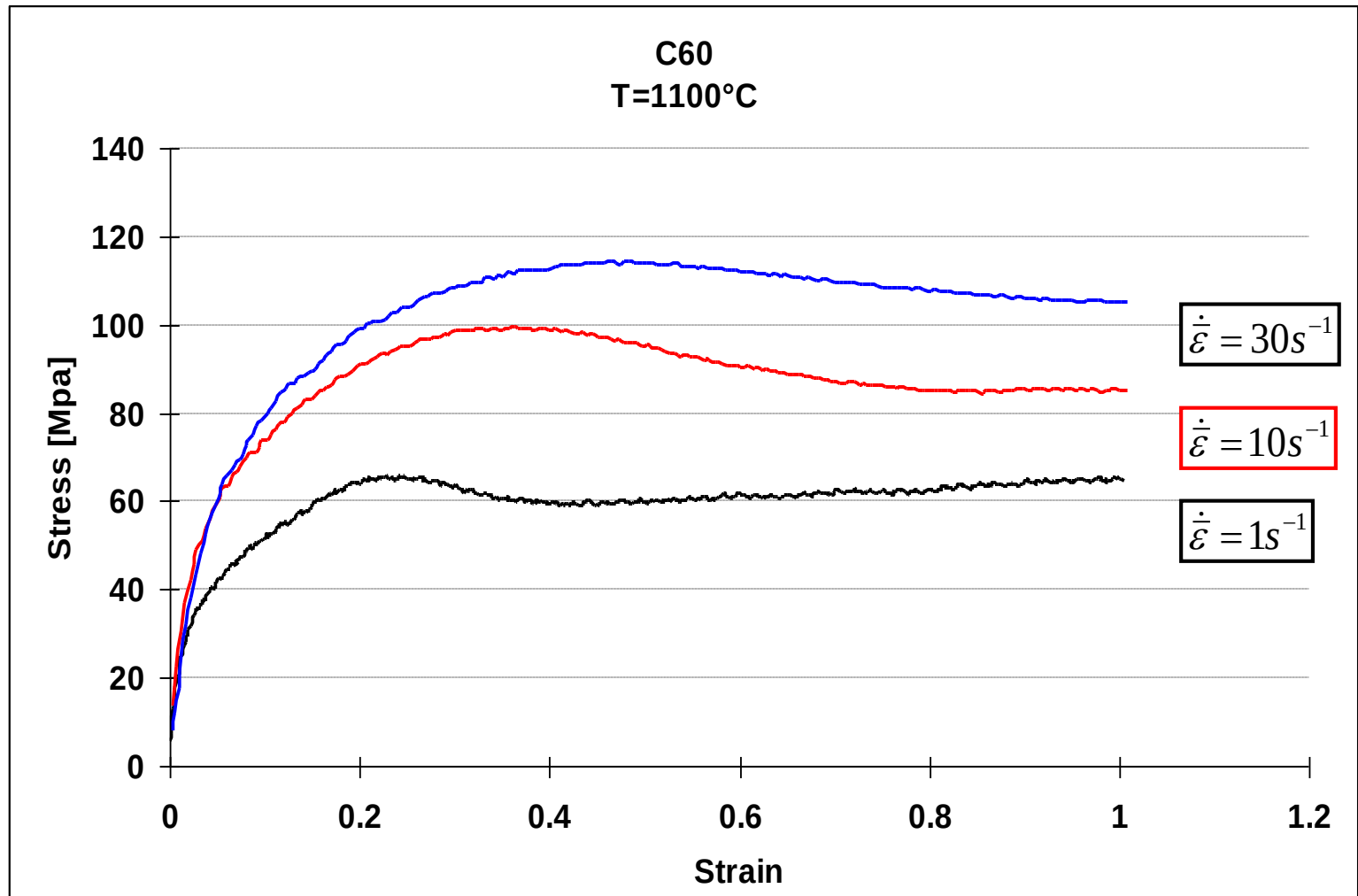
Sensitivity to strain



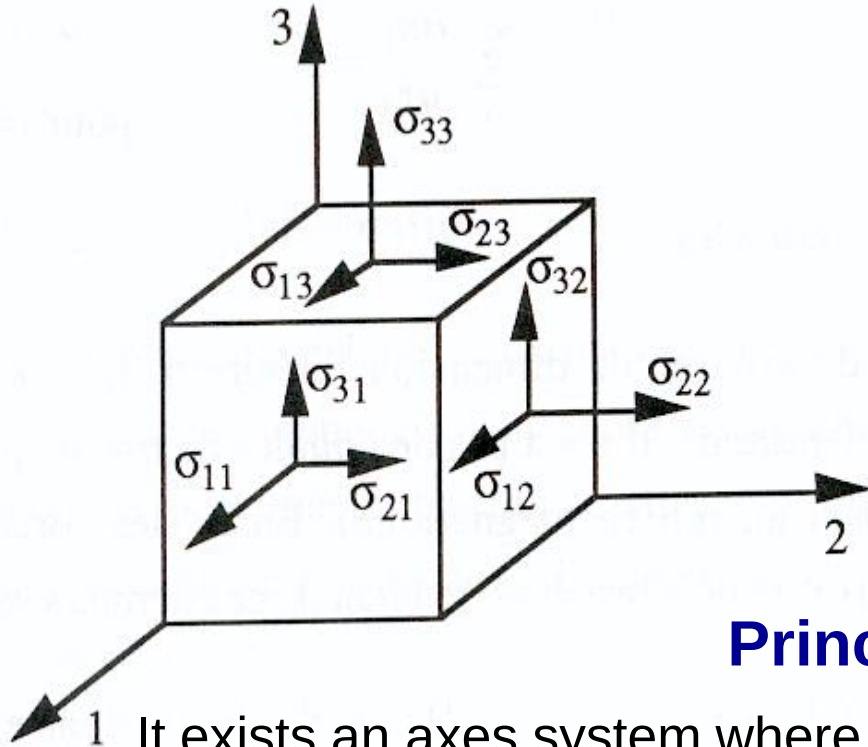
Sensitivity to temperature



Sensitivity to strain rate



Stress tensor



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

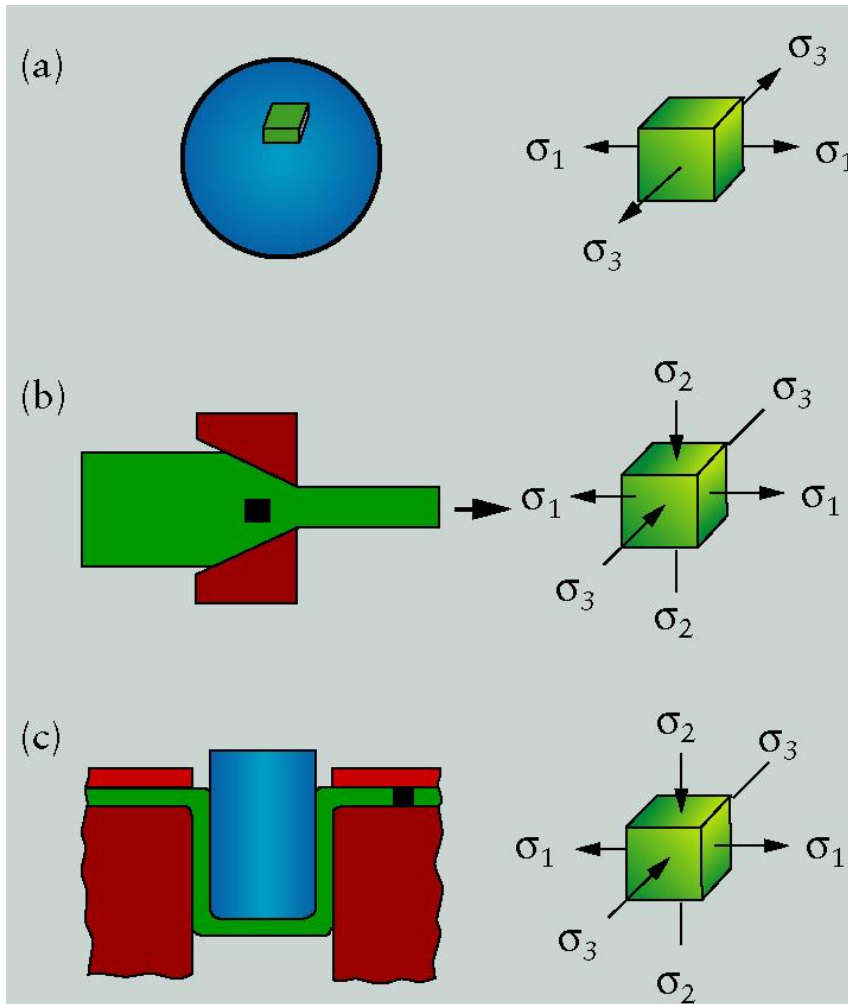
Principal stresses

1 It exists an axes system where the stress tensor is symmetric with only the diagonal values different from zero

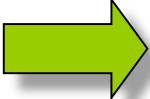
$$\begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix}$$

$$\text{con } \sigma_I > \sigma_{II} > \sigma_{III}$$

State of stress in metalworking processes



The state of stress in various metal working processes. (a) Expansion of a thin-walled spherical shell under internal pressure. (b) Drawing of round rod or wire through a conical die to reduce its diameter. (c) Deep drawing of sheet metal with a punch and die to make a metal cup.

 ***Triaxial state of stress***

Yield criteria

- **TRESCA criterion**: yielding occurs when the maximum shear stress within an element is equal or exceeds a critical value;

this critical value of the shear stress is a material property and is called **shear yield stress k**

For yielding to occur: $\tau_{\max} \geq k$

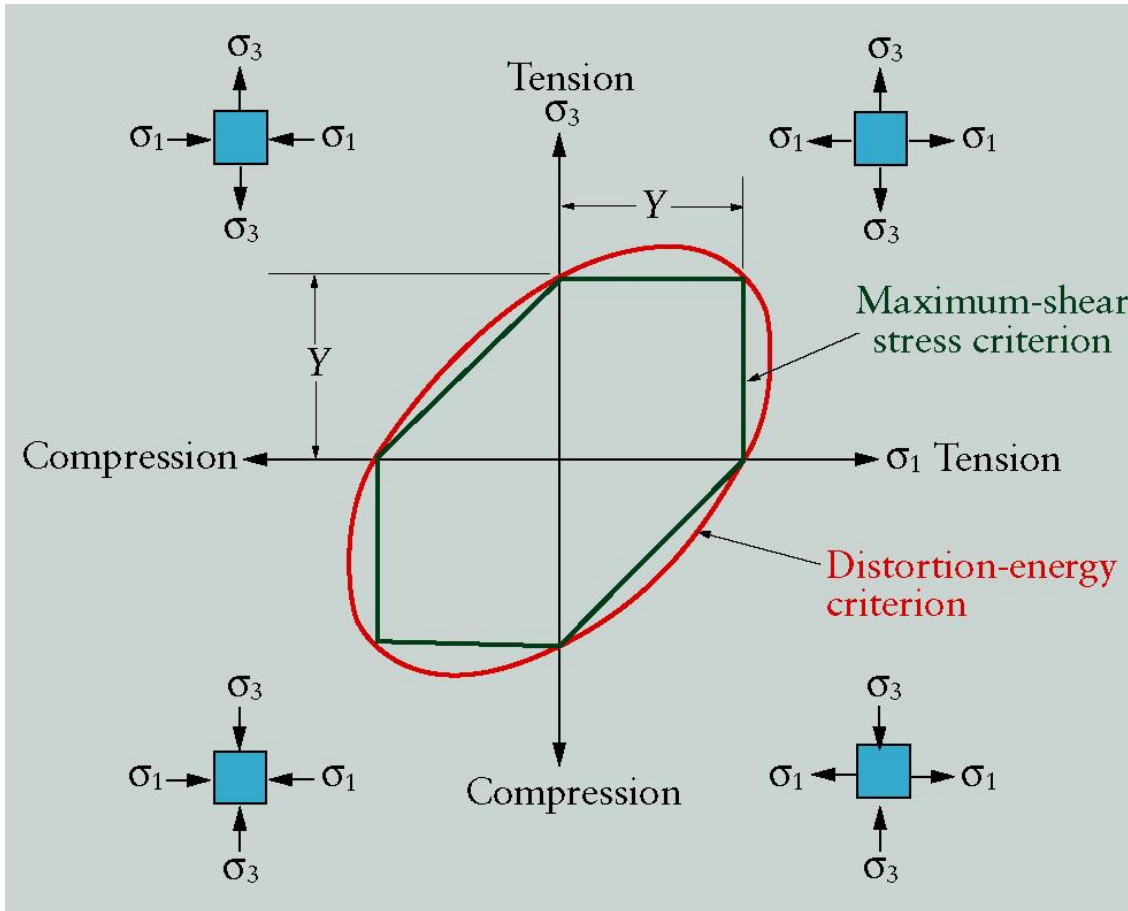
For the simple tension test $k = \frac{Y}{2}$ Y = uniaxial yield stress of the material
 k = shear yield strength of the material

From Mohr's circles
(principal axes): $\sigma_{\max} - \sigma_{\min} = Y$

- **VON MISES criterion**: yielding occurs when the relationship between the principal applied stresses and uniaxial yield stress Y of the material is:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2Y^2$$

Yield criteria (plane stress)



Tresca:

$$\sigma_3 - \sigma_1 = Y$$

Von Mises:

$$\sigma_1^2 + \sigma_3^2 - \sigma_1 \cdot \sigma_3 = Y^2$$

Elastic strains

$$\varepsilon_1 = \frac{1}{E} [\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$\varepsilon_2 = \frac{1}{E} [\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$\varepsilon_3 = \frac{1}{E} [\sigma_3 - \nu(\sigma_1 + \sigma_2)]$$

***Generalised Hooke's
equations***

Plastic strains

$$\left. \begin{aligned} d\varepsilon_1 &= \frac{d\bar{\varepsilon}}{\bar{\sigma}} \left[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3) \right] \\ d\varepsilon_2 &= \frac{d\bar{\varepsilon}}{\bar{\sigma}} \left[\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3) \right] \\ d\varepsilon_3 &= \frac{d\bar{\varepsilon}}{\bar{\sigma}} \left[\sigma_3 - \frac{1}{2}(\sigma_1 + \sigma_2) \right] \end{aligned} \right\} \text{Flow rules – Levy-Mises' equations}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2}$$

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

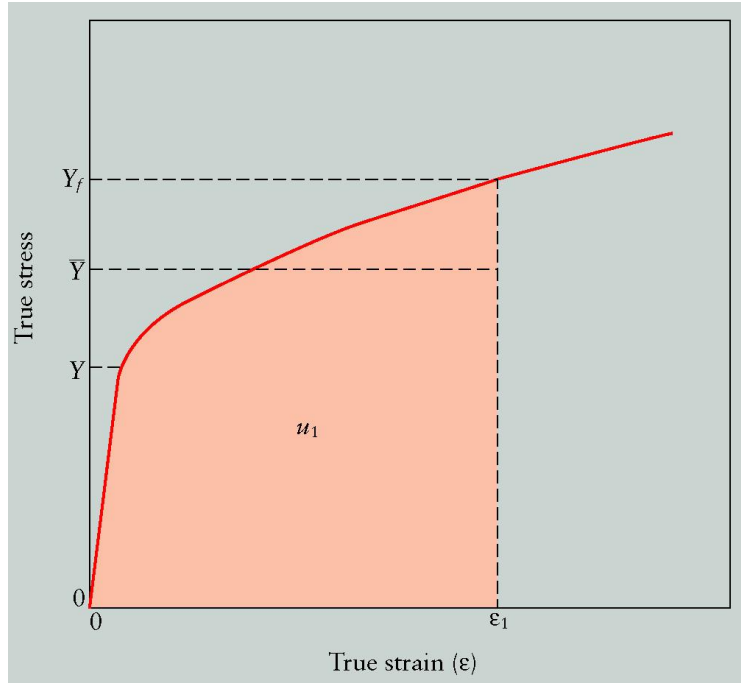
Yield criteria (plane deformation)

$$d\varepsilon_2 = \frac{d\bar{\varepsilon}}{\sigma} \left[\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3) \right]$$

Plane deformation $\varepsilon_2 = 0$  $\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$

Von Mises criterion: $\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}} Y$

Deformation work



Specific energy of the deformed material:

$$u = \int_0^{\varepsilon_1} \sigma d\varepsilon$$

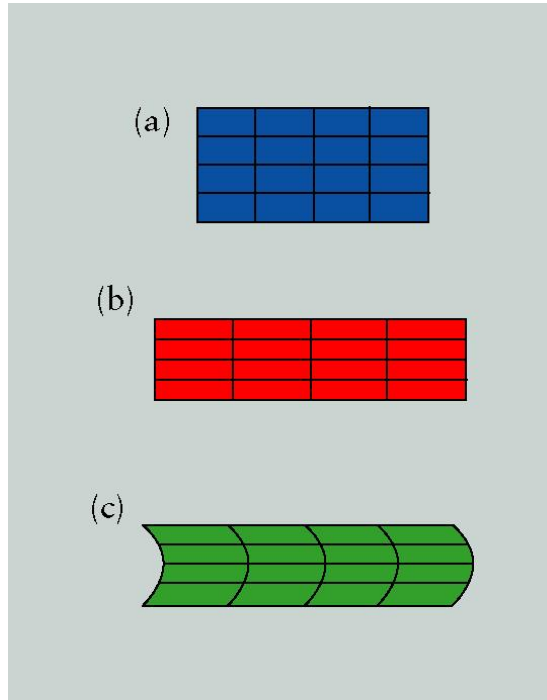


$$u = \int_0^{\varepsilon_1} K \cdot \varepsilon^n d\varepsilon = K \int_0^{\varepsilon_1} \varepsilon^n d\varepsilon$$

$$u = \frac{K \times \varepsilon_1^{n+1}}{n+1} = \bar{Y}_f \times \varepsilon_1$$

Schematic illustration of a true-stress-true-strain curve showing the yield stress Y , the average flow stress $\bar{Y}_f$, the specific energy u_1 , and the flow stress Y_f .

Deformation heating



Deformation of grid patterns in a workpiece: (a) the original pattern; (b) after ideal deformation; (c) after inhomogeneous deformation. Note that grid pattern (c) is basically the same as grid pattern (b), but with additional shearing, especially at the outer layers. Thus, grid pattern (c) requires higher work of deformation than grid pattern (b).

$$Work = (u) \cdot (volume)$$

$$u_{total} = u_{ideal} + u_{friction} + u_{redundant}$$

$$\eta = \frac{u_{ideal}}{u_{total}}$$

Efficiency:

30-60% extrusion

75-95% rolling

$$\Delta T = \frac{u_{total}}{\rho \cdot c}$$