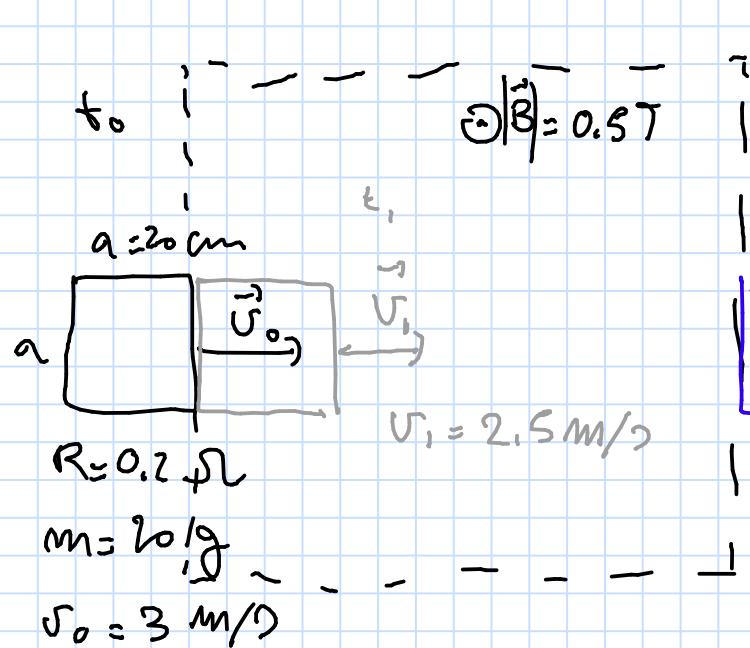


esercizio 8.20 ZN



1) q da t_0 a t_1 attraverso spin

2) energia dissipata $t_0 \rightarrow t_1$

3) v_1 all'uscita completa

1) legge di Faraday

$$q = \frac{1}{R} [\Phi_0 - \Phi_1]$$

scogliamo $\square$ (verso) $\Rightarrow \square \odot \hat{n}$

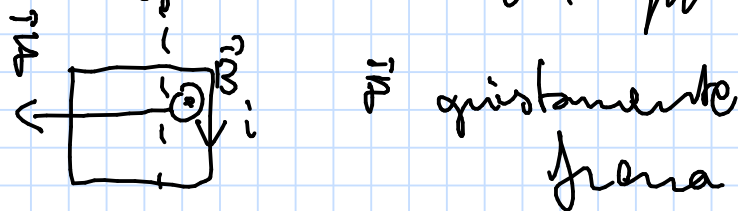
$$\Phi_0 = 0$$

$$\Phi_1 = a^2 B$$

$$a \square \odot \vec{B}$$

$$q = - \frac{a^2 B}{R} = -0.1 \text{ C}$$

regola - : ha fluito nel verso opposto a quello scelto (provare scelta opposta)



2) ene. dissip. : ene cinetica perso

$$W_R = \frac{1}{2} m v_0^2 - \frac{1}{2} m v_1^2 = 2.75 \cdot 10^{-2} \text{ J}$$

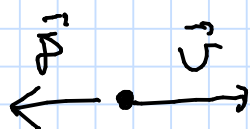
resistenza

3) conosciamo $v_1, v_0 \Rightarrow v_2$

$$F = i a B$$

$$= \frac{\varepsilon_i}{R} a B$$

$$= \frac{B a v}{R} a B \Rightarrow \text{forza di tipo viscosa} \sim v$$



$$\vec{F} = - \frac{B^2 a^2}{R} v$$

$$= - \gamma v$$

$$\gamma = \frac{B^2 a^2}{R}$$

coeff. di smorzamento

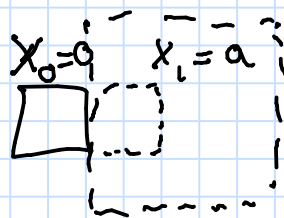
Newton

$$m \frac{d\vec{v}}{dt} = - \gamma \vec{v}$$

$$\text{1d } m \frac{dv}{dt} = - \gamma v = - \gamma \frac{dx}{dt}$$

integrando

$$m \int_{v_0}^{v_1} dv = - \gamma \int_{x_0}^{x_1} dx$$



$$m (v_1 - v_0) = - \gamma (x_1 - x_0) = - \gamma a$$

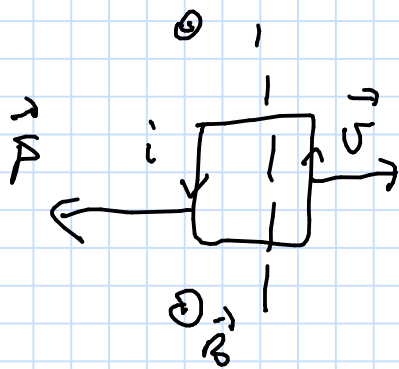
$$v_1 - v_0 = - \frac{\gamma}{m} a$$

unità: $\frac{m}{s}$ $\frac{1}{s}$ m

$\frac{m}{\gamma}$ è costante di tempo

$$\tau = \frac{m}{\gamma} = \frac{m R}{B^2 a^2}$$

in uscita, ragionamento simile



$$\dots \quad \mathcal{E}_2 - \mathcal{E}_1 = -\frac{\gamma}{m} a$$

$$\mathcal{E}_2 - \mathcal{E}_1 = \mathcal{E}_1 - \mathcal{E}_0$$

$$\mathcal{E}_2 = 2\mathcal{E}_1 - \mathcal{E}_0 = 2m\gamma$$

oppure $\mathcal{E}_2 = \mathcal{E}_1 - \frac{\gamma}{m} a = 2m\gamma$

quindi

$$\mathcal{E}_{cin,2} = \frac{1}{2} m \mathcal{E}_2^2 = 4 \cdot 10^{12} \text{ J}$$

altro modo per trovare $\mathcal{P}$: potenza

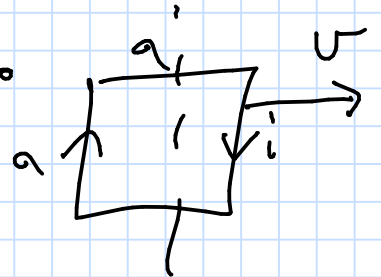
$$P_{\text{diss}} = P_{\text{cin}}$$

dissipata in R fornita da quantità di moto

$$R i^2 = F v$$

$$\Rightarrow \textcircled{*} \boxed{\mathcal{P} = \frac{R}{v} i^2}$$

nel caso di prima, sp. quadrato

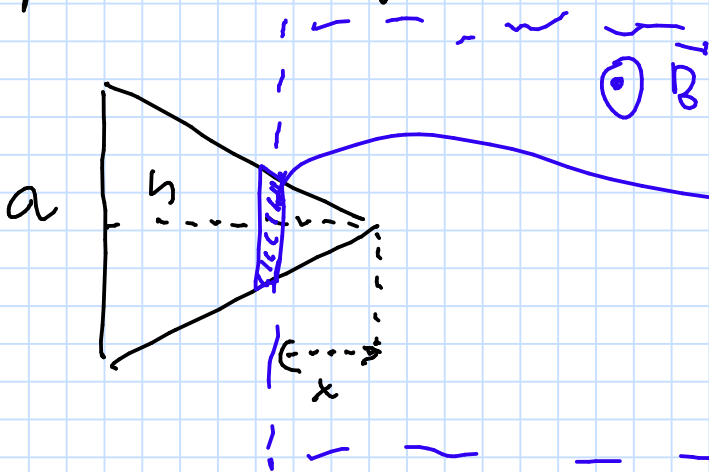


molto

$$i = \frac{1}{R} \mathcal{E}_i = \frac{1}{R} v a B$$

$$\textcircled{*} \mathcal{P} = \frac{R}{v} \left(\frac{1}{R} v a B \right)^2 = \frac{a^2 B^2}{R} v$$

es: spira triangolare



$$d\phi_B = B \frac{x}{h} a dx$$

in tempo dt

$$|\mathcal{E}_i| = \frac{B a x}{h} \frac{dx}{dt} = \frac{B a x}{h} v$$

$$i = \frac{\mathcal{E}_i}{R}$$



$$\textcircled{*} \mathcal{P} = \frac{1}{v} R \left(\frac{B a x v}{R h} \right)^2 = \frac{a^2 x^2 B^2}{R h^2} v$$

$$= \gamma(x) v$$

$$m \frac{dV}{dt} = -\gamma(x) \frac{dx}{dt}$$

$$\dots \quad V_1 - V_0 = -\frac{1}{m} \int_{x_0}^{x_1} \gamma(x) dx$$

$$\dots = -\frac{a^2 B^2}{m h R} \frac{h^3}{3}$$

$$= -\frac{B^2 a^2 h}{3 m R}$$

anell superconductor

$d = 2 \text{ cm}$ (diameter)

$$L = 3.14 \cdot 10^{-8} \text{ H}$$
$$B_0 = B = 0.025 \text{ T}$$
$$b=0 \quad B=13 \rightarrow B \leq 0$$

i) consente i generali nell'ordine?

Leva: flusso di B prima di $t=0$
 = " dopo $t=0$

$$t < 0$$
$$b \rightarrow 0$$

↑ ↑ ↑ ↑ $B_0 = B$

$$\phi_0 = \pi \left(\frac{d}{z} \right)^2 B$$

$B_0 \uparrow \downarrow \Delta\phi$

$$\Delta\phi = -\phi_0$$

A diagram of a closed system, represented by a black oval. Inside the oval, there are four blue arrows pointing upwards. Outside the oval, there are four blue arrows pointing away from the system (upwards and outwards). A black arrow points from the text 'closed system' to the oval. A small black bracket is positioned above the oval.

B non omogeneo

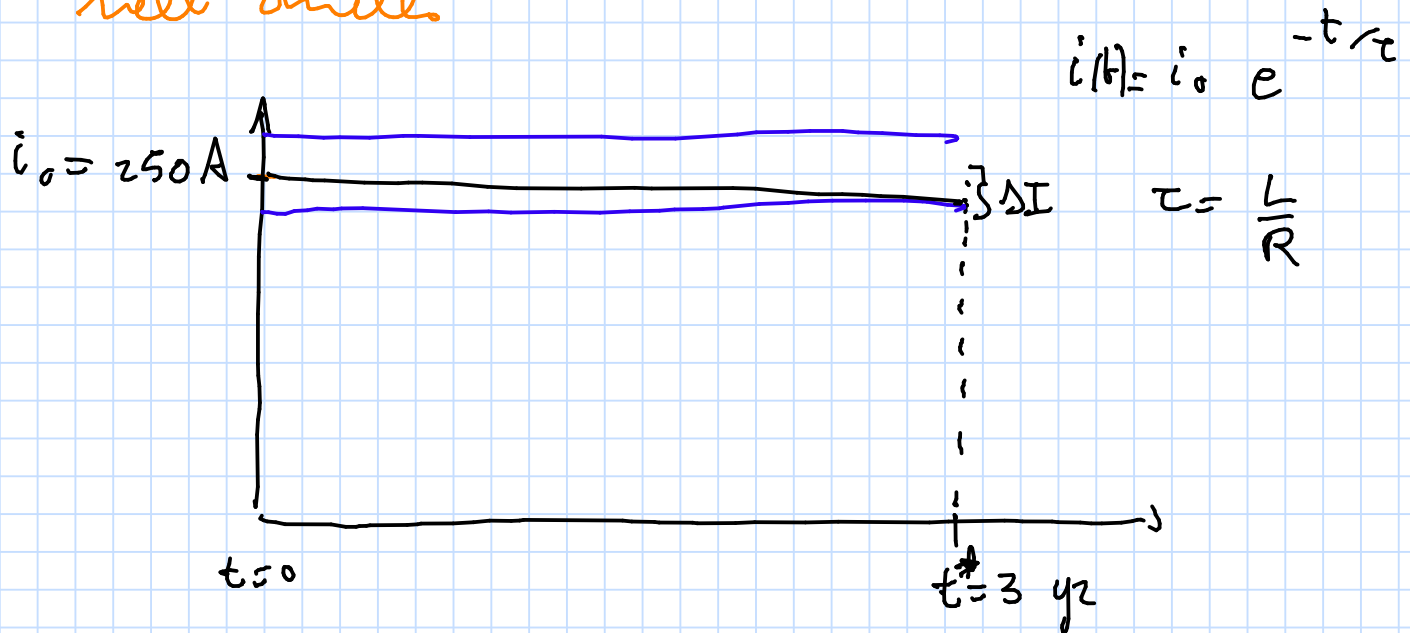
$$\phi_1 = \phi_0 - \Delta\phi$$

$$\Rightarrow i = \frac{1}{L} \pi \left(\frac{d}{2} \right)^2 B$$

$$= \frac{3.14 \cdot 10^{-4} \cdot 0.025}{3.14 \cdot 10^{-8}} \text{ A} = 250 \text{ A}$$

3 anni : non c'è nota variazione di i
sensib. $\frac{\Delta I}{I} = 10^{-9}$

2) calcolare val. max. della resistenza nell'anno



$$e^{-\frac{t}{\tau}} \approx 1 - \frac{t}{\tau} \quad \text{se } t \ll \tau$$

$$\frac{i(t^*)}{i_0} \geq 1 - \frac{\Delta I}{I}$$

$$-\frac{t^*}{\tau} \geq -\frac{\Delta I}{I}$$

↓

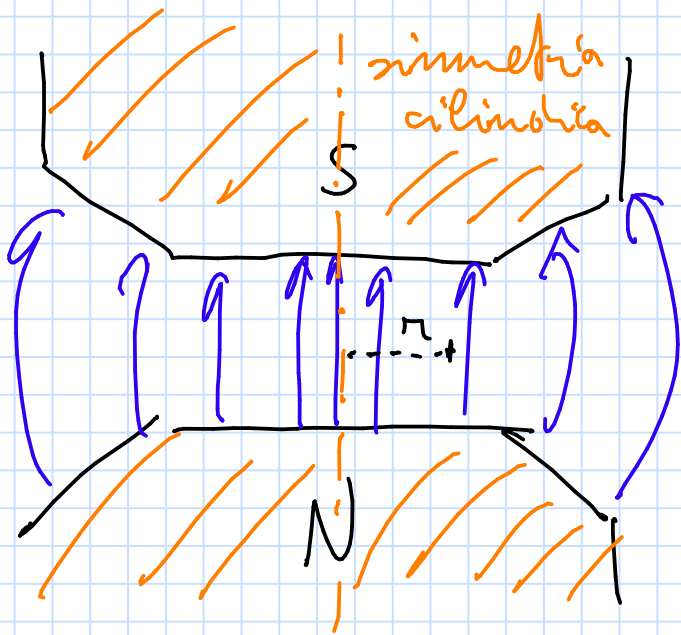
$$R \leq \frac{L}{t^*} \frac{\Delta I}{I} = R_{\max}$$

$$R_{\max} = \frac{3.14 \cdot 10^{-8}}{3(3.15 \cdot 10^7)} \cdot 10^{-9} \Omega = 0.33 \cdot 10^{-24} \Omega$$

Betatron

p. 333 MNV

acceleratore di e^- , 400 MeV



$|\vec{B}(z)|$ più intenso al centro

B varia nel tempo

↓
Faraday

↓
 $\vec{E} = \vec{E}_\phi$ tangente a circonf. Coassiali

$$\int_R \vec{E} \cdot d\vec{s} = 2\pi R E(R) \stackrel{F.}{=} - \frac{d\phi_B}{dt} = - \frac{d}{dt} (\pi R^2 B_m(R))$$

campo B medio in πR^2



(modulo) $E = \frac{R}{2} \frac{dB_m}{dt}$

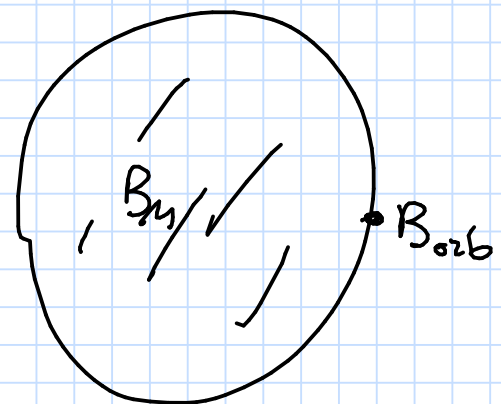


$$\frac{dp}{dt} = e E$$

$$\Delta p = e \frac{R}{2} \Delta B_m$$

variazione
q di moto

in tempo
 Δt



andando più veloce: B_{orb} deve aumentare
per regime F di Lorentz

$$\Delta t \rightarrow \Delta B_{orb}$$

$$t = 0$$

$$t = \Delta t$$

$$R = \frac{p}{e B_{orb}} \longrightarrow R = \frac{p'}{e B'_{orb}} = \frac{p + \Delta p}{e (B_{orb} + \Delta B_{orb})}$$

$$\text{stesso } R \Rightarrow \Delta B_{orb} = \frac{\Delta p}{R e}$$

$$= \frac{1}{R e} \frac{R e}{2} \Delta B_m$$

$$\boxed{\Delta B_{orb} = \frac{1}{2} \Delta B_m}$$

- energia a riposo dell' e^- $m_e c^2 = 0.5 \text{ MeV}$
- raggiungendo $100 \text{ MeV} \Rightarrow$ relativistico
- trattazione relat. corretta

formule
valide

$$\left\{ \begin{array}{l} \frac{dp}{dt} = F \\ R = \frac{p}{e B} \end{array} \right.$$

in buona approssim. $E_{cin} = p \cdot c$

$$\text{e } R = 1 \text{ m}$$

$$B_{orb} = 0.5 \text{ T}$$

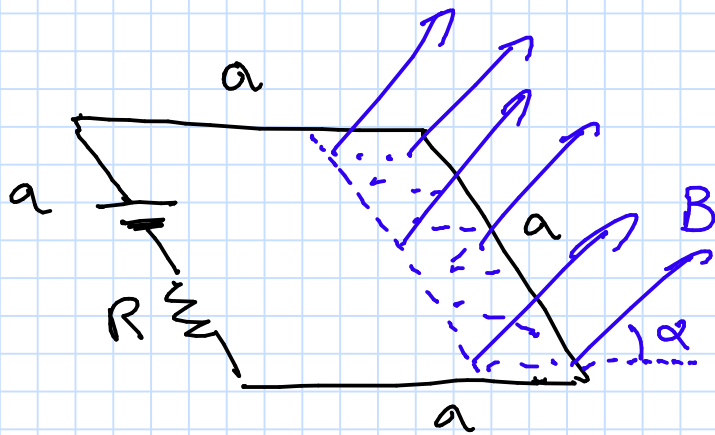
$$B_{orb} = 0.5 \text{ T} \cdot \alpha(t)$$

$$\alpha(t): 0 \rightarrow 1$$

$$\alpha(t) = \frac{t}{\tau}$$

$$E_{cin, max} = p_{max} \cdot c = \Delta p \cdot c$$

$$= e R \Delta B_{orb} c = e \cdot 1.5 \cdot 10^8 \frac{\text{J}}{\text{C}} = 1.5 \cdot 10^8 \text{ eV} = 150 \text{ MeV}$$



$$a = 3 \text{ m}$$

$$\mathcal{E} = 0.5 \text{ V}$$

$$R = 15 \text{ }\Omega$$

$$2 = 60^\circ$$

$t=0$: B cresce
linearmente

annullandosi a
 $t = 4 \text{ sec.}$
poi rimane nulla

- 1) i nella spira per $t < \tau$
- 2) $P(t < \tau)$ dissipata
- 3) energia diss. in calore
in R da $t=0$ a $t = 10 \text{ s}$

1) $B(t) = B_0 \left(1 - \frac{t}{\tau}\right)$ dai dati

$$B_{\text{perp}}(t) = B_0 \left(1 - \frac{t}{\tau}\right) \text{ con } 2 = \frac{\sqrt{3}}{2} B_0 \left(1 - \frac{t}{\tau}\right)$$

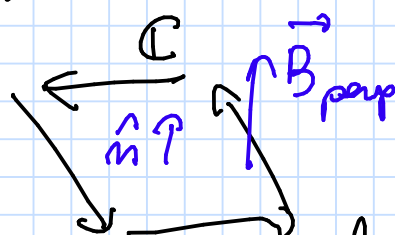
$$\Phi_B(t) = \Sigma_B \cdot B_{\text{perp}}(t) = \Sigma \frac{\sqrt{3}}{2} \left(1 - \frac{t}{\tau}\right)$$

$$\downarrow \Sigma_B = \frac{1}{3} a^2$$

$$\mathcal{E}_i = - \frac{d\Phi_B}{dt} = \frac{a^2}{3} \frac{\sqrt{3}}{2} B_0 \cdot \frac{1}{\tau} = 1.3 \text{ V}$$

ovvero di $\mathcal{E}_i$:

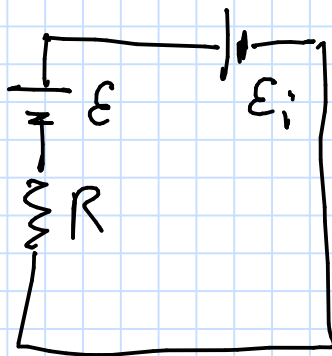
scegliamo



flusso di B_{perp} e $\vec{L}$ e $\vec{v}$ cala nel tempo $\Rightarrow$ segno +
 $\mathcal{E}_i$ concorde $\square$

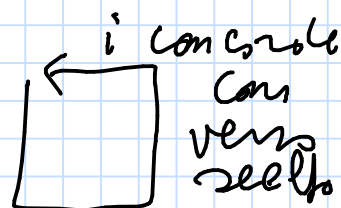
$$0 \leq t \leq \tau$$

$\Leftrightarrow$



con orient.  Kirchhoff: $-\varepsilon + \varepsilon_i = R i$

$$i = \frac{-\varepsilon + \varepsilon_i}{R} = \frac{-0.5 + 1.3}{15} \text{ A} = 53 \text{ mA} \quad i > 0$$



2) $P(t=1s) \quad t < \tau$

$$P = R i^2 = 43 \text{ mW}$$

3) ene. dissip. $W_R = \int_0^{t_f} P(t) dt$

$$= \int_0^{\tau} P(t) dt + \int_{\tau}^{t_f} P'(t) dt$$

$$= W_1 + W_2$$

$\uparrow$
i solo dal generatore

$$P' = \frac{\varepsilon^2}{R}$$

$$W_1 = P \tau = 0.170 \text{ J}$$

$$W_2 = (t_f - \tau) P' = 0.1 \text{ J}$$

$$W_R = W_1 + W_2 = 0.27 \text{ J}$$

Δ i cambia verso dopo τ