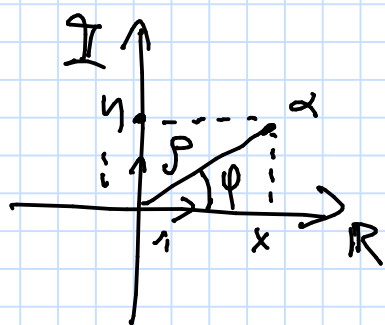
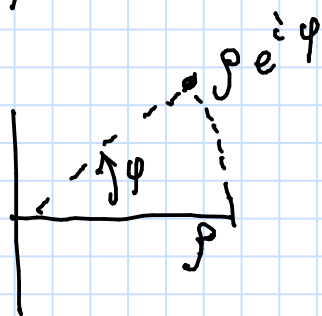


Numeri complessi

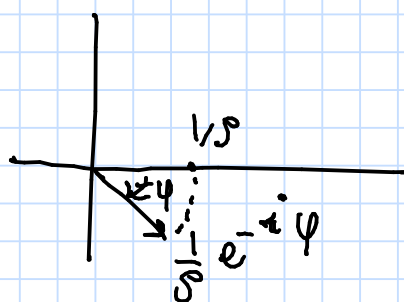


$$\alpha = x + iy$$

$$\alpha = \rho e^{i\varphi}$$



$$\alpha^{-1} = \frac{1}{\alpha} = \frac{1}{\rho} e^{-i\varphi}$$



$$\frac{1}{\alpha} = \frac{1}{x+iy} \cdot \frac{x-iy}{x-iy} = \frac{x-iy}{x^2 - \underbrace{(i^2)}_{-1} y^2} = \frac{x-iy}{x^2 + y^2}$$

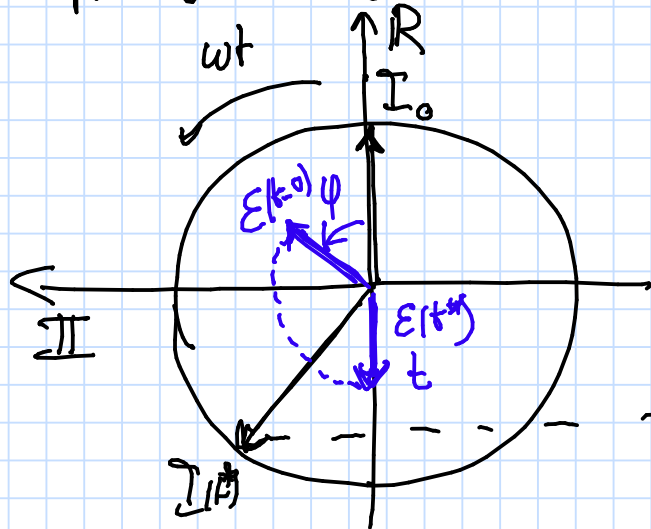
$$\frac{1}{iy} = \frac{-i}{y}$$

Circuiti in corrente alternata

- R, L, C sono lineari (non variano la ω in ingresso) ma introducono sfasamento (φ)
- trattati anche con "metodo simbolico"

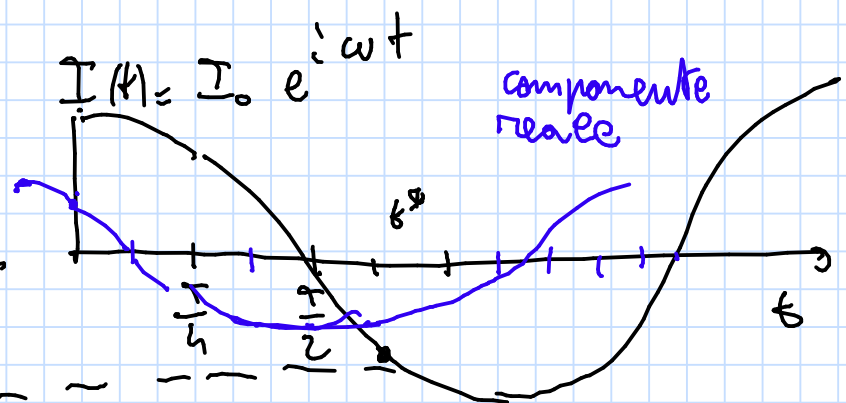
$$\begin{aligned} \mathcal{E} = \mathcal{E}_0 \cos(\omega t + \varphi) &\xrightarrow{\Delta} \mathcal{E} = \mathcal{E}_0 e^{i(\omega t + \varphi)} \\ &= \mathcal{E}_0 \cos(\omega t + \varphi) + \underbrace{i \mathcal{E}_0 \sin(\omega t + \varphi)}_{\text{...}} \end{aligned}$$

misure reali:
parte reale

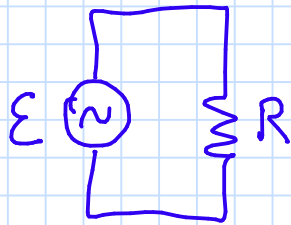


↓ studia sistema

$\leftarrow R$



R



$$\varepsilon = \varepsilon_0 \cos(\omega t)$$

sotto 1 MHz, unico effetto
riscaldamento da dissipazione

$$I(t) = \frac{\varepsilon(t)}{R}$$

$$\Rightarrow I = I_0 \cos(\omega t)$$

$$P_R(t) = R I(t)^2 = R I_0^2 \cos^2(\omega t)$$

$$\overline{P}_R = \frac{1}{2} R I_0^2 = \frac{1}{2} \frac{\varepsilon_0^2}{R}$$

... -

$$\text{def } I_{\text{eff}} = \frac{1}{\sqrt{2}} I_0$$

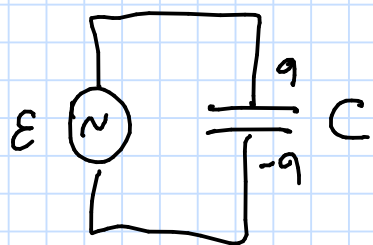
$$\varepsilon_{\text{eff}} = \frac{1}{\sqrt{2}} \varepsilon_0$$

$\Rightarrow$

$$P_D = R I_{\text{eff}}^2 = \frac{\varepsilon_{\text{eff}}^2}{R}$$

facilmente confrontabili
con regime in I continua

C



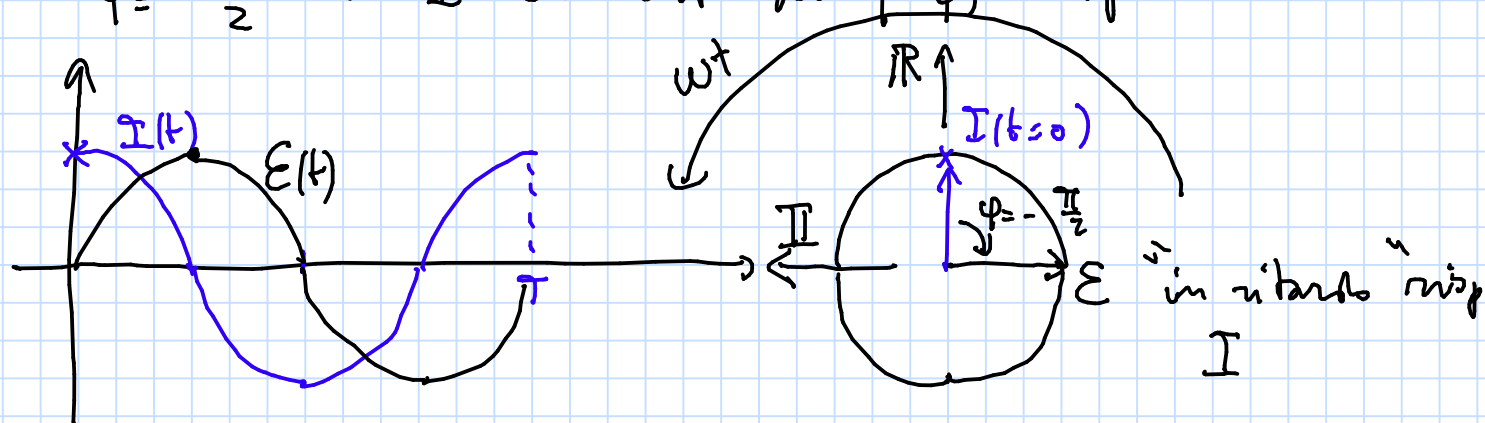
$$\mathcal{E} = \frac{q}{C} \quad \forall t!$$

$$\mathcal{E} = \mathcal{E}_0 \cos(\omega t + \varphi)$$

$$\begin{aligned} I &= \frac{dq}{dt} = C \mathcal{E}_0 \frac{d}{dt} (\cos) \\ &= \omega C \mathcal{E}_0 (-\sin(\omega t + \varphi)) \\ &= \omega C \mathcal{E}_0 \cos(\omega t + \varphi + \frac{\pi}{2}) \end{aligned}$$

per scegliere fase nulla per $I(t=0) \Rightarrow \varphi = -\frac{\pi}{2}$

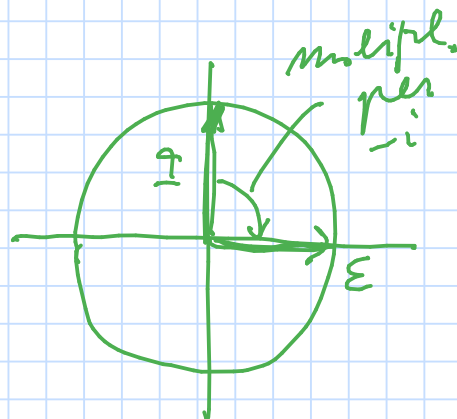
$\varphi = -\frac{\pi}{2}$: I in anticipo $(-\varphi)$ risp. $\mathcal{E}$



metodo simboli

$$\mathcal{E} = \mathcal{E}_0 e^{i(\omega t + \varphi)}$$

$$\begin{aligned} I &= \frac{dq}{dt} = C \mathcal{E}_0 \frac{d}{dt} e^{i(\omega t + \varphi)} = \\ &= i \omega C \underbrace{\mathcal{E}_0 e^{i(\omega t + \varphi)}}_{\mathcal{E}} \end{aligned}$$



$$\mathcal{E} = \underbrace{\frac{1}{i\omega C}}_{\rightarrow \frac{-i}{\omega C}} I$$

$$-i = e^{-i\frac{\pi}{2}} \Rightarrow$$

per trovare fase $\mathcal{E}$ risp quella di I

$\frac{1}{\omega C} = \text{reattanza capacitiva}, X_C$

$\frac{1}{i\omega C} = \text{impedenza del C}$
 $= Z_C$

$$[Z_C] = [R] = \Omega$$

impedenza generalizzata resistenza al caso
con I alternate, q. complessa

$$E = Z I$$

tutti $\mathbb{C}$

ci dice come riscolare ($C: \frac{1}{\omega C}$) la corrente
e che fase applicare ($-i \Rightarrow -\frac{\pi}{2}$) per ottenere E

ammettenza $Y = \frac{1}{Z}$

$$I = Y E$$

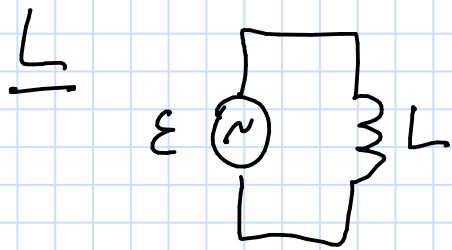
per condens: $Y_C = \frac{1}{Z_C} = i\omega C$

Di per frequenze basse ($\omega \rightarrow 0$), $Y_C \rightarrow 0$

potenza dissipata

$$\begin{aligned} P(t) &= I V_C = I E = I_0 \cos(\dots) \cdot E_0 (-\sin(\dots)) \\ &= I_0 E_0 \frac{1}{2} \sin(\dots) \end{aligned}$$

media nel tempo di $\sin(\dots) = 0 \Rightarrow \overline{P}_C = 0$



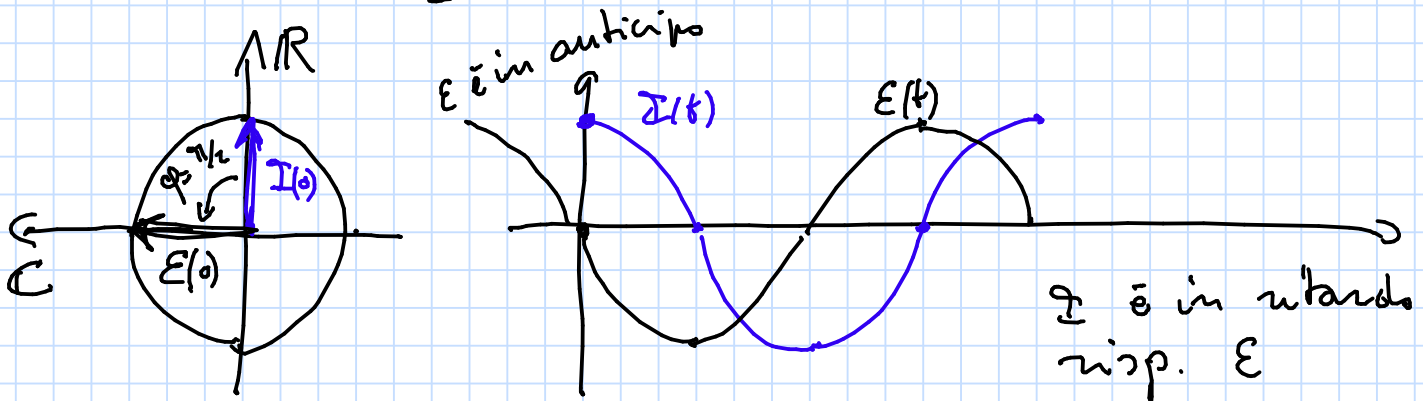
$$\mathcal{E} + \mathcal{E}_i = 0$$

$$\mathcal{E} = -\mathcal{E}_i = -\mathcal{E}_0 e^{i(\omega t + \varphi)}$$

dato che $\mathcal{E}_i = -L \frac{dI}{dt} \Rightarrow I = \frac{\mathcal{E}_0}{\omega L} \sin(\omega t + \varphi)$

$$= \frac{\mathcal{E}_0}{\omega L} \cos(\omega t + \varphi - \frac{\pi}{2})$$

scegliamo $\varphi = \frac{\pi}{2}$ per ritrovare $I = I_0 \cos(\omega t)$



anche qui quadratura di fase ($\varphi = \frac{\pi}{2}$)

reattanza $\omega L = X_L$ reattanza induttiva.

metodo simbolico

$$\mathcal{E}_i = -L \frac{d}{dt} I = -L \frac{d}{dt} I_0 e^{i\omega t} = -i\omega L I_0 e^{i\omega t} = -\mathcal{E}$$

$$\Rightarrow \mathcal{E} = \overbrace{i\omega L}^{X_L} I$$

Z_L : impedenza di L

$\overbrace{i\omega L}^{X_L}$: $\mathcal{E}$ è "reattiva" di $\frac{\pi}{2}$ risp I , amplif. ωL

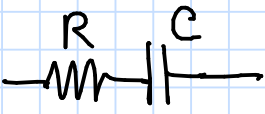
	Resistenza e reattanza	sfasamento φ	Impedenza Z	Ammettenza Y
R	R	0	R	$\frac{1}{R} = G$
L	ωL	$\pi/2$	$i\omega L$	$\frac{1}{i\omega L}$
C	$\frac{1}{\omega C}$	$-\pi/2$	$\frac{1}{i\omega C}$	$i\omega C$

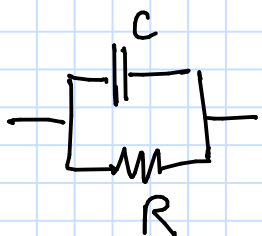
in serie si sommano impedenze



in parallelo si sommano le ammettenze



es)  $\Rightarrow Z = R + \frac{1}{i\omega C}$

 $\Rightarrow Y = \frac{1}{R} + i\omega C \Rightarrow Z = \frac{1}{Y}$

$[R] = [Z]$

$\Rightarrow \left[\frac{1}{C\omega}\right] = [\omega L] = \left[\sqrt{\frac{L}{C}}\right] = [R] = \Omega$

RLC con metodo simbolico

$$\frac{d^2 I}{dt^2} + \frac{R}{L} \frac{dI}{dt} + \frac{1}{LC} I = \frac{1}{L} \frac{d}{dt} \mathcal{E}$$

oggi $\mathcal{E} = \mathcal{E}_0 e^{i(\omega t + \varphi)}$
 $I = I_0 e^{i\omega t}$

oggi $\frac{d}{dt}$ porta un $i\omega$

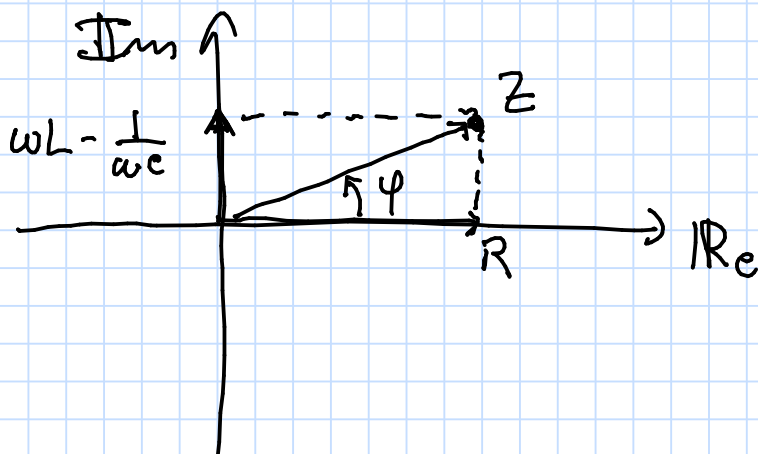
$$\left((i\omega)^2 + \frac{R}{L} (i\omega) + \frac{1}{LC} \right) I_0 \cancel{e^{i\omega t}} = \frac{1}{L} (i\omega) \mathcal{E}_0 e^{i\varphi} \cancel{e^{i\omega t}}$$

$$L \left(-\omega^2 + \frac{R}{L} (i\omega) + \frac{1}{LC} \right) I_0 = i\omega \mathcal{E}_0 e^{i\varphi}$$

$$\left[R + i \left(L\omega - \frac{1}{\omega C} \right) \right] I_0 = \mathcal{E}_0 e^{i\varphi}$$

$\bar{z}$ del circuito RLC

$$z = z_0 e^{i\varphi}$$



$$z_0 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$\tan \varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

ieri: $\frac{i_0}{\mathcal{E}_0}$ avevamo disegnato $\frac{1}{z_0} = Y_0$

Potenza in regime alternato

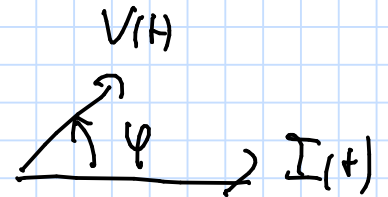
R dissipa, I in fase con E

L, C NON dissip. (magnatura di fase)

$\mathcal{E} \rightarrow V$ $I \rightarrow i$

$$P(t) = V(t) i(t)$$

$$= V_0 \underbrace{\cos(\omega t + \varphi)}_{\downarrow} i_0 \cos(\omega t)$$



$$\bar{P}(t) = V_0 i_0 \left(\underbrace{\cos \omega t}_{\text{green}} \cos \varphi \underbrace{\sin \omega t \sin \varphi}_{\text{orange}} \right) \underbrace{\cos \omega t}_{\text{green}}$$

$$= V_0 i_0 \cos \varphi \cos^2 \omega t$$

$$= \frac{1}{2} V_0 i_0 \cos \varphi$$

$$\bar{P} = V_{\text{eff}} i_{\text{eff}} \cos \varphi$$

$\cos \varphi =$ fattore di potenza