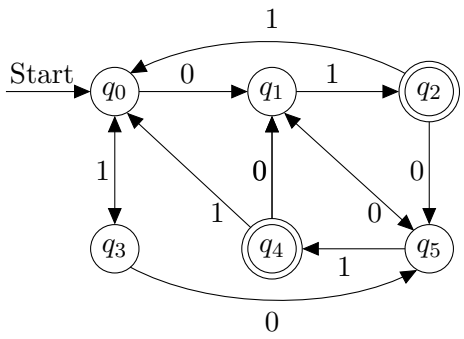
Master Degree in Computer Engineering

Final Exam for Automata, Languages and Computation

September 10th, 2026

1. **[5 points]** Consider the DFA A whose transition function is graphically represented below (arcs with double direction represent two arcs in opposite directions)



   (a) Provide the definition of equivalent pair of states for a DFA.

   (b) Apply to A the tabular algorithm presented in the textbook for detecting pairs of equivalent states, reporting all the **intermediate steps**.

   (c) Specify the minimal DFA equivalent to A and provide a graphical representation of its transition function.

2. **[7 points]** Let $\Sigma = \{a, b\}$. For $w \in \Sigma^*$ and $X \in \Sigma$, we write $\#_X(w)$ to denote the number of occurrences of X in w. Consider the following languages

$$\begin{aligned} L_1 &= \{u \mid u \in \Sigma^*,\ \#_a(u) = \#_b(u)\}\ ; \\ L_2 &= \{uv \mid u, v \in \Sigma^*,\ \#_a(u) = \#_a(v)\}\ . \end{aligned}$$

   For each of the above languages, state whether it belongs to REG, to CFL$\setminus$REG, or else whether it is outside of CFL. Provide a mathematical proof for all of your answers.

3. **[5 points]** With reference to push-down automata (PDA), answer the following questions.

   (a) Provide the definition of language accepted by final state and language accepted by empty stack.

   (b) Prove that if P_F is a PDA accepting by final state, then there exists a PDA P_N accepting by empty stack such that $L(P_F) = N(P_N)$.

(please turn to the next page)

4. **[7 points]** Assess whether the following statements are **true** or **false**, providing clear explanations for all your answers.

   (a) There exists a language L in CFL and a finite language L' such that $L \setminus L'$ is not in CFL.

   (b) There exists a language L in CFL and an infinite language L' such that $L \setminus L'$ is not in CFL.

   (c) Let $\mathcal{P}$ be the class of languages that can be recognized in polynomial time by a TM. There exist languages L and L' both in $\mathcal{P}$ such that $L \cup L'$ is not in $\mathcal{P}$.

   (d) Let $\mathcal{NP}$ be the class of languages that can be recognized in polynomial time by a nondeterministic TM. There exist languages L and L' both in $\mathcal{NP}$ such that $L \cup L'$ is not in $\mathcal{NP}$.

5. **[6 points]** In relation to the theory of undecidability, answer the following questions. All the TMs introduced below are defined over the input alphabet $\Sigma = \{0, 1\}$.

   Consider the following property of the RE languages

   $$\mathcal{P} \quad = \quad \{L \in \text{RE} \mid \text{ there exists } u, v \in L \text{ such that } u \neq v,\ |u| = |v|\}$$

   and define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Use Rice's theorem to prove that $L_{\mathcal{P}}$ is not in REC.

   (b) Prove that $\overline{L_{\mathcal{P}}} = L_{\overline{\mathcal{P}}}$ is not in REC, **without** using Rice's theorem.

   (c) Asses whether the language $L_{\mathcal{P}}$ belongs to RE$\setminus$REC or else is not in RE.

6. **[3 points]** In relation to the theory of intractability, answer the following questions.

   (a) Define the notions of Boolean expression and satisfiability, and introduce the SAT decision problem.

   (b) Prove that SAT is in $\mathcal{NP}$, the class of languages that can be recognized in polynomial time by a nondeterministic TM.