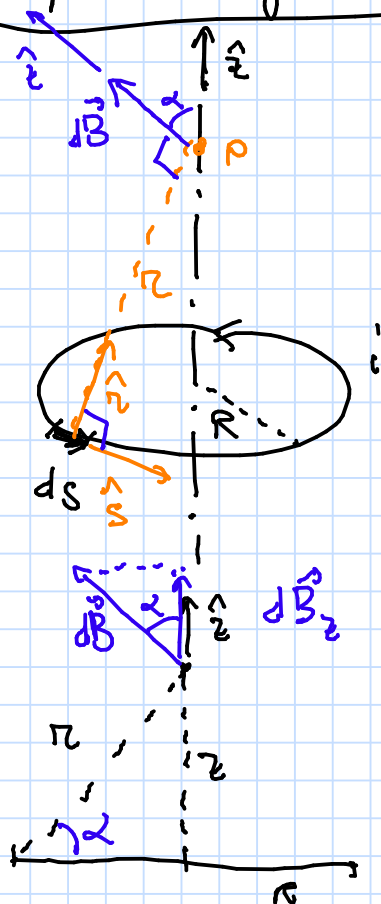


Campo magn. in asse di spira circolare



$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{ds}{r^2} \underbrace{\hat{s} \times \hat{r}}_{\hat{e}}$$

per simmetria, componenti
dei $d\vec{B}$ fuori asse si elidono

$$\begin{aligned} dB_z &= \frac{\mu_0 i}{4\pi} \frac{ds}{r^2} \cos \alpha \\ &= dB \frac{R}{r} \\ &= dB \frac{R}{\sqrt{R^2 + z^2}} \end{aligned}$$

(a)

$$B_z(z) = \int_{s=0}^{s=2\pi R} dB_z = \frac{\mu_0 i}{4\pi} \frac{2\pi R^2}{(R^2 + z^2)^{3/2}} = \frac{\mu_0 i}{2R} \frac{1}{\left(1 + \left(\frac{z}{R}\right)^2\right)^{3/2}}$$

$z=0$ centro spira $B_z(0) = \frac{\mu_0 i}{2R}$

(b) $\Rightarrow dB_z = \frac{\mu_0 i}{4\pi} \frac{R ds}{r^3}$

$$B_z(z) = \int dB_z = \frac{\mu_0 i R^2}{2\pi^3} = \frac{\mu_0}{4\pi} \frac{2m}{\pi^3} \text{ nell'asse}$$

$m = \sum i$

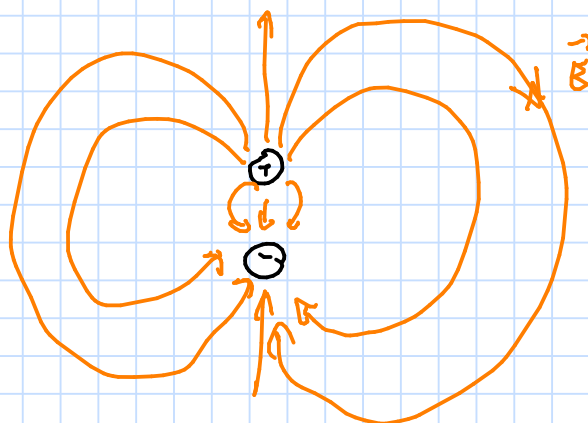
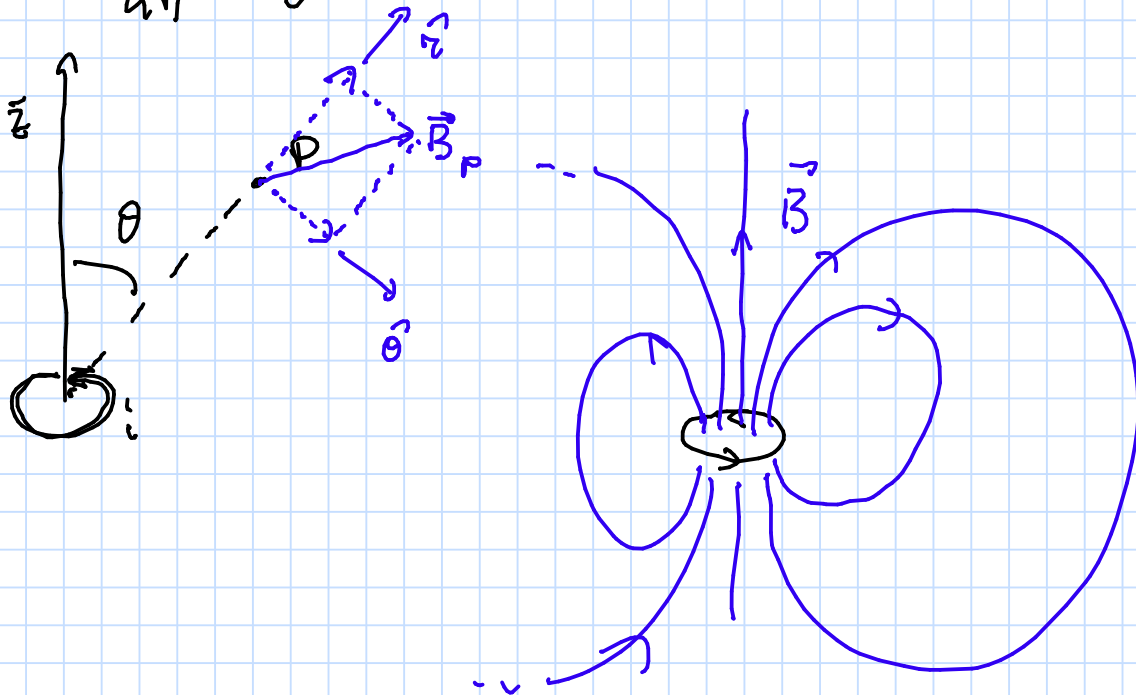
a grande distanza

$$B_z(z) \sim \frac{1}{r^3} \sim \frac{1}{z^3}$$

raggiungia al
decadimento con $1/r^3$
di E del dipolo el.

$r \gg R$

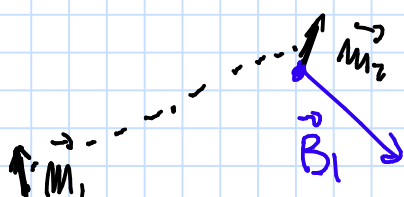
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$$



campo dipolo el.

anche
$$\vec{B} = \frac{\mu_0}{4\pi r^3} [-\vec{m} + 3(\vec{m} \cdot \hat{r}) \hat{r}]$$

interazione dipolo-dipolo magnetico

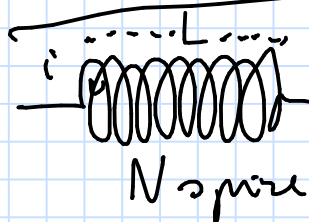


$$U_p = -\vec{m}_2 \cdot \vec{B}_1$$

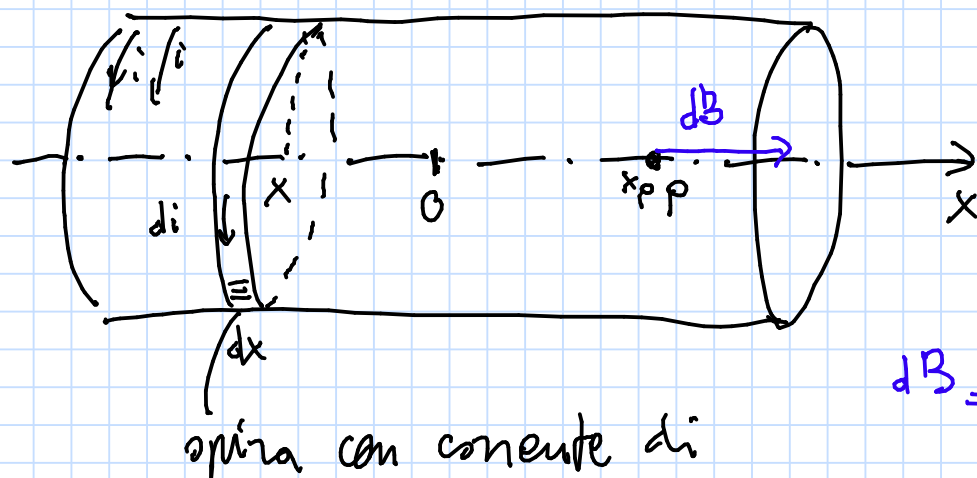
$$= \frac{\mu_0}{4\pi r^3} [\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})]$$

minima energia se $\vec{m}_1 \parallel \vec{m}_2$

campo magnetico su asse di solenoide



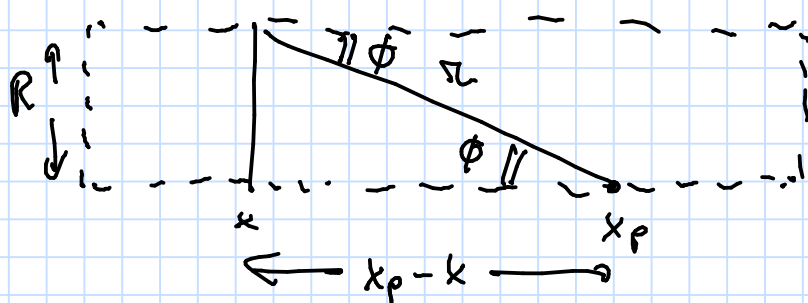
$$n = \frac{N}{L} \quad \text{spire/m}$$



$$di = (ni) \cdot dx$$

$$dB = \frac{\mu_0 R^2}{2\pi^3} \cdot (ni \, dx)$$

spira con corrente di



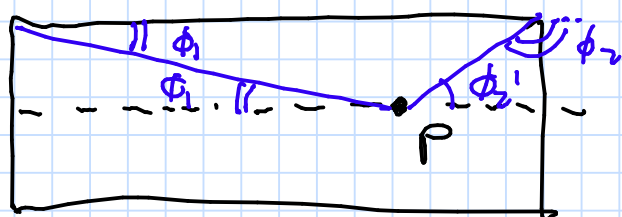
$$\sin \phi = \frac{R}{x}$$

$$R = \tan \phi (x_p - x)$$

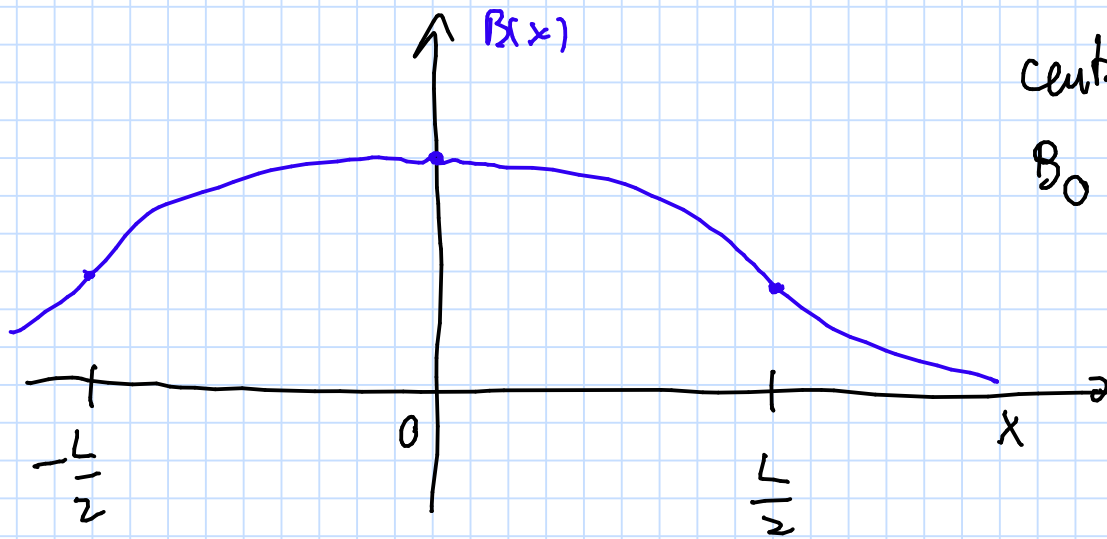
$$\rightarrow x = x_p - \frac{R}{\tan \phi}$$

$$dx = \frac{R}{\sin^2 \phi} d\phi$$

$$\begin{aligned} dB &= \frac{\mu_0}{2} \left(\frac{R}{x} \right)^3 ni \frac{d\phi}{\sin^2 \phi} \\ &= \frac{\mu_0}{2} \sin^3 \phi ni \frac{d\phi}{\sin^2 \phi} \\ &= \frac{\mu_0}{2} ni \sin \phi d\phi \end{aligned}$$



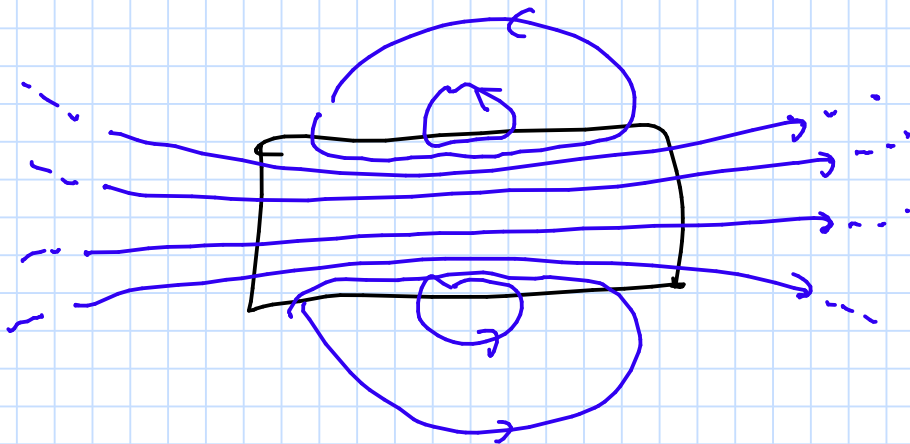
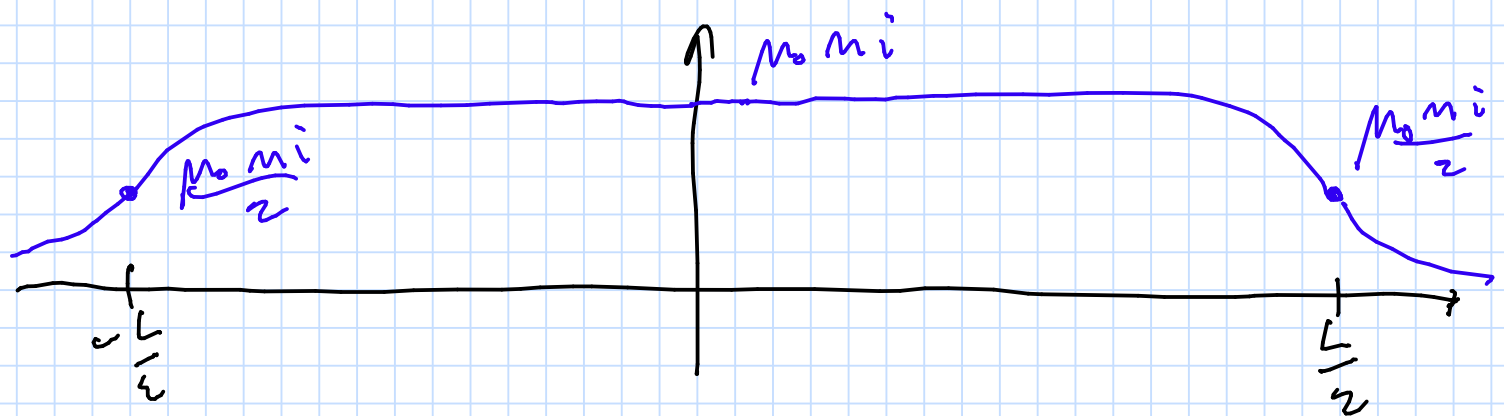
$$\begin{aligned} B &= \int dB = \frac{\mu_0 ni}{2} (-\cos \phi_2 + \cos \phi_1) \\ &= \frac{\mu_0 n i}{2} (\cos \phi_2' + \cos \phi_1') \end{aligned}$$



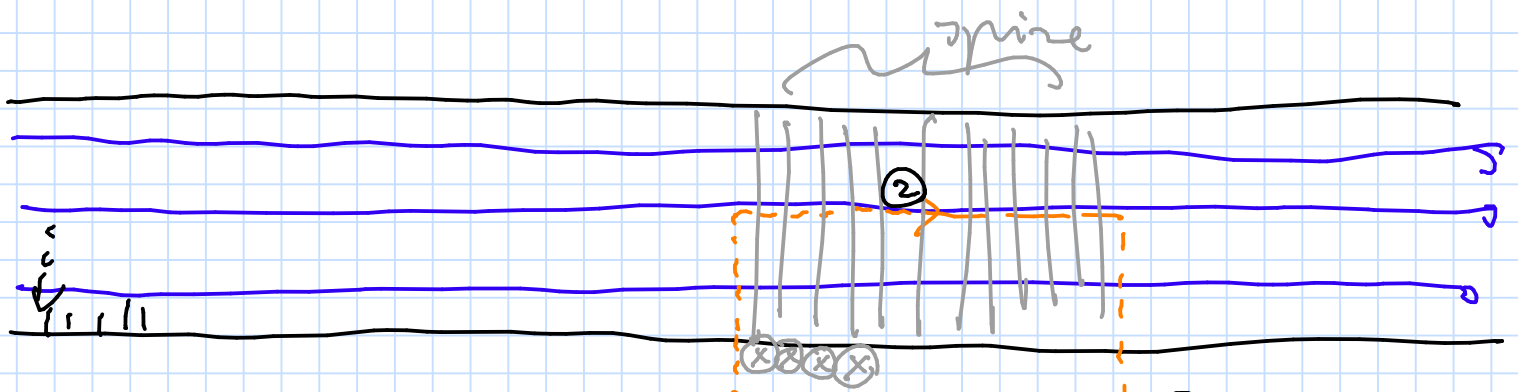
centro:

$$B_0 = \mu_0 n i \frac{L}{\sqrt{L^2 + 4R^2}}$$

solenoide molto lungo (Risp. R) $L \gg R$



solenoide ideale, infinito



calcolo con Ampere

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_c$$

$$i_c = l \cdot n \cdot i$$

lato ① $\vec{B} \perp d\vec{s} \Rightarrow \int_1 \vec{B} \cdot d\vec{s} = 0$ stesso su lato ③

lato ② $\int_2 \vec{B} \cdot d\vec{s} = l B$

lato ④ $B = 0 \Rightarrow 0$

$$\oint = \oint_1 + \oint_2 + \oint_3 + \oint_4 = \int_2 \vec{B} \cdot d\vec{s} = l B$$

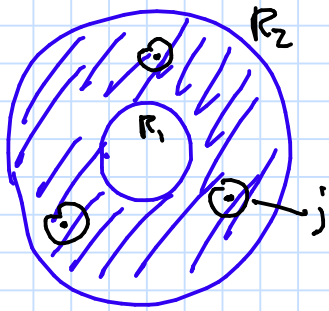
Ampere $B = \mu_0 i_c = \mu_0 n i$

$$\boxed{B = \mu_0 n i} \text{ sol. ideale}$$

Filo cavo: legge di Ampere $\Rightarrow \vec{B}$

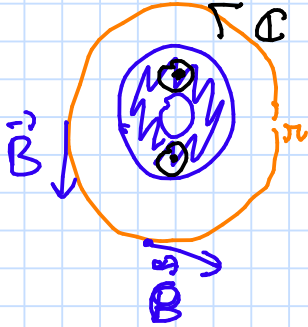
densità di corrente j uscente

$$i = j (\pi R_2^2 - \pi R_1^2)$$



simmetria cilindrica: ci si aspetta $\vec{B}$ di Biot-Savart all'esterno

$r > R_2$ (esterno)



curva orientata C

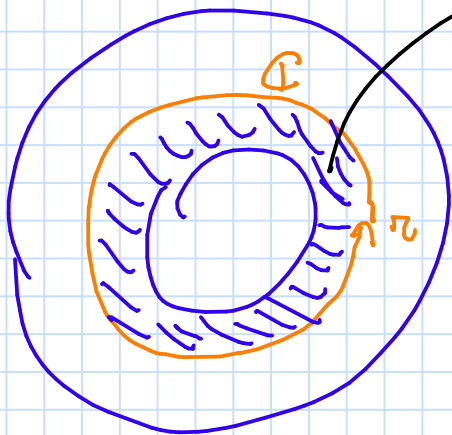
$$\oint_C \vec{B} \cdot d\vec{s} = B 2\pi r = \mu_0 i_c \quad \text{qui } i_c = i$$

direzione: tangente

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

stema di B-S.

$R_1 < r < R_2$



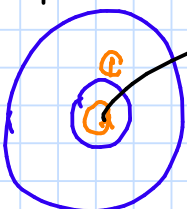
$$i_c = \int_C j$$

$$\Sigma_c = \pi r^2 - \pi R_1^2$$

Ampere: $B 2\pi r = \mu_0 j \pi (r^2 - R_1^2)$

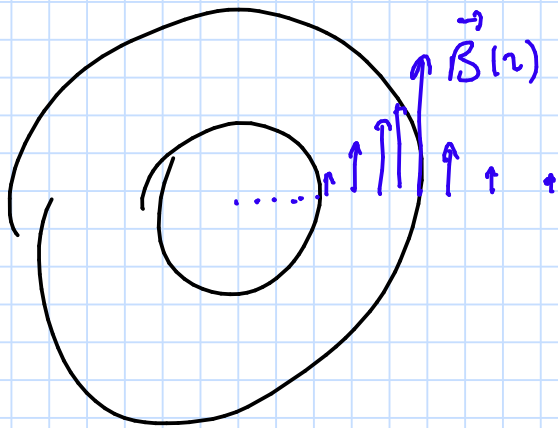
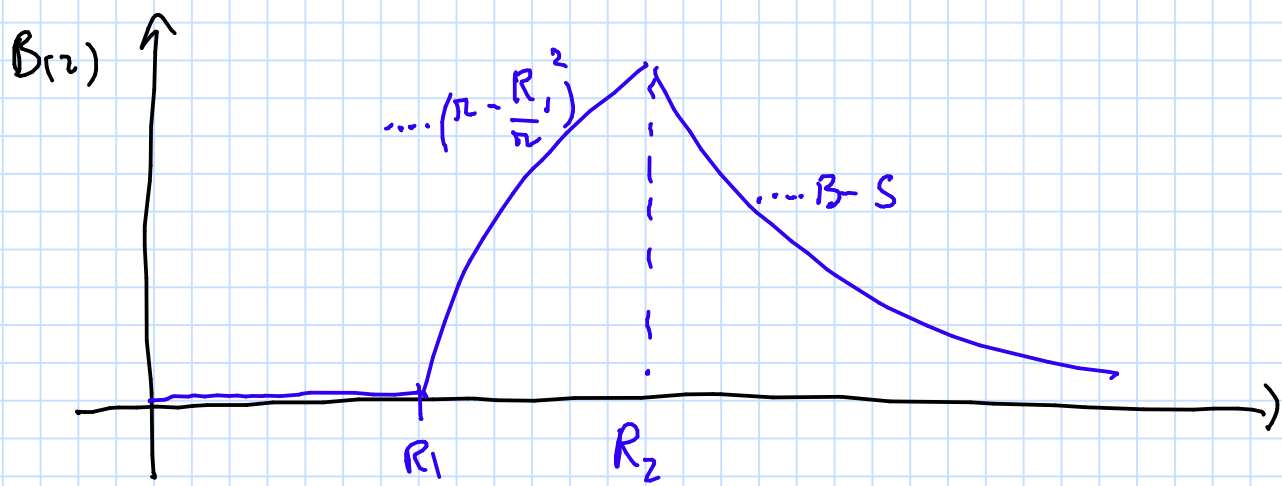
$$B(r) = \frac{\mu_0 j}{2} \frac{r^2 - R_1^2}{r} = \frac{\mu_0 i}{2} \left(r - \frac{R_1^2}{r} \right)$$

$r < R_1$

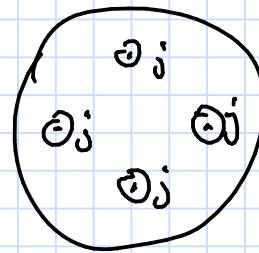
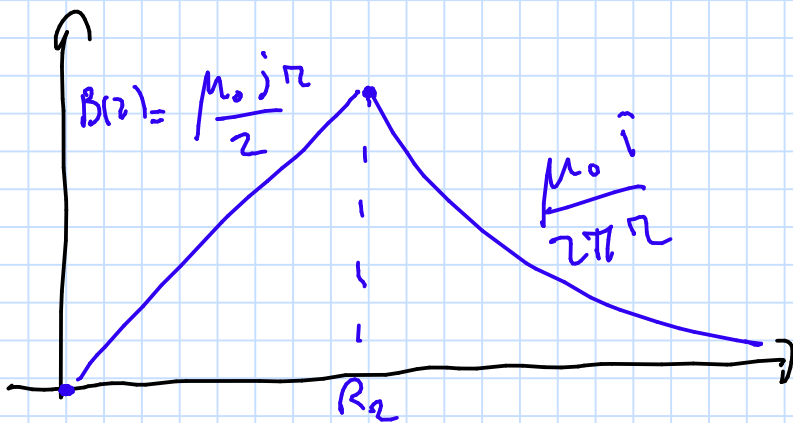


C non concatena corrente, per simmetria $\Rightarrow B=0$ all'interno

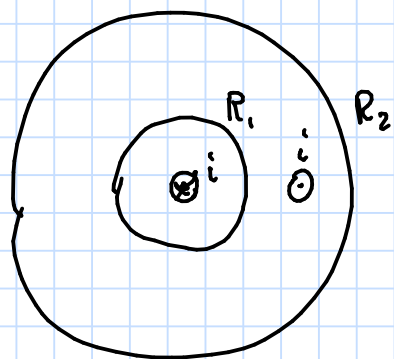




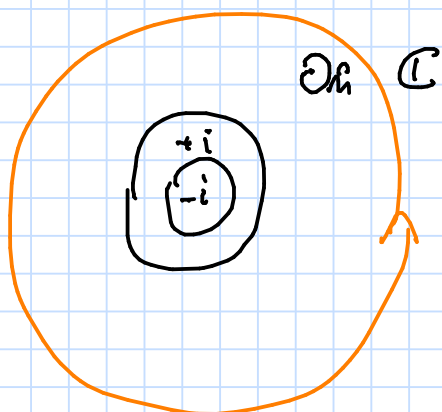
zenna carviri: $R_1 \leq 0$



filo coassiale :



i entrante nel filo interno
i uscente $\rightarrow$ esterno



intern / estern
 $i_c = -i + i = 0$
per simmetria
 $B = 0$ all'esterno

si può sommare $\vec{B}_1$ (i entrante, int.) con
 $\vec{B}_2$ (i uscente, cavo)

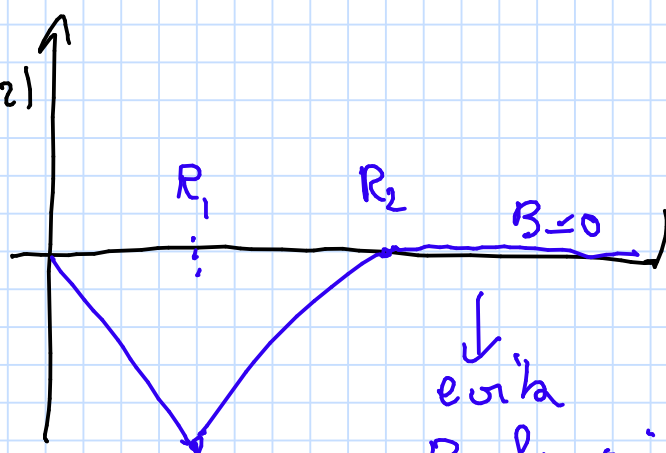
$r < R_1$

solo B_1

$B(r)$

$R_1 < r < R_2$

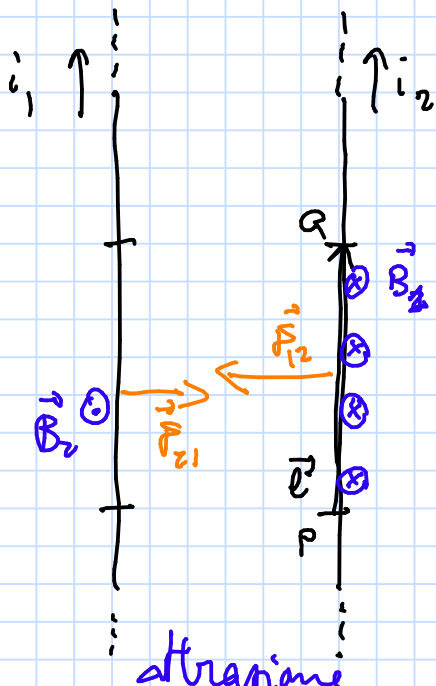
$B_1 + B_2$
 $B \neq 0$ filo cavo
 $B \leq 0$



$\downarrow$
evita
B fuori
e quindi interferenza

$r > R_2$

forza tra fili paralleli percorsi da
correnti



$$\vec{B}_1, i_2$$

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r} \hat{t}$$

in tutto il
tratto l

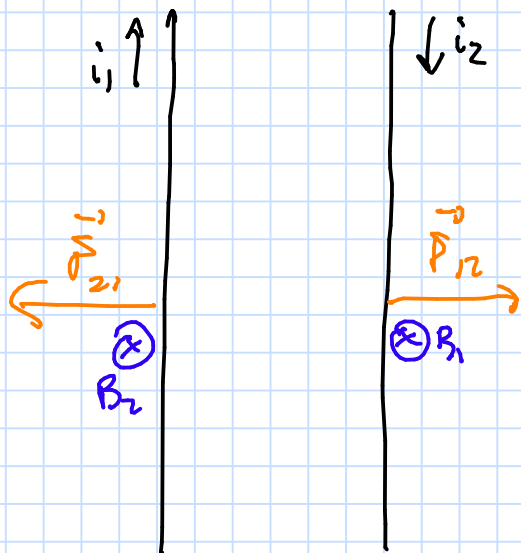
$$\vec{F}_{12} = \vec{F}_e = i_2 \vec{l} \times \vec{B}_1$$

$$F_{12} = \frac{\mu_0 i_1 i_2}{2\pi r} l$$

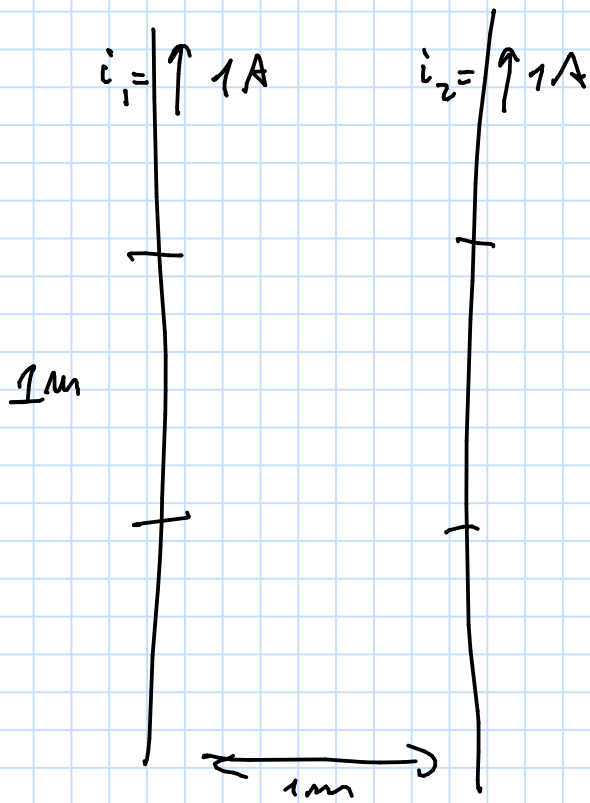
forza per unità di lunghezza
($\frac{N}{m}$)

$$f_{12} = F_{12}/l$$

$$f_{12} = \frac{\mu_0 i_1 i_2}{2\pi r} = f_{21} \quad \left(\frac{N}{m}\right) \text{ Ampere}$$



repulsione
tra fili



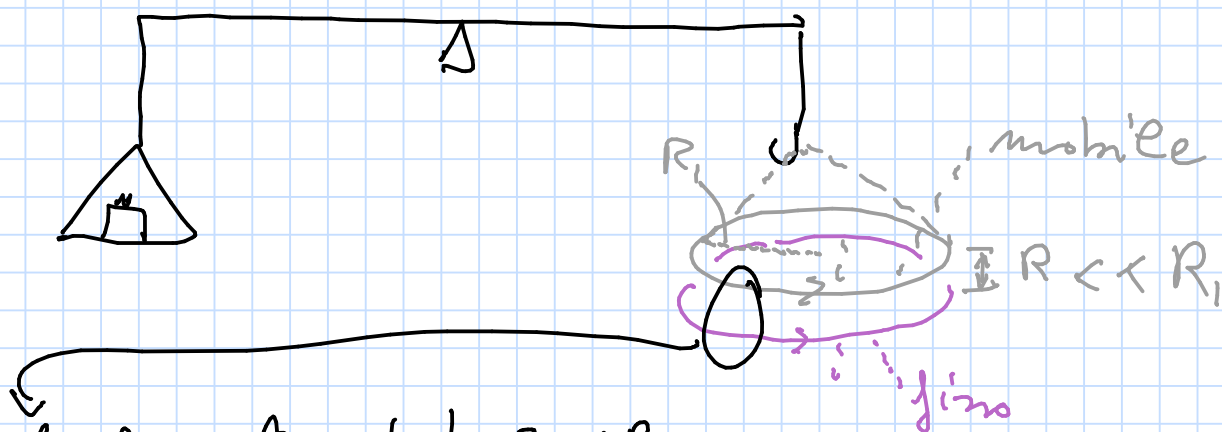
$$r = 1m$$

$$i_1 = i_2 = i = 1A$$

$1A$ genera una
forza / m di
 $2 \cdot 10^{-7} N/m$
tra i fili

$$\left[\frac{\mu_0}{2\pi} = 2 \left(\frac{\mu_0}{4\pi} \right) = 2 \cdot 10^{-7} \dots \right]$$

Elettrodinamometro



localmente, dato $R \ll R_1$,
vale B-S $\Rightarrow$ Ampere (lo usiamo come se
fossero fili rettilinei)

$$R_1 = 30 \text{ cm}$$

$$i_1 = i_2 = i \approx 2 \text{ A}$$

$$R = 1 \text{ mm}$$

$$= 10^{-3} \text{ m}$$

$$F_{\text{tot}} = \oint i_2 \cdot l$$

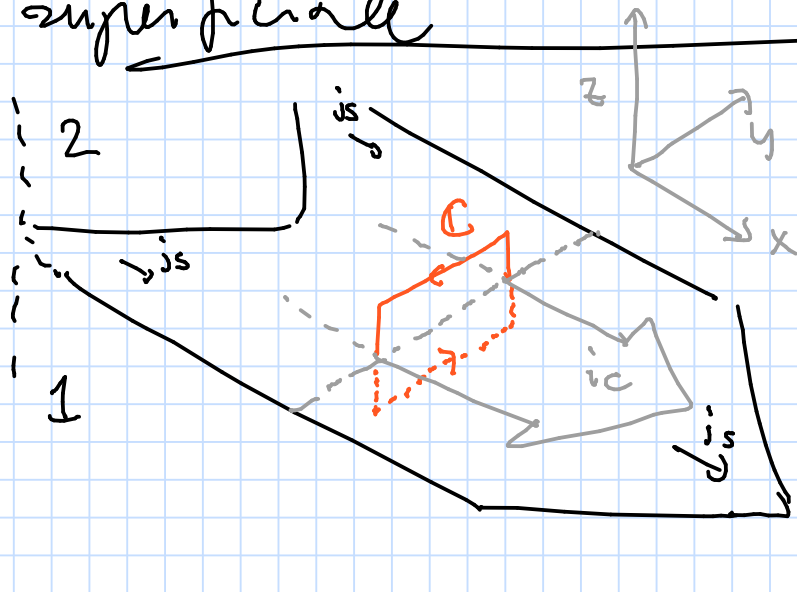
$$l = 2\pi R_1$$

$$F_{\text{tot}} = \left(\frac{\mu_0 i^2}{2\pi R} \right) \cdot 2\pi R_1$$

$$= \frac{2 \cdot 10^{-7} \cdot 4 \cdot 6.28 \cdot 0.3}{10^{-3}} = 1.5 \text{ mN}$$

$\underbrace{\hspace{10em}}_{R_1}$

Discontinuità di $\vec{B}$ nei pressi di corrente superficiale

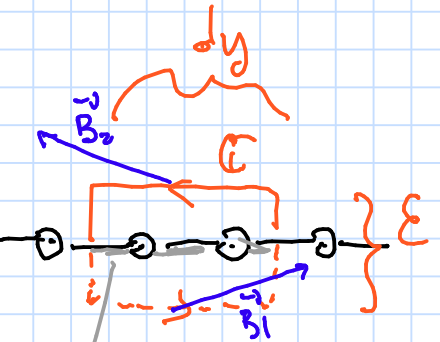


$$[j_s] = \frac{A}{m}$$

$$\oint_C \vec{B} \cdot d\vec{s} = \underbrace{\vec{B}_1 \cdot d\vec{y} + \vec{B}_2 \cdot (-d\vec{y})}_{\text{vertical sides}} + O(\epsilon)$$

C

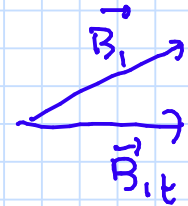
$$\lim_{\epsilon \rightarrow 0} \oint_C \vec{B} \cdot d\vec{s} = (B_{1t} - B_{2t}) dy$$



$$i_c = dy \cdot j_s$$

aliquota + $\vec{j}_s$

Ampere $(B_{1t} - B_{2t}) dy = \mu_0 j_s dy$

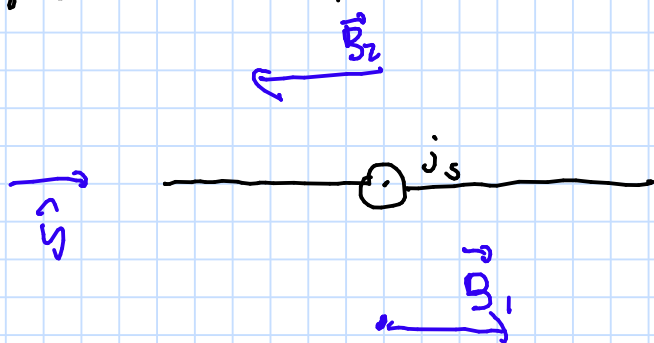


Discontinuità della componente // piano di j_s del campo $\vec{B}$

$$B_{1t} - B_{2t} = \mu_0 j_s$$

se non c'è B esterno, B_1 e B_2 generati da j_s

per simmetria

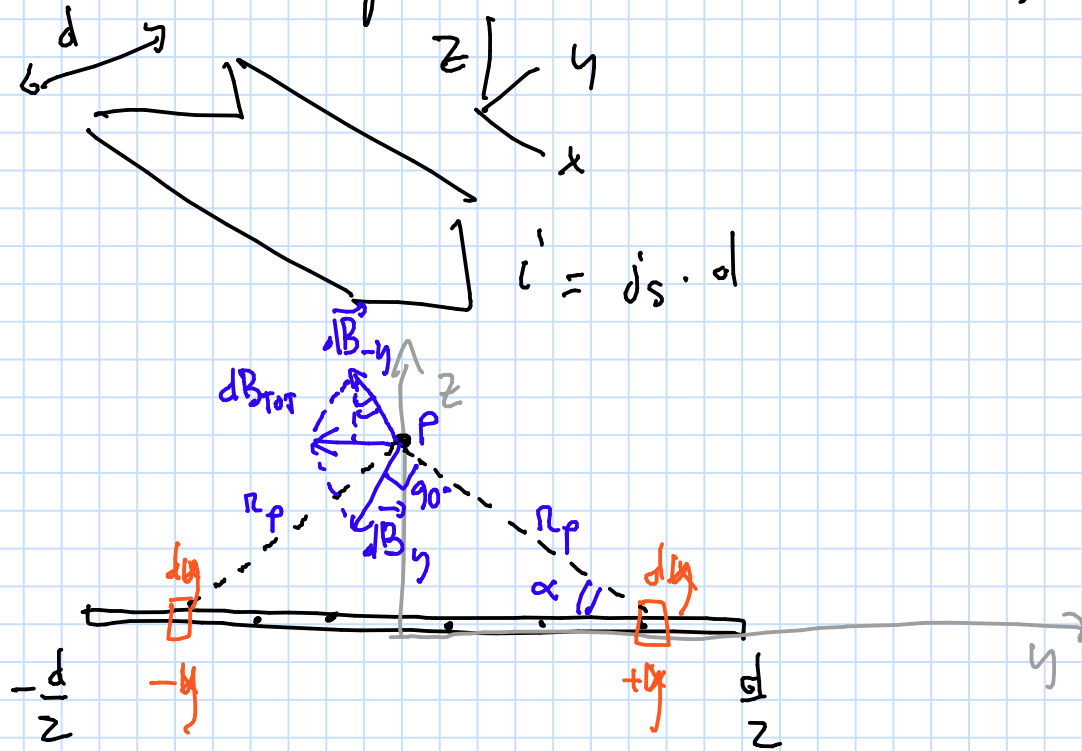


$$\vec{B}_1 = -\vec{B}_2$$

$$\begin{cases} \vec{B}_1 = \frac{\mu_0 j_s}{2} \hat{y} \\ \vec{B}_2 = -\frac{\mu_0 j_s}{2} \hat{y} \end{cases}$$

laminina percorsa da corrente, larghezza d

es. 8.7 MNV
p 411



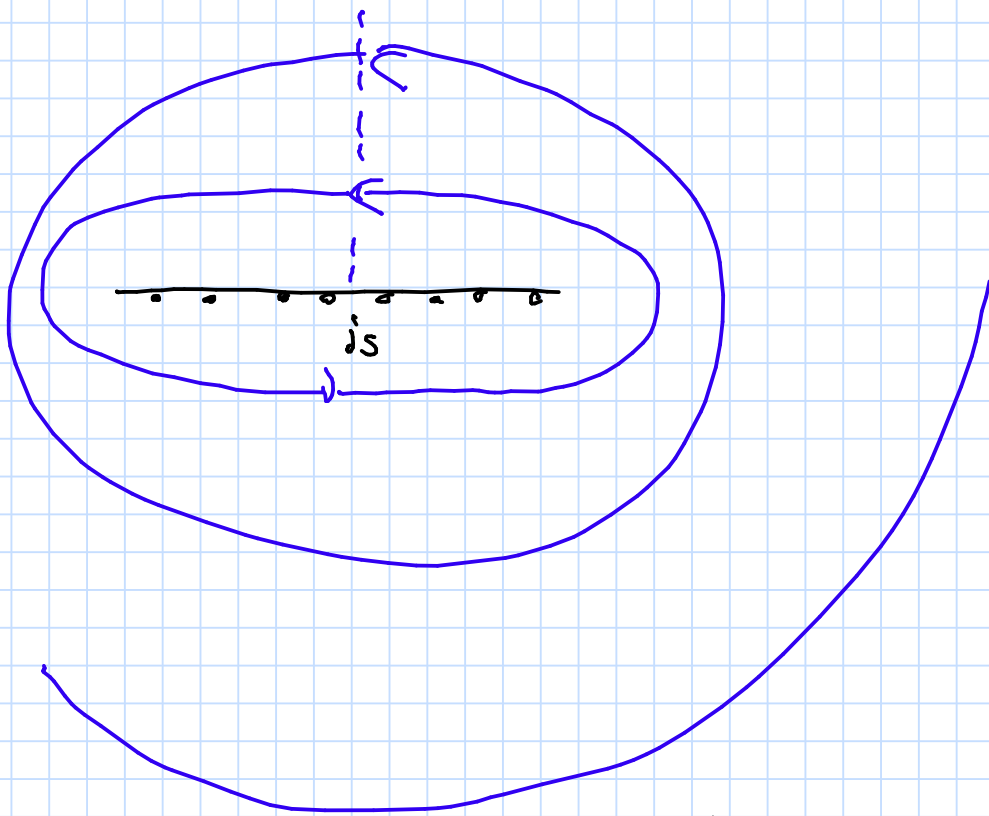
$$dB_y^P = dB_{-y}^P = \frac{\mu_0 (di)}{2\pi r_p} = \frac{\mu_0 j_s}{2\pi} \frac{dy}{(y^2 + z^2)^{1/2}}$$

$B-S$

$$dB_{tot}^P = 2 \cdot dB_y^P \cdot \underbrace{\sin \alpha}_{\frac{z}{(z^2 + y^2)^{1/2}}}$$

$$B^P = \int_0^{d/2} dB_{tot}^P = 2 \frac{\mu_0}{2\pi} j_s z \int_0^{d/2} \frac{dy}{(y^2 + z^2)^{1/2}}$$

$$B^P = \frac{\mu_0 j_s}{\pi} \arctan \left(\frac{d}{2z} \right) \quad (*)$$



$$\text{se } z \gg d/2 \quad \arctan\left(\frac{d}{2z}\right) = \frac{d}{2z}$$

$$B_p \approx \frac{\mu_0 i_s d}{2\pi z} = \frac{\mu_0 i}{2\pi z} \quad B-S$$

$$\text{se } z \ll d/2, \text{ conto di prima}$$

$$\arctan\left(\frac{d}{2z}\right) \approx \frac{\pi}{2}$$

$$B_p \approx \frac{\mu_0 i_s}{2}$$

$$\text{es) } \left. \begin{array}{l} d = 8 \text{ cm} \\ i = 3 \text{ A} \end{array} \right\} \quad i_s = \frac{3 \text{ A}}{0.08 \text{ m}} = 37.5 \text{ A/m}$$

$$z_1 = 1 \text{ mm}$$

$$B_1 = 23.2 \mu\text{T}$$

⊗

$$z_2 = 2 \text{ mm}$$

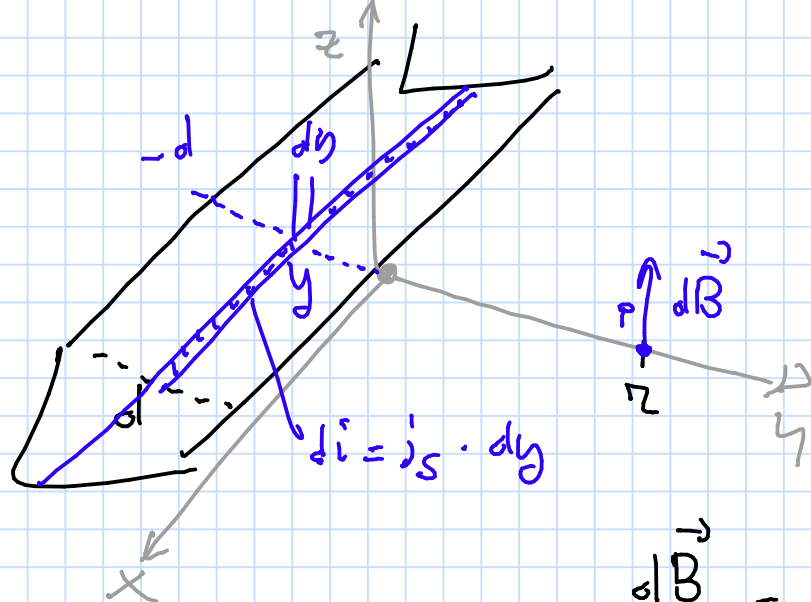
$$B_2 = 22.8 \mu\text{T}$$

⊗

$$z_3 = 12 \text{ mm}$$

$$B_3 = 4.8 \mu\text{T}$$

$$\left| \frac{\mu_0 i_s}{2} = 23.5 \mu\text{T} \right.$$



campo $\vec{B}$ nel
piano di
carriera
con corrente

$$d\vec{B} = \frac{\mu_0 di}{2\pi(r-y)} \hat{z} \quad (r > 0)$$

$$\vec{B}_P = \int d\vec{B} = \frac{\mu_0 j_s}{2\pi} \log\left(1 + \frac{d}{r}\right) \hat{z} \quad (r > 0)$$

$r \gg d$, $\log\left(1 + \frac{d}{r}\right) \simeq \frac{d}{r}$

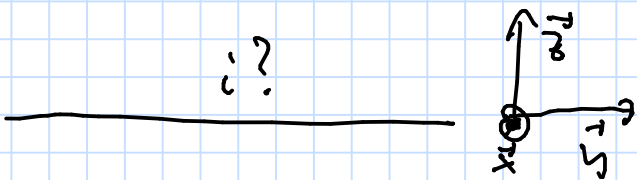
$$\vec{B}_P \simeq \frac{\mu_0 i}{2\pi d} \cdot \frac{d}{r} \simeq \frac{\mu_0 i}{2\pi r}$$

B-S

stessi d, i di prima:
 $y = 1 \text{ mm} = r$

$$B = \left(\frac{\mu_0}{2\pi}\right) j_s \log\left(1 + \frac{d}{r}\right) =$$

$$2 \cdot 10^{-7} \cdot 37.5 \cdot \log\left(1 + \frac{80 \text{ mm}}{1 \text{ mm}}\right) = 33.0 \mu\text{T}$$

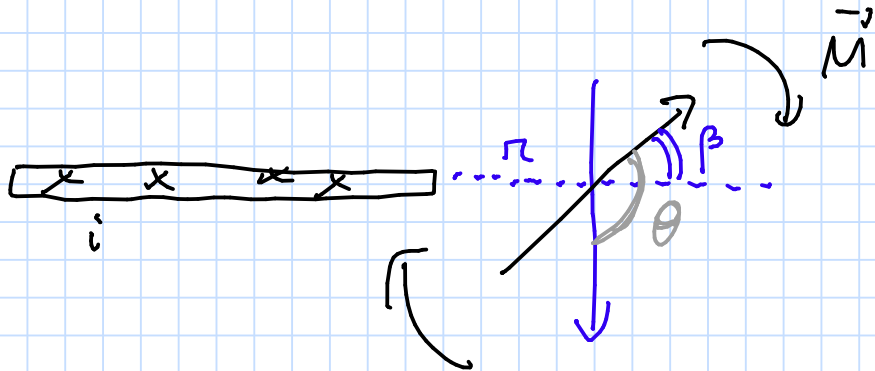
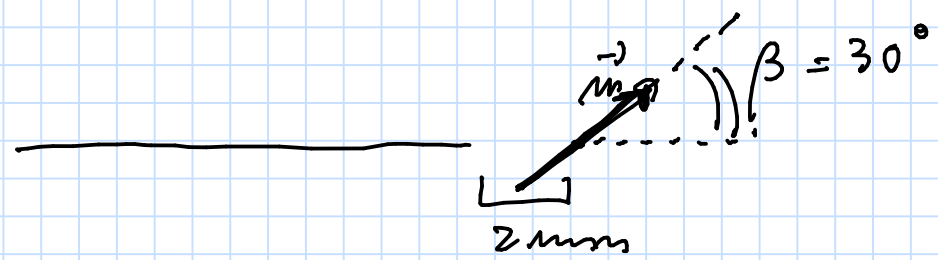


$$\vec{m} = 3 \cdot 10^{-6} \text{ A m}^2$$

mem. forte

$$\vec{M} = 6.5 \text{ pNm}$$

$i = ?$



deduciamo che $\vec{B}$ è verso il basso
↓
i entrante

$$\vec{M} = \vec{m} \times \vec{B}$$

modulo $M = m B \sin \theta$

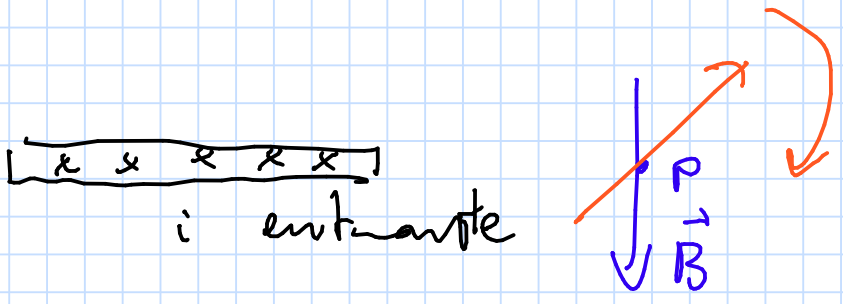
$$\beta = 30^\circ$$

$$\theta = 120^\circ = \frac{2}{3} \pi$$

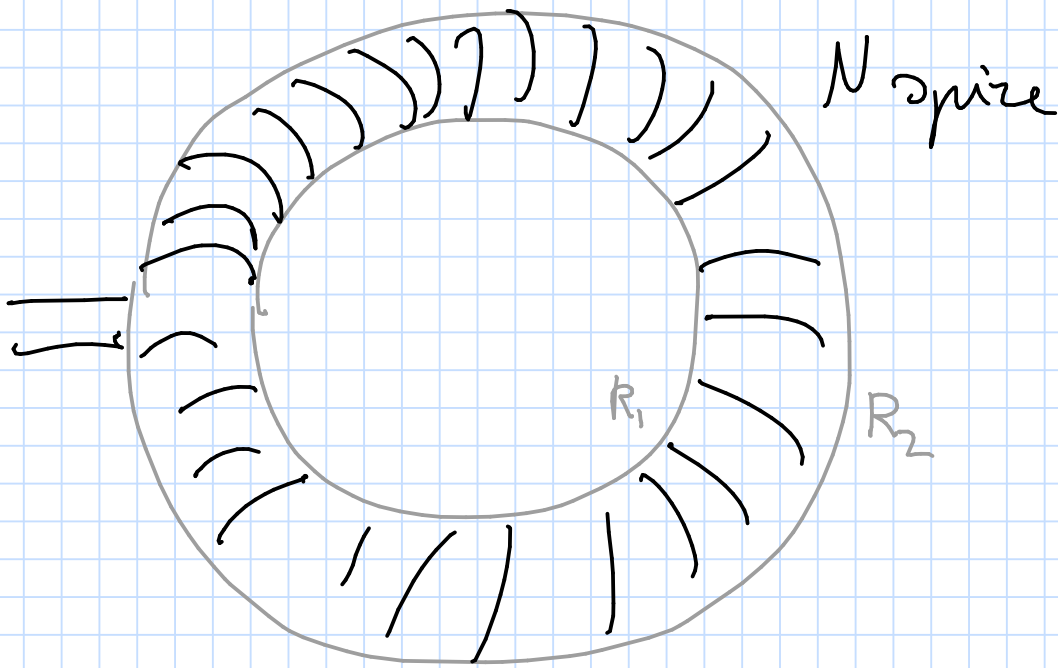
$$M = m \sin \theta \frac{\mu_0 i_s}{2\pi} \log\left(1 + \frac{d}{r}\right)$$

$$i_s = i / d$$

$$i = \frac{M 2\pi d}{m \sin \theta \mu_0 \log\left(1 + \frac{d}{r}\right)} = 0.269 \text{ A}$$



campo $\vec{B}$ in un toroide



spire μ_0
 B segue
 circonferenza

$$R_1 < r < R_2$$

Ampere

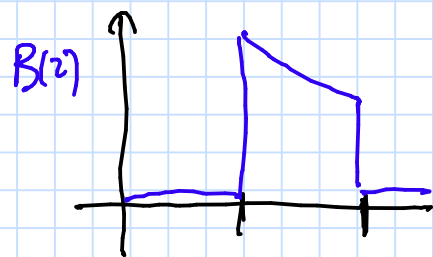
$$\oint_C \vec{B} \cdot d\vec{s} = B(r) 2\pi r$$

$$i_c = Ni$$

Ampere

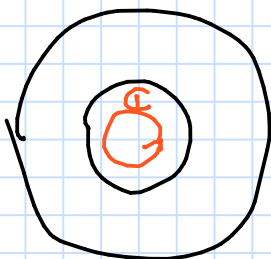
$$B(r) \cdot 2\pi r = \mu_0 Ni$$

$$B(r) = \frac{\mu_0 Ni}{2\pi r}$$



$$r < R_1$$

$$i_c = 0$$

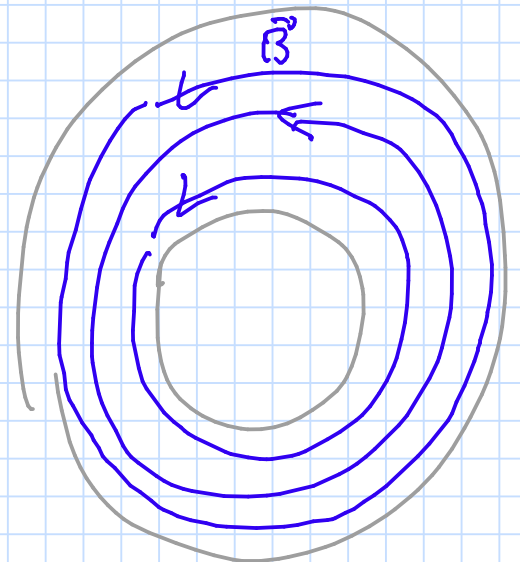


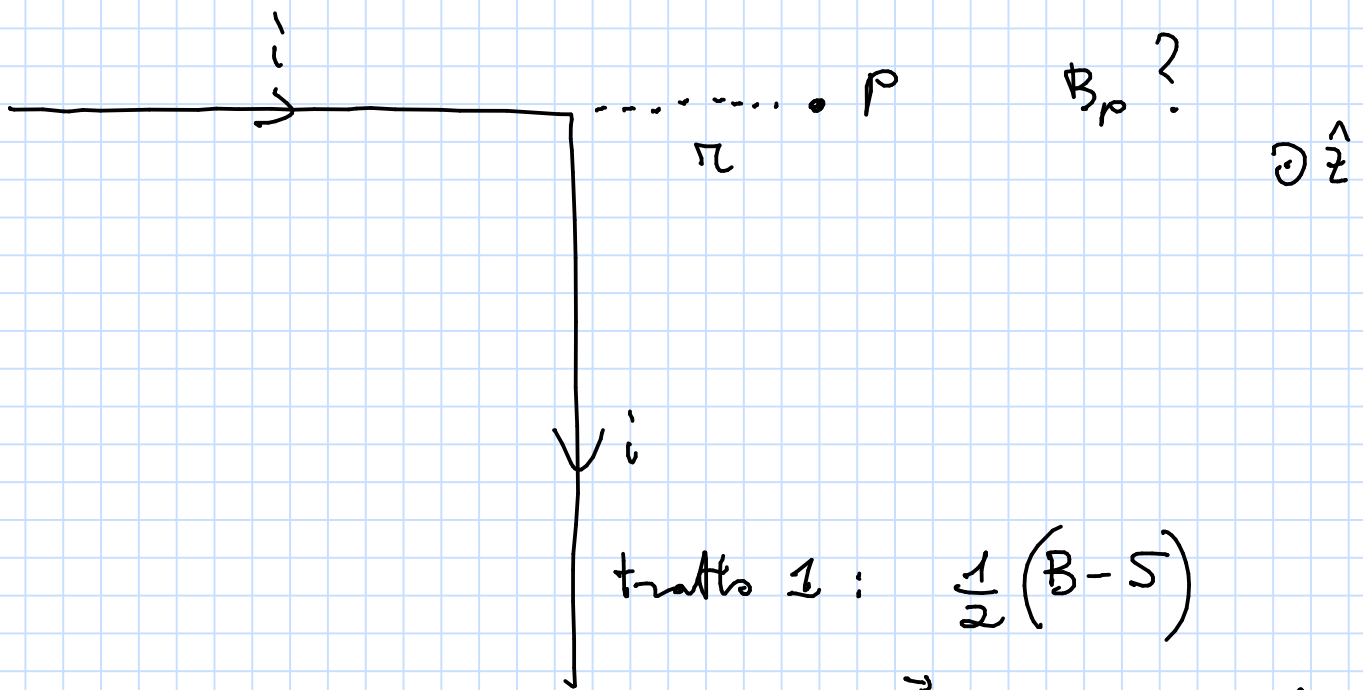
$$B(r) = 0$$

$$r > R_2$$

...

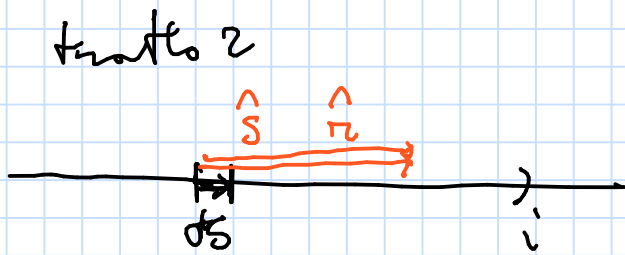
$$B(r) = 0$$





tratto 1: $\frac{1}{2} (B - S)$

$$\vec{B}_1 = \frac{1}{2} \frac{\mu_0 i}{2\pi r} \hat{z}$$

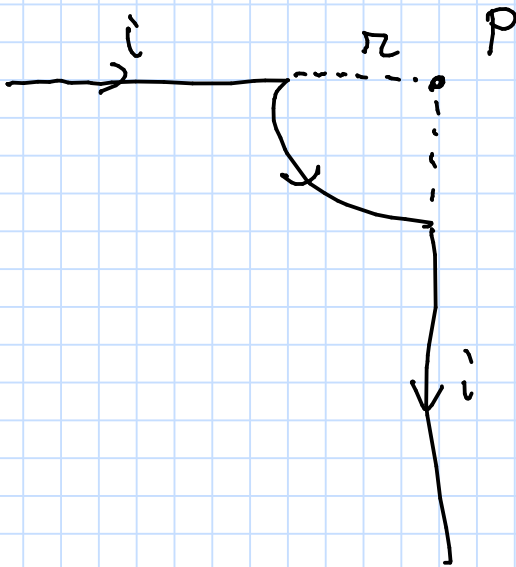


$dB_p \cdot P$

$$dB_p = \frac{\mu_0 i}{4\pi} \frac{ds}{r^2} \underbrace{\hat{s} \times \hat{r}}_{=0}$$

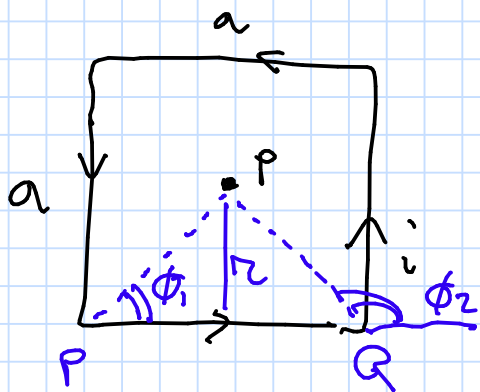
non si genera $\vec{B}$ nella dir. del filo

B totale solo da tratto 1



$$B_P = \frac{1}{4} B_{\text{spira}}$$

$$\approx \frac{1}{4} \frac{\mu_0 i}{2\pi r}$$



B_P ?

cogni lato:

$$r = \frac{a}{2}$$

$$B_{PQ}(r) = \frac{\mu_0 i}{4\pi r} (-\cos \phi_2 + \cos \phi_1)$$

$\cos 135^\circ \quad \cos 45^\circ$

1 lato

$$= \frac{\mu_0 i}{4\pi r} \left(-\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} \right)$$

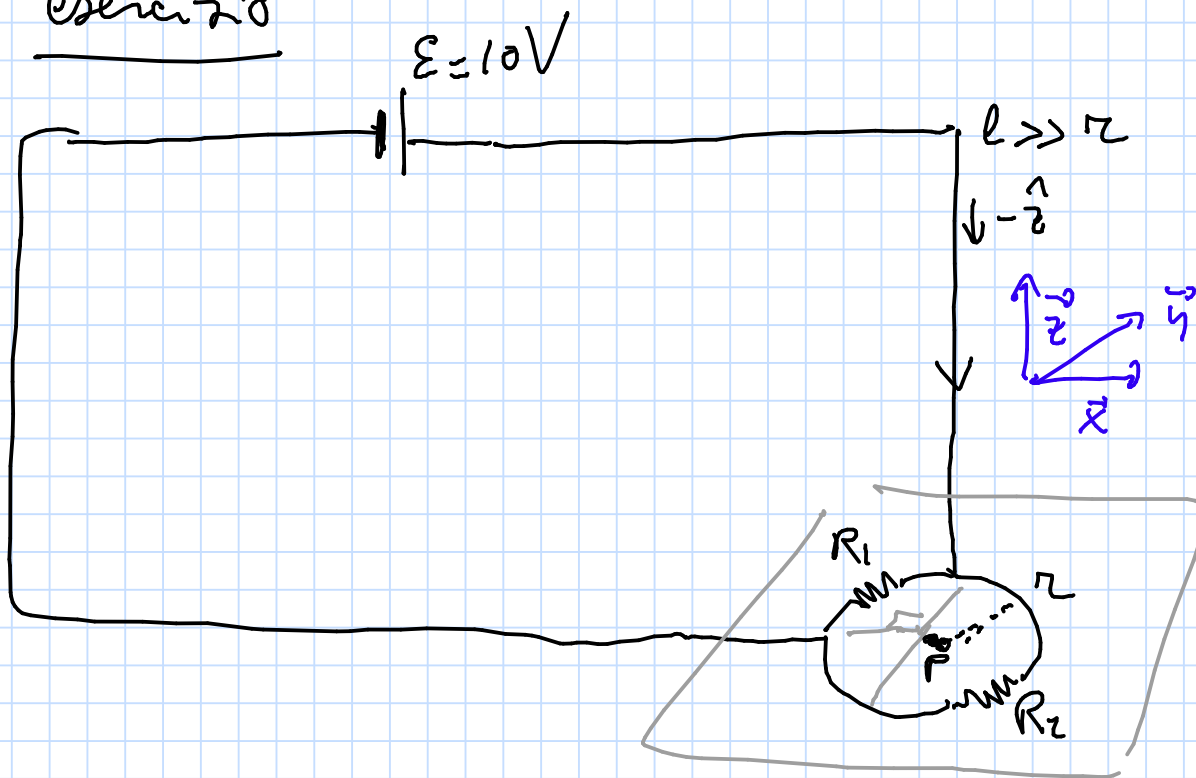
$$= \frac{\sqrt{2}}{4} \frac{\mu_0 i}{\pi r}$$

spira

$$B_P = 4 B_{\text{lato}}$$

$$= \sqrt{2} \frac{\mu_0 i}{\pi r} = 2\sqrt{2} \frac{\mu_0 i}{\pi a}$$

esercizio



$\varepsilon = 10V$

$$\varepsilon = 10V$$

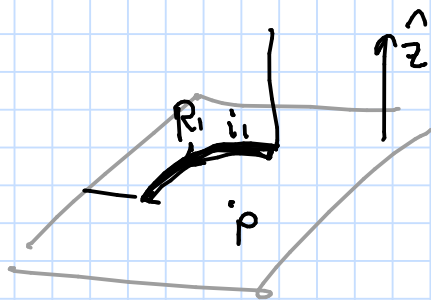
$$R_1 = 2\Omega$$

$$R_2 = 2.5\Omega$$

$$R_3 = 1\Omega$$

1) B_p (centro spira)?

i contributo

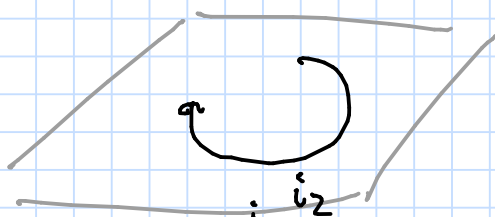


$$i_1 = \frac{\varepsilon}{R_1} = 5A$$

$$\frac{1}{4} B_{\text{spira}}$$

$$\vec{B}_1 = \frac{1}{4} \left(\frac{\mu_0 i_1}{2\pi} \right) \hat{z}$$

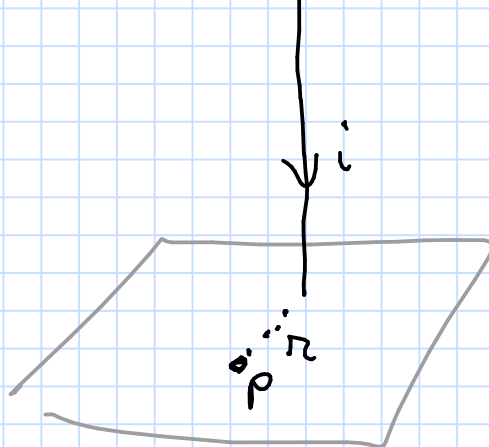
i_2 contributo



$$i_2 = \frac{\varepsilon}{R_2} = 4A$$

$$\vec{B}_2 = \frac{3}{4} \left(\frac{\mu_0 i_2}{2\pi} \right) (-\hat{z})$$

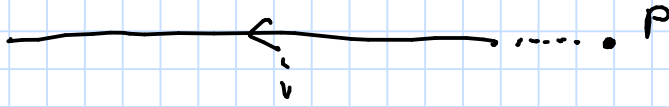
i_3 contributo



$$\vec{B}_3 = \frac{1}{2} \left(\frac{\mu_0 i_3}{2\pi} \right) (-\hat{x})$$

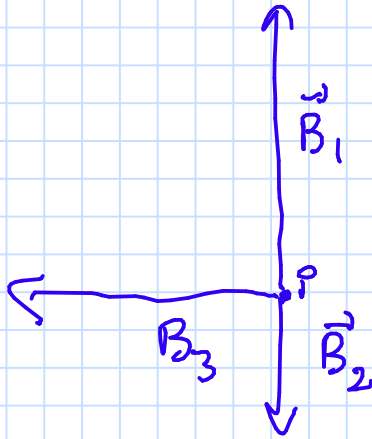
$$(i = i_1 + i_2)$$

4° contrib



$$\vec{B}_L = 0$$

$\vec{B}_L$



$$B_p = \sqrt{(B_1 - B_2)^2 + B_3^2}$$

$$= \sqrt{(110 \mu T)^2 + (90 \mu T)^2}$$

$$= 142 \mu T$$

2) $M = 6 \text{ mA m}^2$

U_x , U_y , U_z

$\rightarrow$ $\nearrow$ $\uparrow$

nel punto P?