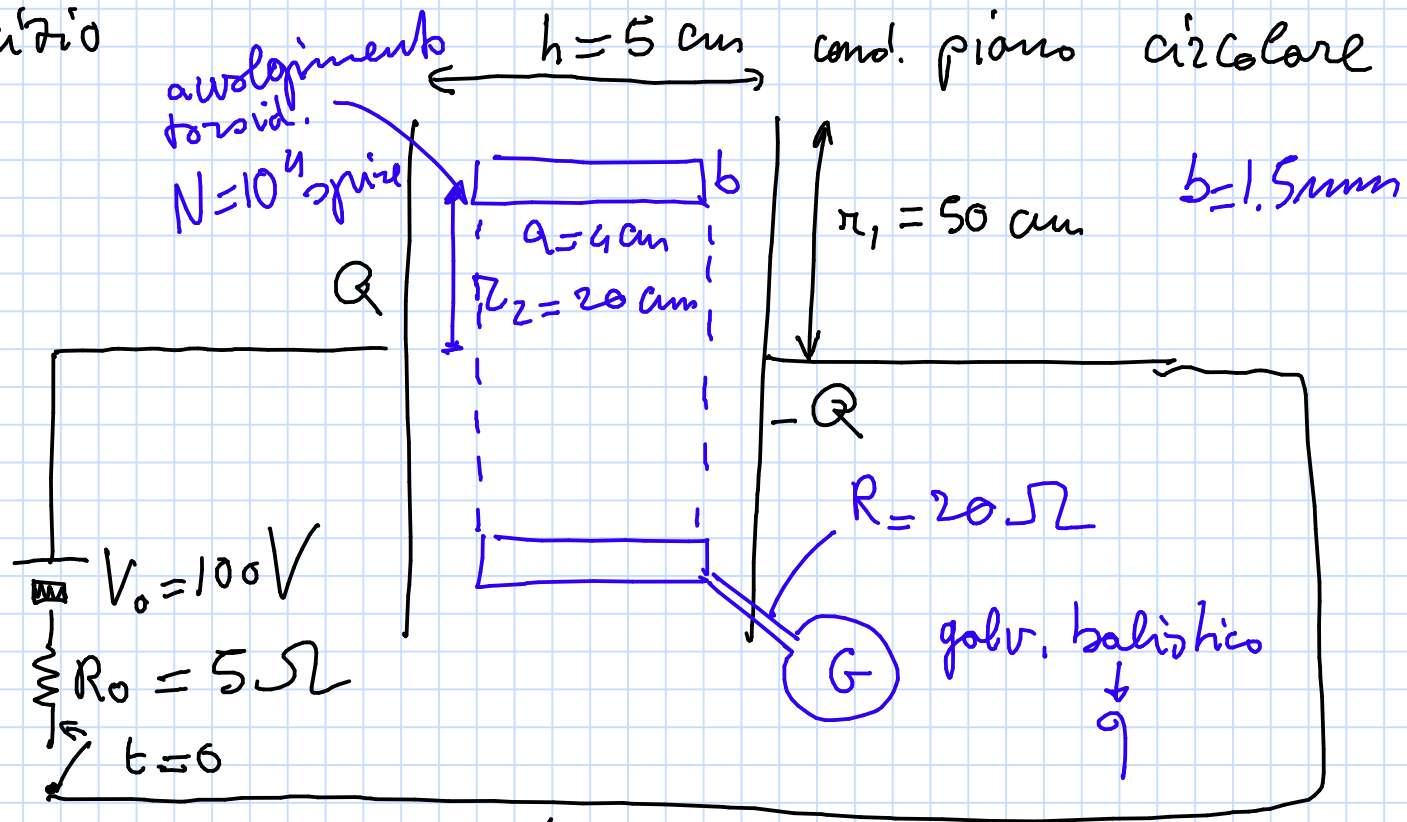
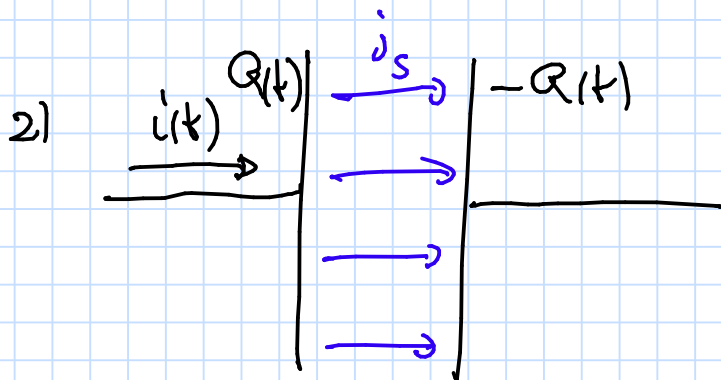


esercizio



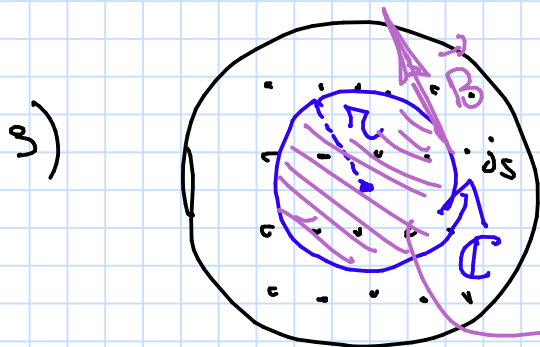
$q \neq Q = ?$

- 1) flusso varia nelle spire $\Rightarrow$ $\vec{E}$ indotti ... q
 $\vec{B}$ variabile da $\vec{E}$ var.



$i(t) \Rightarrow j_s = \epsilon_0 \frac{\partial E}{\partial t}$
 omogenea

↓
 simmetria cilindrica



$\oint_C \vec{B} \cdot d\vec{s} = 2\pi r B(r) = \mu_0 i_c$
 Ampere
 $= j_s \Sigma(r) = \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right) \pi r^2$

$\Rightarrow B(r) = \mu_0 \epsilon_0 \frac{r}{2} \frac{\partial E}{\partial t}$

4) $b \ll \pi_2$, assumiamo $B(z_2)$ costante sulla sezione dell'avvolgimento

$$\begin{aligned} 5) \quad \phi_{sol} &= N a b B(z_2) \\ &= N a b \frac{\pi_2}{2} \epsilon_0 \mu_0 \frac{\partial B}{\partial t} \quad \left(\epsilon_0 \mu_0 = \frac{1}{c^2} \right) \end{aligned}$$

$$6) \quad E = \frac{V}{h}$$

$$R_0 C: \quad V(t) = V_0 [1 - e^{-t/\tau}], \quad \tau = R_0 C$$

$$7) \quad t=0 \quad \frac{\partial B}{\partial t} = \frac{1}{h} V_0 \frac{1}{\tau} e^{-t/\tau} \Big|_{t=0} = \frac{V_0}{R_0 C h}$$

$$t \rightarrow \infty \quad \frac{\partial B}{\partial t} = 0$$

$$8) \quad \text{Felicì} \quad q = \frac{\phi_{sol}^{INI} - \cancel{\phi_{sol}^{FIN}}}{R} = \frac{1}{R} N a b \frac{\pi_2}{2} \epsilon_0 \mu_0 \frac{V_0}{R_0 C h}$$

$$9) \quad Q = C V_0 \quad \Rightarrow \quad \frac{q}{Q} = \frac{N a b \pi_2 \epsilon_0 \mu_0}{2 R R_0 C^2 h}$$

$$\text{vuoto} \quad C = \frac{\epsilon_0 \Sigma}{h} = 1.39 \cdot 10^{-10} \text{ F}$$

$$\frac{q}{Q} = 11.5$$

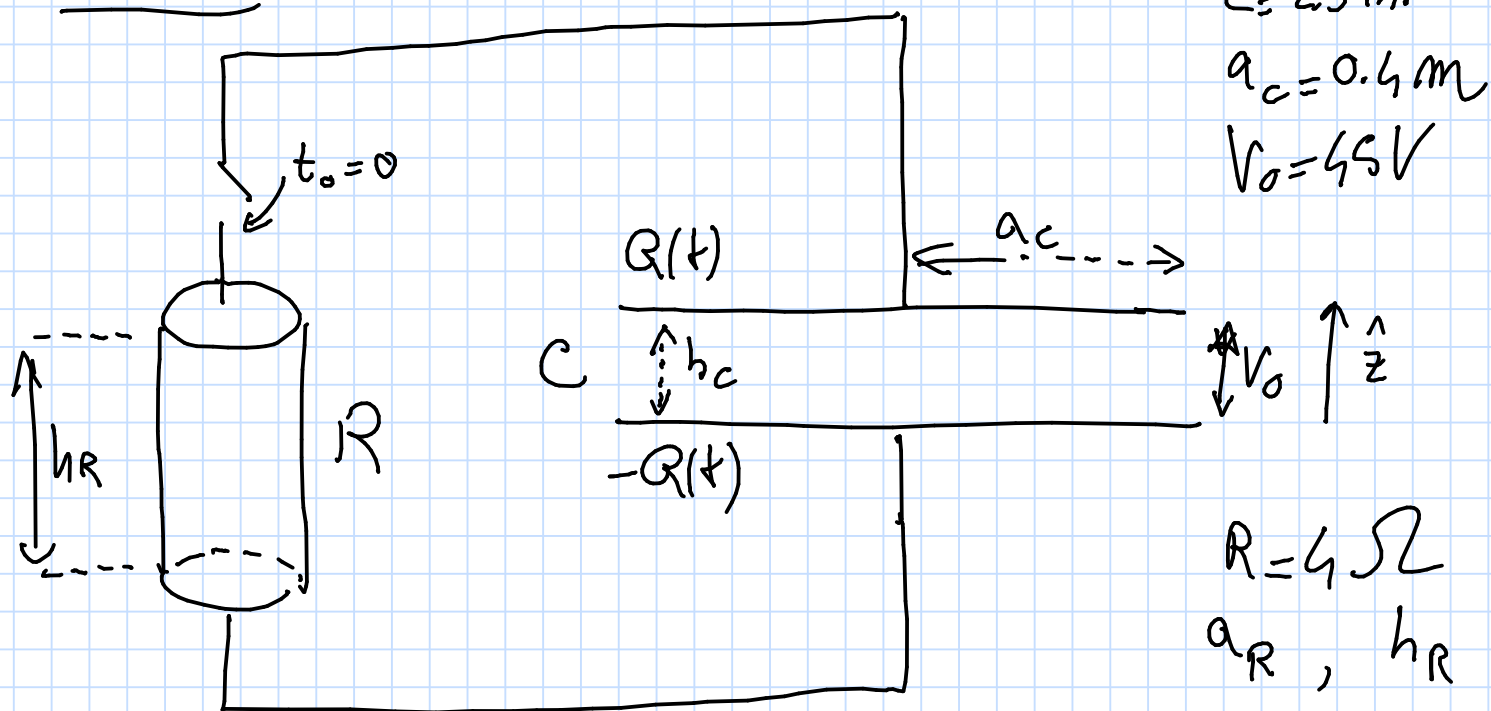
dielettrico $K_e = 5$ nel C

$$\epsilon_0 \rightarrow \epsilon = K_e \epsilon_0$$

$$C \rightarrow K_e C$$

$$\left| \frac{q'}{Q'} = \frac{K_e q}{K_e^2 Q} = \frac{1}{K_e} \frac{q}{Q} = 2.3 \right.$$

esercizio



1) $\vec{E}_c(t)$ $V(t) = V_0 e^{-t/\tau}$ $\tau = RC$

$\vec{E}_c(t) = -\frac{V_0}{h_c} e^{-t/\tau} \hat{z}$ $h_c = \epsilon_0 \frac{\pi a_c^2}{C} = 1.78 \text{ mm}$

$t = 2\tau$ $|\vec{E}(t)| = \frac{V}{h_c} e^{-2} = 3.42 \text{ V/mm}$

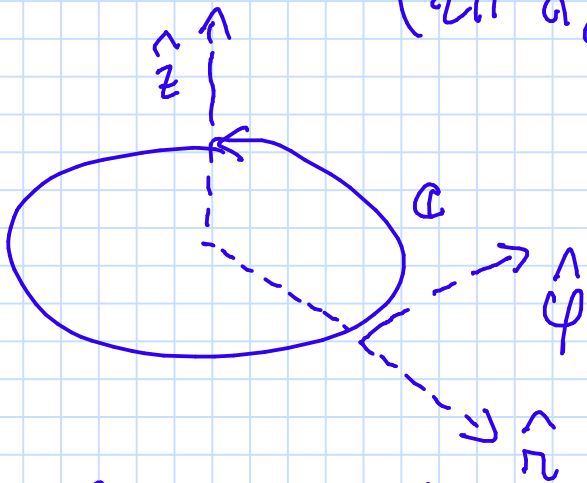
2) $B_c(t)$

$\oint_C \vec{B} \cdot d\vec{s} = 2\pi r B_c(z) = \epsilon_0 \mu_0 \frac{\partial B}{\partial t} \pi r^2 C$

$= \epsilon_0 \mu_0 \pi r^2 \frac{1}{RC} \frac{V_0}{h} e^{-t/\tau}$

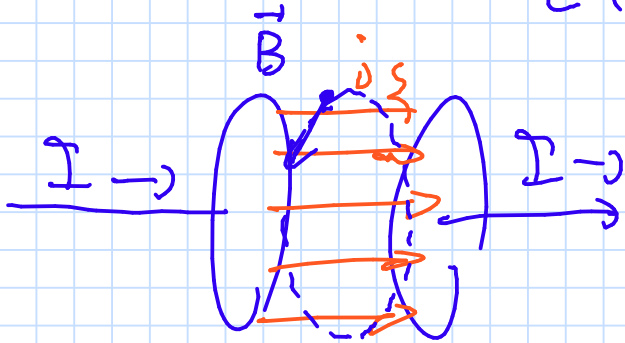
> 0 con questa scelta

$$B_c(t, r) = \left(\frac{\mu_0 \epsilon_0 V_0 e^{-t/\tau_c}}{2\pi a_c^2 R} r \right) \hat{\varphi}$$



$$B_c(t=2\tau, r=a_c) = 7.61 \cdot 10^{-7} \text{ T} \quad \text{molto piccolo}$$

alternativa $B_c(t=2\tau, a_c) = \frac{\mu_0 I}{2\pi a_c}$

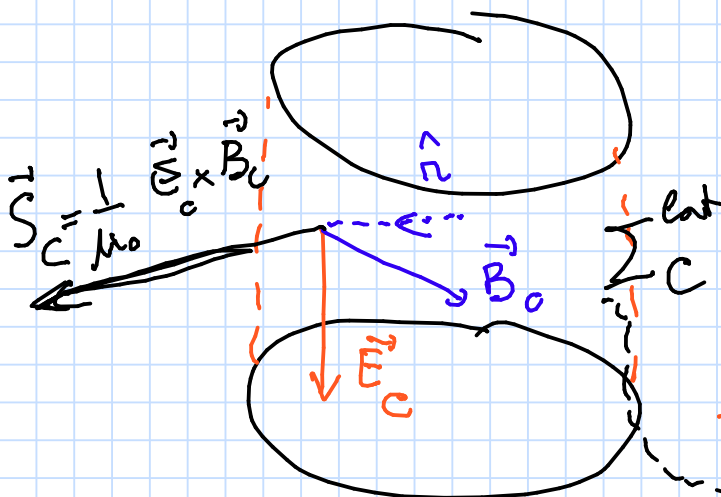


$$\pi a_c^2 \cdot j_s = I$$

c) Poynting su superficie laterale del cond.

$$\vec{S}_c(t, r=a_c)$$

$P_c(t)$ potenza e.m. uscente



$$\vec{S}_c = \frac{1}{\mu_0} \left(-\frac{V_0}{h_c} \right) e^{-t/\tau_c} \hat{z} \times$$

$$\times \frac{V_0}{2\pi a_c R} e^{-t/\tau_c} \hat{\varphi}$$

$$= \frac{V(t)^2}{(2\pi a_c h_c) R}$$

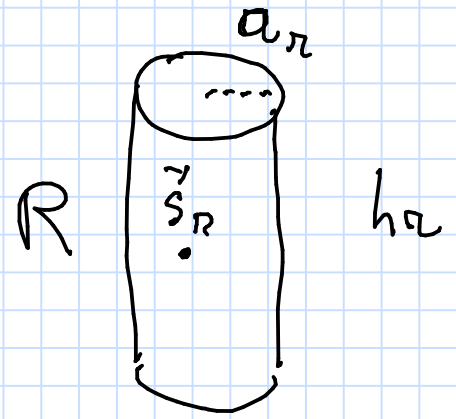
$$P_c(t) = \int_{\sum_c^{et}} \vec{S}_c \cdot \hat{n} d\Sigma = \sum_c^{et} S_c$$

$$= \frac{V(t)^2}{R}$$

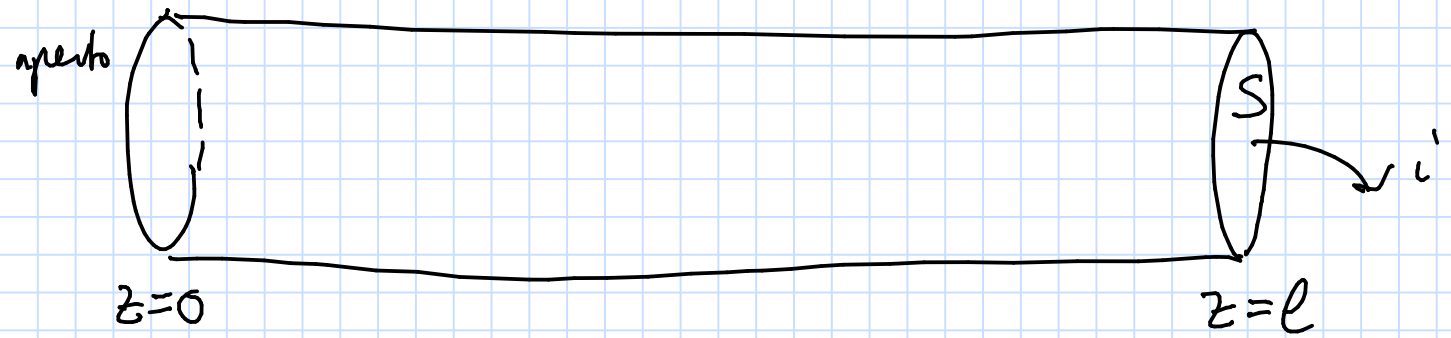
(d) $\vec{S}_R(t, r = a_R)$

$P_R(t)$ entzogene

....



filo aperto che si scarica dalla S a destra



densità di carica ρ uniforme

$$\rho(t) = \rho_0 e^{-t/\tau}$$

— la corrente deve aumentare con z per trasferire carica da $z=0$ a $z=l$ con ρ costante (acqua...)

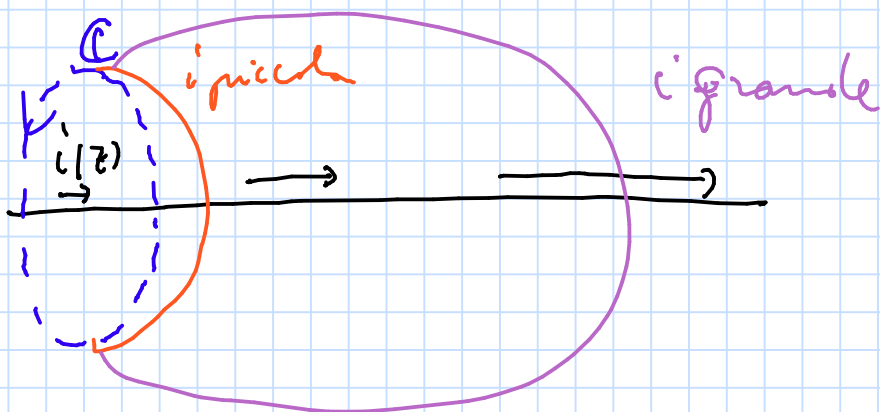
— dens. corrente $\vec{j}$: eq. continuità

$$\vec{j} \Rightarrow j_z$$

$$\frac{\partial j_z}{\partial z} = -\dot{\rho}$$

$$j_z = -z \dot{\rho} \quad (\text{se } \dot{\rho} < 0)$$

corrente $i(t, z) = S j_z = -z S \dot{\rho}$



$$i_S = S j_S = S E_0 \frac{\partial E}{\partial t}$$

sistema da corrente di spostamento

1st Maxwell

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

contribution σ

$$\sigma E_z = j_z$$

$$E_z = -\frac{z}{\epsilon_0} \int \Rightarrow E_z = 0 \text{ for } z=0$$

$$E_z = \frac{\rho}{\epsilon_0} z$$

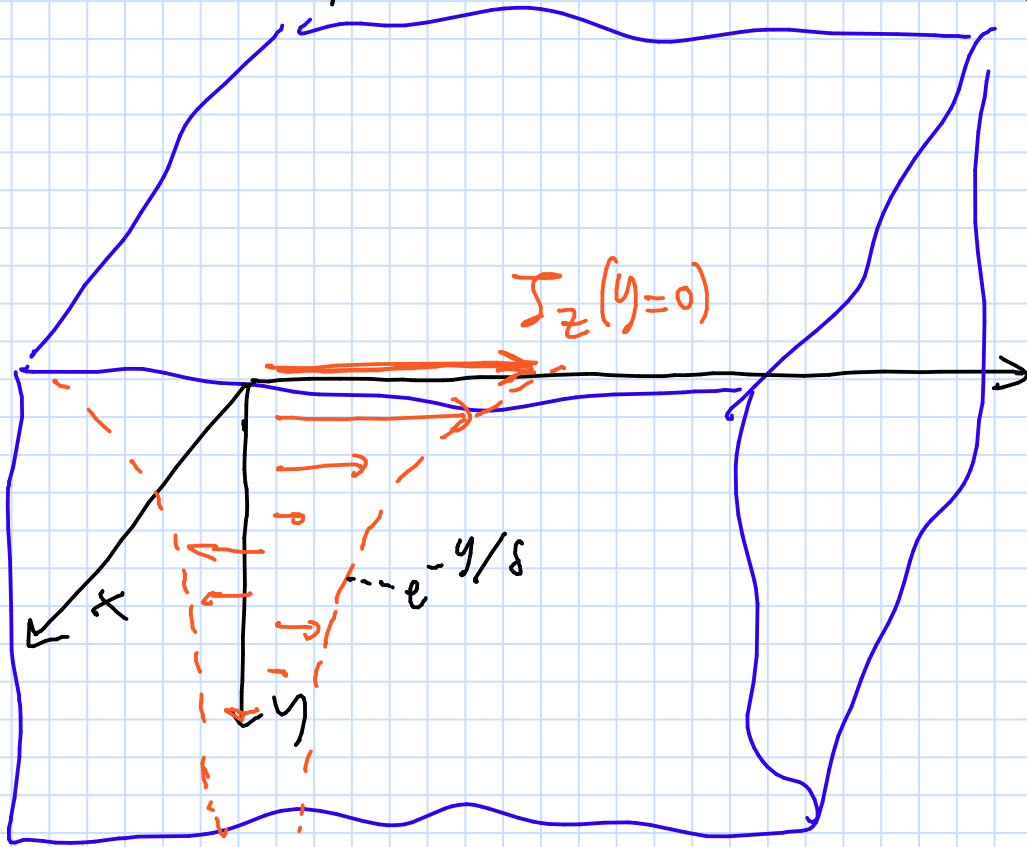
$$i_{s,z} = \int \epsilon_0 \frac{\partial E_z}{\partial t} = \int \cancel{\epsilon_0} z \cancel{\frac{\partial}{\partial t}} = -i_z \text{ di contribution}$$

$$i_{\text{tot}} = i_z + i_{s,z} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{tot}} = 0 \Rightarrow \vec{B} = 0$$

Effetto pelle

- molte ricadute pratiche
- alte frequenze $\Rightarrow$ i sulla superficie dei fili



- intensità della corrente $J_z(y)$ decresce con y
- fase cambia con y

- J_z alternata, pulsata ω
- materiale $\left\{ \begin{array}{l} \text{conduttività } \sigma \\ \text{perm. m. } \mu \end{array} \right.$

- $\vec{J} \parallel \vec{z}$ J_z

- $\vec{B} e^{i\omega t}$ $\frac{\partial \vec{B} e^{i\omega t}}{\partial t} = i\omega \vec{B} e^{i\omega t}$

- Faraday $\vec{\nabla} \times \vec{E} = -i\omega \vec{B}$ (1)

- Ampere Maxwell $\vec{\nabla} \times \vec{B} = \mu \vec{J}$ (2)

Biot-Savart $\Rightarrow \vec{B} \parallel \hat{x} \quad B_x$
 $\vec{J} \parallel \hat{z}$

$\Rightarrow \vec{B} = \frac{\mu_0}{2} \vec{J} \parallel \hat{z}$

① $\vec{\nabla} \times \vec{J} = -i\omega\sigma\vec{B}$

$$\begin{cases} \frac{dJ_z}{dy} = -i\omega\sigma B_x \\ -\frac{dB_x}{dy} = \mu J_z \end{cases}$$

comp. dei
rotori non
nulle

$\left(\frac{d}{dy} = \frac{\partial}{\partial y} \right)$

elim. B_x :

$$\left(\frac{d^2 J_z}{dy^2} = +i\omega\sigma\mu J_z \right)$$

soluzione $J_z(y) = A_1 e^{Ky} + A_2 e^{-Ky}$

$K = \sqrt{i\omega\sigma\mu} = \sqrt{i} \sqrt{\omega\sigma\mu} = \frac{1+i}{\sqrt{2}} \sqrt{\omega\sigma\mu}$



$A_1 = 0$ Non può divergere

$$\underline{K} = (1+i) \sqrt{\frac{\omega \sigma \mu}{2}} = \frac{1+i}{\delta}$$

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

lunghezza (proporzionalità)
di penetrazione

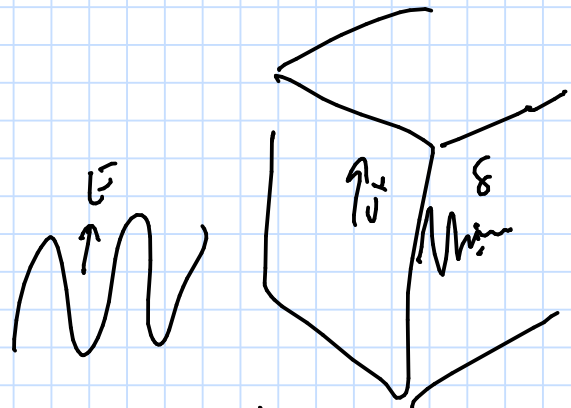
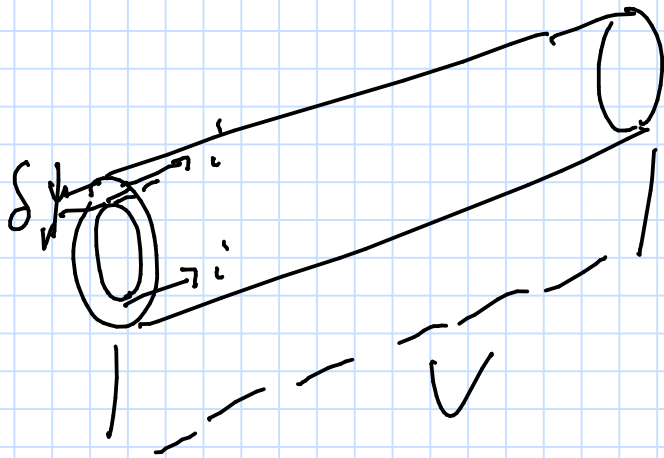
$$A_z = J_z(0)$$

sol.

$$J_z(y) = J_z(0) \underbrace{e^{-y/\delta}}_{\text{amplitude}} \underbrace{e^{-iy/\delta}}_{*}$$

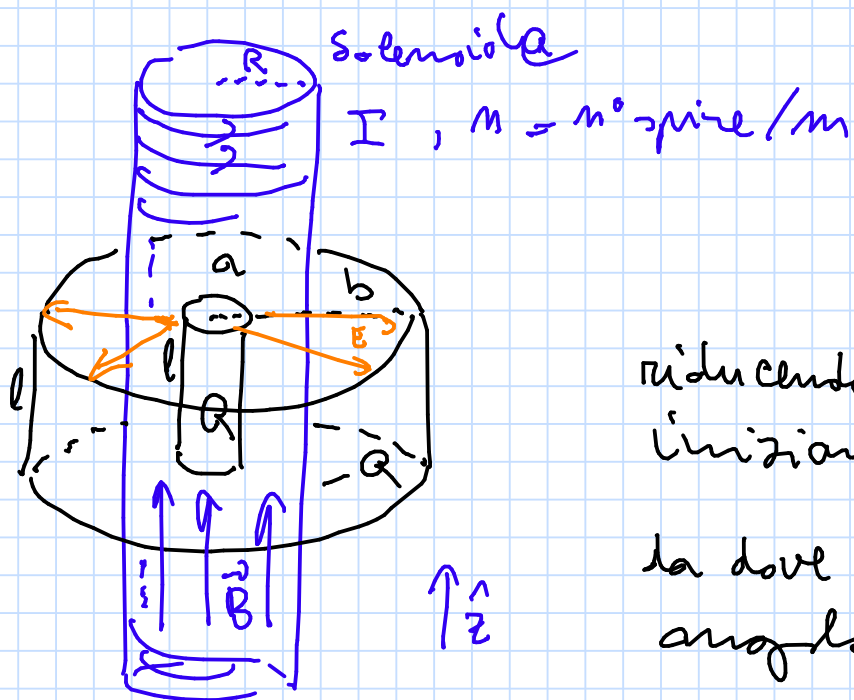
amplitude
decade esp.
con penetrazione y

*
forse: a $y = \delta \pi$ fase si è invertita



penetrazione di
onde e.m. dentro
metalli fino a δ

Momento angolare del campo e.m.



$$l \gg b$$

riducendo $I \rightarrow 0$ i cilindri iniziano a ruotare
da dove deriva il mom. angolare

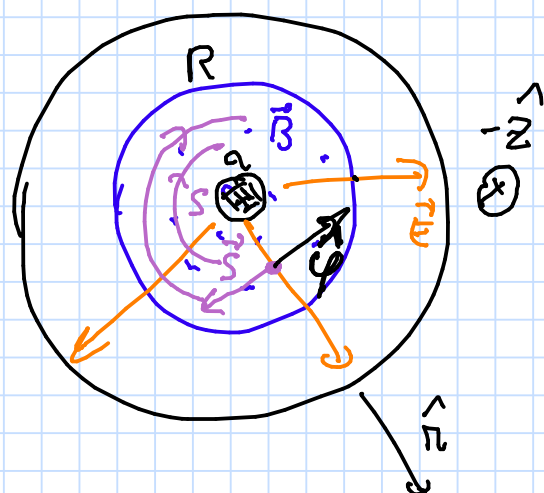
inizialmente ($I \neq 0$)

$$\vec{E} = \frac{Q}{2\pi\epsilon_0 l \pi} \hat{r} \quad a < r < b$$

$$\vec{B} = \mu_0 n I \hat{z} \quad r < R$$

densità di q. di moto del campo e.m.

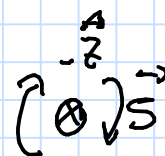
$$\begin{aligned} \vec{P}_{em} &= \epsilon_0 \mu_0 \vec{S} = \frac{1}{c^2} \vec{S} \\ &= \epsilon_0 \mu_0 \frac{1}{\cancel{\mu_0}} \vec{B} \times \vec{B} \\ &= \frac{\mu_0 n I R}{2\pi \epsilon l \pi} (-\hat{\varphi}) \quad a < r < R \end{aligned}$$



$\vec{S}$ è solenoidale $\Rightarrow$ non esce energia

densità di momento angolare

$$\vec{l}_{em} = \vec{r} \times \vec{P}_{em} = - \frac{\mu_0 n I R}{2\pi \epsilon \pi} \hat{z}$$



$\vec{l}_{em}$ è costante $\Rightarrow$ mom angolare totale
 $\vec{L}_{em} = \vec{l}_{em} \cdot \text{volume}$
 $= \vec{l}_{em} \cdot \pi(R^2 - a^2) l$

$$\vec{L}_{em} = -\frac{1}{2} \mu_0 m I Q (R^2 - a^2) \hat{z}$$

quando c'è I

riducendo $I \Rightarrow$ trasferimento di $\vec{L}$ ai
cilindri coassiali

$\Rightarrow \vec{B}$ variabile $\Rightarrow$ Paramag: $\vec{E}_i$:

$$\vec{E}_i = \begin{cases} -\frac{1}{2} \mu_0 m \frac{dI}{dt} \frac{R^2}{r} \hat{\varphi} & \text{se } r > R \\ -\frac{1}{2} \mu_0 m \frac{dI}{dt} r \hat{\varphi} & \text{se } r < R \end{cases}$$



mom. delle forze sul cilindro esterno

$$\vec{M}_b = \left. \vec{r} \times (-Q \vec{E}_i) \right|_{r=R} = \frac{1}{2} \mu_0 m Q R^2 \frac{dI}{dt} \hat{z}$$

mom angolare cil. esterno

$$\vec{L}_b = \int \vec{M}_b = \int_I \frac{dI}{dt} dt = -\frac{1}{2} \mu_0 m I Q R^2 \hat{z}$$

sul cil interno

$$\dots \vec{M}_a = -\frac{1}{2} \mu_m Q a^2 \frac{dI}{dt} \hat{z}$$

$$\Rightarrow \vec{L}_a = \frac{1}{2} \mu_m I Q a^2 \hat{z}$$

$$\triangle \vec{L}_{em} = \vec{L}_a + \vec{L}_b$$

e.m. all'inizio

meccanica alla fine

onde e.m.

eq. Maxwell nel vuoto ($\rho=0$, $j=0$)

$$\vec{\nabla} \cdot \vec{E} = 0$$

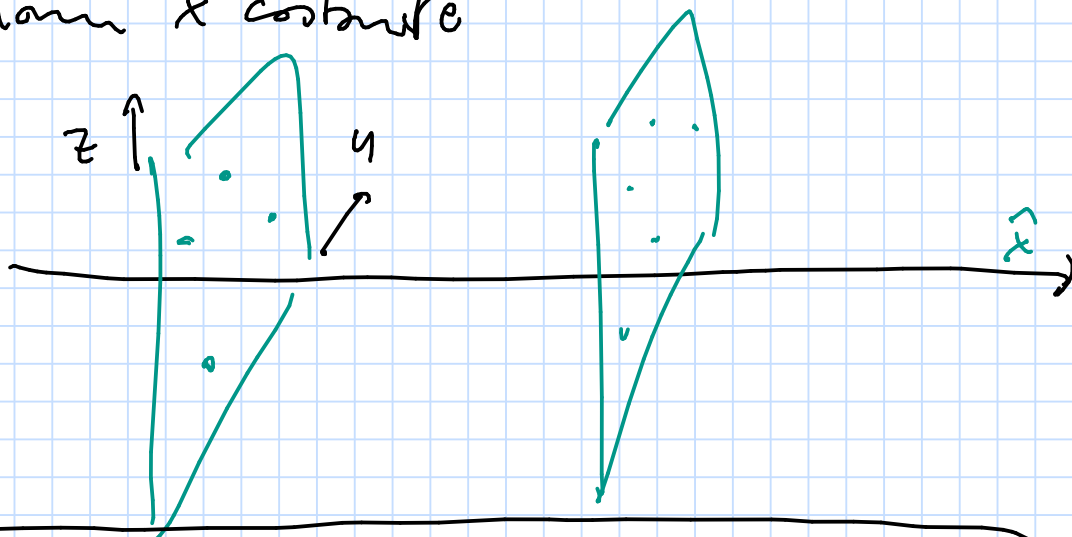
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

simmetria tra eq. per $\vec{E}$ e $\vec{B}$

— assumiamo propagazione $\parallel \hat{x}$, $\vec{E}$, $\vec{B}$ costanti
in piani x costanti



$$\frac{\partial \vec{E}}{\partial y} = 0 = \frac{\partial \vec{E}}{\partial z} \quad \frac{\partial \vec{B}}{\partial y} = 0 = \frac{\partial \vec{B}}{\partial z} \quad \otimes$$

Gauss $\frac{\partial E_x}{\partial x} + \cancel{\frac{\partial E_y}{\partial y}} + \cancel{\frac{\partial E_z}{\partial z}} = 0$

$\Rightarrow E_x$ è costante in x

$\Rightarrow E_y$ e E_z portano le onde e.m.
 $\Rightarrow$ onde e.m. sono trasversali

stesse conclusioni per $\vec{B}_x$

— —

rotori

comp. x

$$\cancel{\frac{\partial E_z}{\partial y}} - \cancel{\frac{\partial E_y}{\partial z}} = -\frac{\partial B_x}{\partial t} = 0$$

B_x è costante anche nel tempo

stesso per B_x

— —

rotori

comp. y

comp. z

Paradossi

$\boxed{y_1}$

$$\cancel{\frac{\partial E_x}{\partial z}} - \cancel{\frac{\partial E_z}{\partial x}} = -\frac{\partial B_y}{\partial t}$$

$\boxed{z_1}$

$$\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}} = -\frac{\partial B_z}{\partial t}$$

AM.

$\boxed{y_2}$

$$\cancel{\frac{\partial B_x}{\partial z}} - \frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

$\boxed{z_2}$

$$\frac{\partial B_y}{\partial x} - \cancel{\frac{\partial B_x}{\partial y}} = \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t}$$

$\boxed{y_1}$ e $\boxed{z_2}$

solo B_z e B_y

$\boxed{y_2}$ e $\boxed{z_1}$

solo B_y e B_z

$$\frac{\partial}{\partial t} \boxed{y_1} \Rightarrow \boxed{\frac{\partial}{\partial t} \frac{\partial}{\partial x} E_z} = \frac{\partial^2}{\partial t^2} B_y$$

$$\frac{\partial}{\partial x} \boxed{z_2} \Rightarrow \frac{\partial^2}{\partial x^2} B_y = \mu_0 \epsilon_0 \boxed{\frac{\partial}{\partial x} \frac{\partial}{\partial t} E_z}$$

$$\frac{\partial^2}{\partial x^2} B_y = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} B_y$$

eq. di D'Alembert
per B_y

velocità v , $\frac{1}{v^2} = \epsilon_0 \mu_0$ | vuoto $v = c \Rightarrow c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

stessa c per tutte le onde e.m.

stessa dim. $B_z, E_y, B_z \Rightarrow$ campi che viaggiano a vel. $v = c \approx 3 \cdot 10^8 \frac{m}{s}$ nel vuoto

relazione tra $\vec{E}$ e $\vec{B}$

$$\boxed{y_1} \quad \frac{\partial}{\partial x} E_z = \frac{\partial}{\partial t} B_y$$

$$\text{se } E_z = \zeta(x - ct)$$

$$\rightarrow \frac{\partial}{\partial x} E_z = \frac{\partial}{\partial x} \zeta = \zeta'$$

$$= \boxed{-\frac{1}{c}} \frac{\partial}{\partial t} \zeta$$

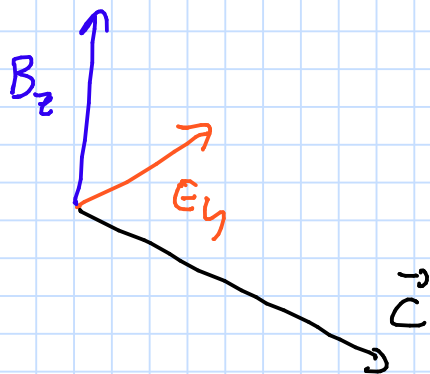
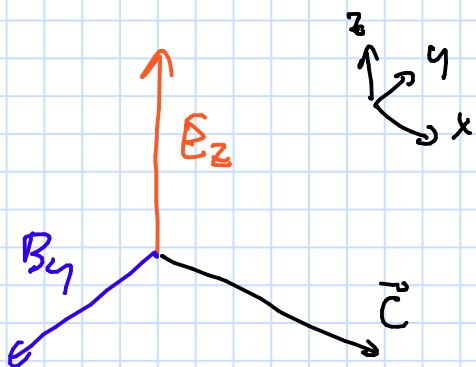
$$\boxed{B_y = -\frac{1}{c} E_z}$$

stessa consid. per $\boxed{z_1}$

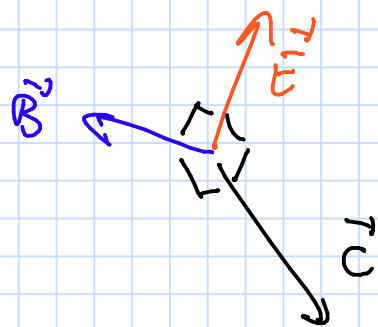
$$\frac{\partial}{\partial x} B_y = -\frac{\partial}{\partial t} B_z$$

...

$$\Rightarrow \boxed{B_z = \frac{1}{c} E_y}$$



in generale $\vec{E} \perp \vec{B}$, fasi t. c.
 $\vec{E} \times \vec{B} \parallel \vec{v}, \vec{c}$



- onde che viaggiano a $v = c$
- $E = Bc$ $B = E/c$
- $\vec{E} \perp \vec{B}$ $\vec{E}, \vec{B} \perp \vec{c}$
- $\vec{E} \times \vec{B} \parallel \vec{v}$

materiali

$$\epsilon_0 \rightarrow \epsilon$$

senza

$$\mu_0 \rightarrow \mu$$

coniche

$$c \rightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

libere

energia delle onde e.m. (armoniche)

in generale le onde trasportano energia
densità di energia e.m.

$$u_{em} = u_e + u_m$$
$$= \frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2\mu} \vec{B}^2$$

sappiamo $B = \frac{E}{c}$

$$B^2 = \frac{E^2}{c^2} = \epsilon \mu E^2$$

$$u_{em} = 2u_e = \epsilon \vec{E}^2 = 2u_m = \frac{B^2}{\mu}$$

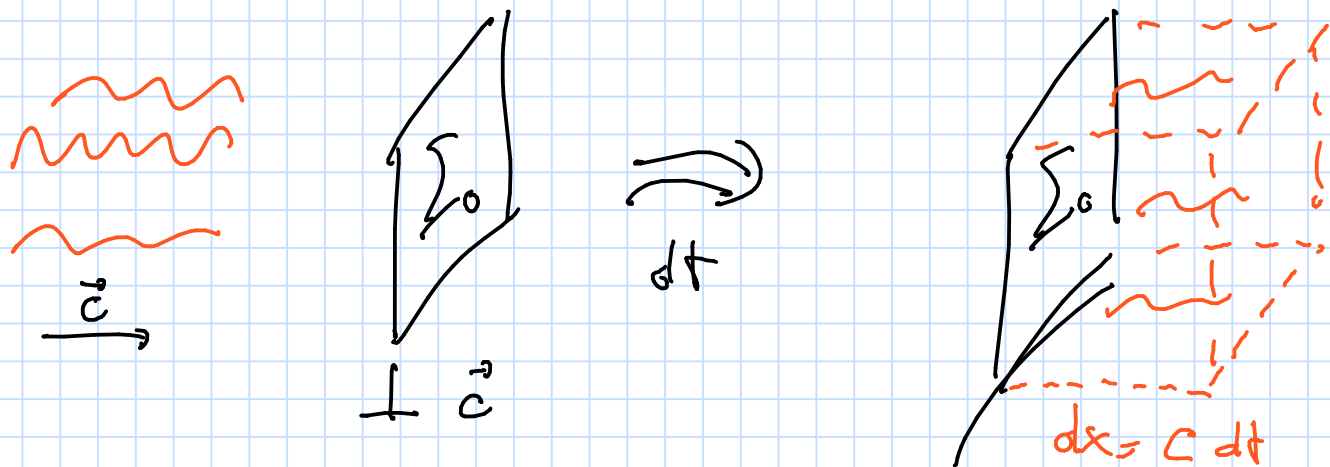
in un punto a un tempo dato

$$\vec{E} = E_0 \sin(kx - \omega t)$$

$$\langle u_{em} \rangle = \frac{1}{2} \epsilon E_0^2$$

($\frac{1}{2}$ qui da media
temporale)

Potenza delle onde e.m.



energia ^{media} $\langle u \rangle$ che attraversa Σ_0 nel tempo dt

$$dU = \langle u_{em} \rangle \cdot dx \cdot \Sigma_0$$

$$= \left(\frac{1}{2} \epsilon_0 E_0^2 \right) \Sigma_0 c \cdot dt$$

potenza $P = \frac{dU}{dt} = \underbrace{\frac{1}{2} \epsilon_0 E_0^2 c}_{\langle S \rangle} \underbrace{\Sigma_0}_{A}$

intensità

$$I = \frac{P}{\Sigma_0} = \left(\frac{1}{2} \epsilon_0 E_0^2 \right) c = u_{em} \cdot c$$

...

vettore di Poynting $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

$$|\vec{S}| = S = \frac{1}{\mu_0} E B = \frac{1}{\mu_0} E \frac{E}{c} = \epsilon_0 E^2 c$$

$$\langle S \rangle = \frac{1}{2} \epsilon_0 E_0^2 c = I$$

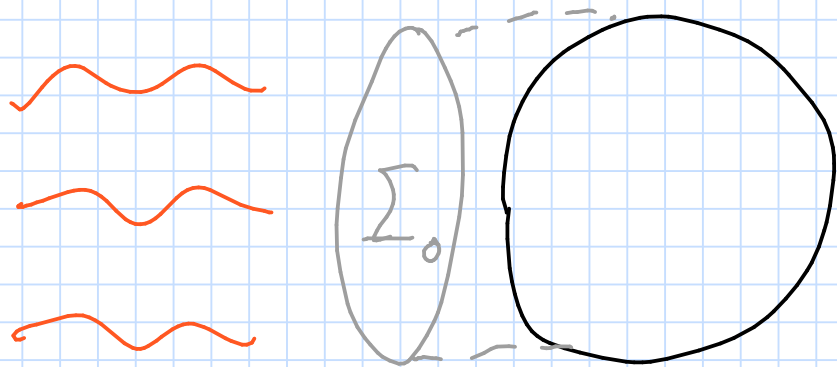
intensità I è già mediata nel tempo per def.

energia solare

intensità nello spaz. vicina alla Terra
di ene. solare

$$I \approx 1300 \frac{W}{m^2}$$

potenza assorbita dalla Terra



$$P_{\text{terra}} = I \cdot \Sigma_0 = I \cdot \pi R_T^2 \approx \left(10^{14} m^2 \right) 1300 \frac{W}{m^2}$$

$\downarrow$
 $6.35 \cdot 10^6 m$

$$P_{\text{terra}} \approx 1.3 \cdot 10^{17} W$$

25% disperso nell'atmosfera

$$\tilde{P}_{\text{terra}} \approx 10^{17} W$$

$$\tilde{I} \approx 1000 \frac{W}{m^2}$$

Quanti pannelli solari servono per far
funzionare un microonde da 800 W.

efficienza, rendimento $\eta = 15\%$

$$\Sigma_p = 1.5 m^2, \quad \frac{m}{45^\circ} = 2$$

$$\Sigma_{p0} = \cos 45^\circ \cdot \Sigma_p$$

$$P = 800 \text{ W} = n \Sigma_{p0} \cdot \eta \cdot \tilde{I}$$

$$n = \frac{P}{\Sigma_{p0} \cdot \eta \cdot \tilde{I}} = \frac{800 \text{ W}}{\frac{\sqrt{2}}{2} \cdot 1.5 \text{ m}^2 \cdot 0.15 \cdot 1000 \frac{\text{W}}{\text{m}^2}}$$

$$= 5,03$$