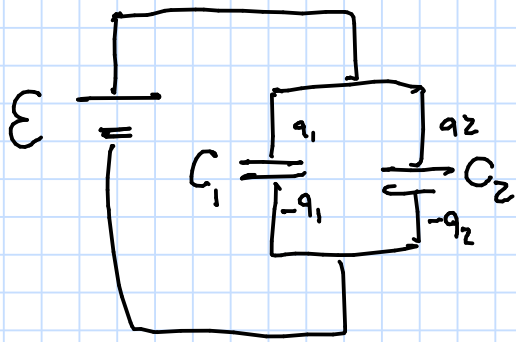
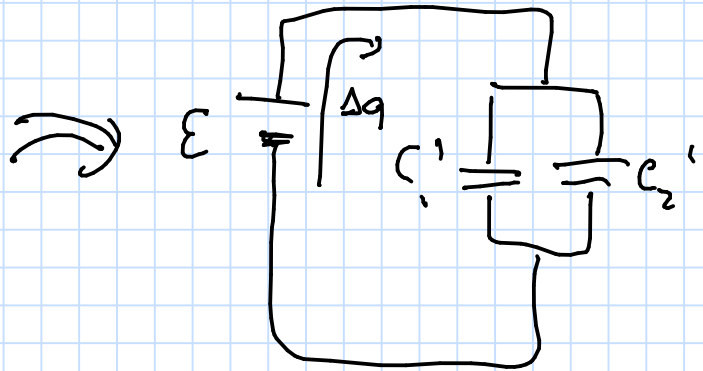


forniamo al caso ΔV costante



$$q = q_1 + q_2 = \varepsilon (C_1 + C_2) = \varepsilon C_{eq}$$



$$q' = q_1' + q_2' = \varepsilon C_{eq}'$$

se $q' \neq q \rightarrow$ carica $\Delta q = q' - q$
 fluide $\frac{+|\varepsilon|}{\Delta q}$

$$\Delta U^{gen} = -W^{gen} = -\Delta q \varepsilon$$

ΔU_e nei cond:

$$\Delta U_e = \frac{1}{2} C' \varepsilon^2 - \frac{1}{2} C \varepsilon^2 = \dots = \frac{1}{2} (q' - q) \varepsilon = \frac{1}{2} \Delta q \varepsilon$$

$$\Delta U^{tot} = \Delta U^{gen} + \Delta U_e = \left(-1 + \frac{1}{2}\right) \Delta q \varepsilon = \boxed{-\frac{1}{2} \Delta q \varepsilon} = -\Delta U_e$$

Lavoro fatto sul sistema

$$W = -\frac{1}{2} \Delta q \varepsilon$$

$$< 0$$

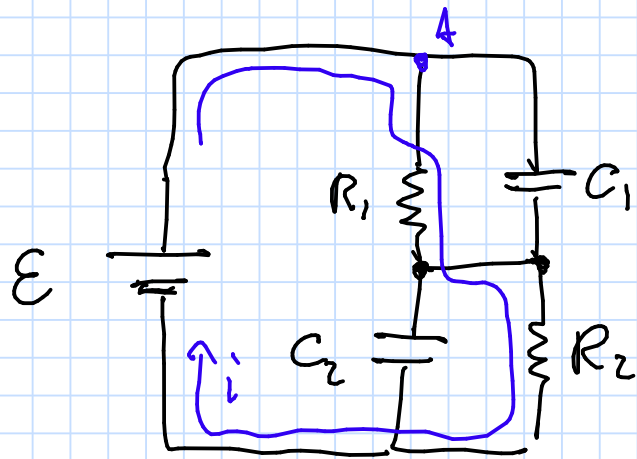
se $\Delta q > 0$
(generatore eroga...)

$$> 0$$

se $\Delta q < 0$
(generatore assorbe)

esercizio :

(simile 2N 5.11)



$$E = 120 \text{ V}$$

$$R_1 = 50 \, \Omega$$

$$R_2 = 70 \, \Omega$$

$$C_1 = 40 \, \mu\text{F}$$

$$C_2 = ?$$

a regime, nei C c'è energia $U_e = 0.295 \text{ J}$

1) $C_2 = ?$

a regime $i = 0$ nei C
 i la R_1 , R_2

$$i = \frac{E}{R_1 + R_2} = \frac{120 \text{ V}}{120 \, \Omega} = 1 \text{ A}$$

$$\Delta V_1 = i R_1 = 50 \text{ V}$$

$$\Delta V_2 = i R_2 = 70 \text{ V}$$

anche su C_1 c'è ΔV_1

$$U_1 = \frac{1}{2} C_1 \Delta V_1^2 = \frac{1}{2} 50^2 \cdot 40 \cdot 10^{-6} = 0.05 \text{ J}$$

$$U_2 = U_e - U_1 = 0.245 \text{ J}$$

$$C_2 = \frac{2 U_2}{\Delta V_2^2} = 10^{-4} \text{ F}$$



2) dielettrico in $C_1 \Rightarrow \Delta U_C = 0.2 \text{ J}$ nei cond.
 K del diel.?

$\Delta U_1 = \text{come prima}$
 $\Delta U_2 = \text{...}$ } varia $\rightarrow U_1$

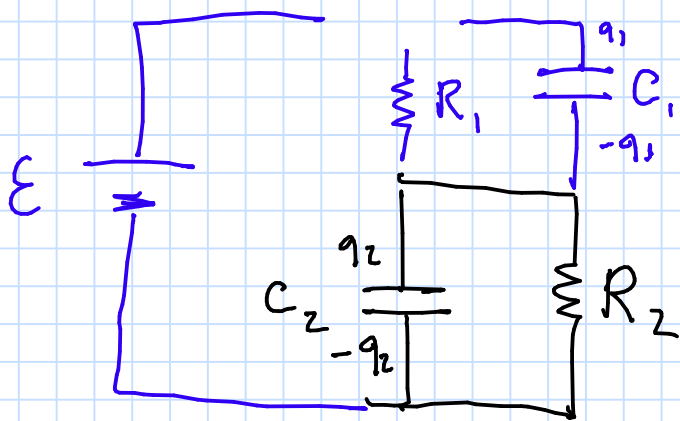
$$U_1 = \frac{1}{2} C_1 \Delta V_1^2 \longrightarrow \frac{1}{2} \underbrace{K C_1}_{C'} \Delta V_1^2 = K U_1$$

$$\Delta U_C = \Delta U_1 = (K-1) U_1$$

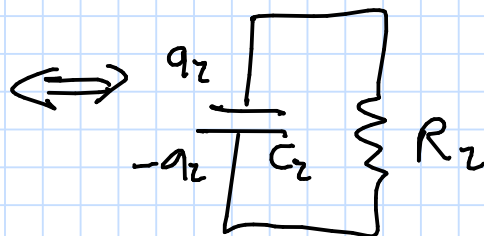
$$K-1 = \frac{0.2 \text{ J}}{0.05 \text{ J}} = 4 \rightarrow K = 5$$

c'è più ene in C_1 perché $C'_1 = K C_1 > C_1$
 $\Delta V \Rightarrow$ immagazzina più ene.

al tempo $t=0$ si rompe nodo A



$\Rightarrow$ rami blu
 influenti



3) dopo quanto tempo $q_2 \rightarrow q_2/4$
 t^*

$$R_2 \cdot C_2 = \tau = 70 \cdot 10^{-9} = 7 \text{ ns}$$

$$q(t) = q_2 e^{-t/\tau}$$


$$q_2 e^{-t^*/\tau} = \frac{q_2}{4}$$

$$-\frac{t^*}{\tau} = \ln \frac{1}{4}$$

$$t^* = \tau \ln 4 = 1.38 \tau = 1.7 \text{ ms}$$

4) energia nel sistema per $t \rightarrow \infty$

o $U_1 = (0.05 \text{ J}) \times$ (con dielettrico in C_1)
 $= 0.25 \text{ J}$

5) energia dissipata in R_2 fino t^* ?

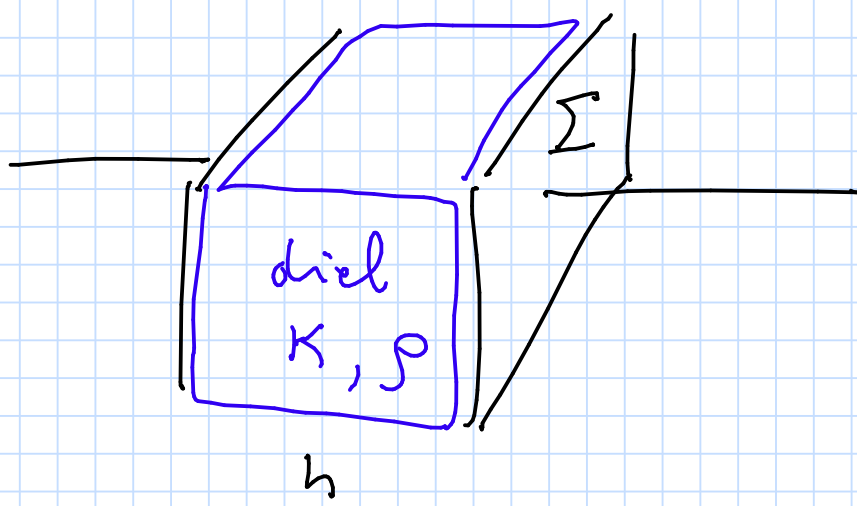
$$W^{\text{diss}} = \int_0^{t^*} P^{\text{diss}} dt = \int_0^{t^*} R_2 [i(t)]^2 dt$$

$$i(t) = \frac{dq}{dt} = \frac{d}{dt} (q_2 e^{-t/\tau}) = q_2 \left(-\frac{1}{\tau}\right) e^{-t/\tau}$$

segno di i è irrilevante, i^2 in W^{diss}
 $W^{\text{diss}} \geq 0$ sempre

$$W^{\text{diss}} = \frac{R_2}{\tau^2} q_2^2 \int_0^{t^*} e^{-2t/\tau} dt$$

$$= \frac{R_2}{\tau^2} q_2^2 \left(-\frac{\tau}{2}\right) e^{-2t/\tau} \Big|_{t=0}^{t=t^*} = \frac{1}{2C_2} q_2^2 \left[1 - e^{-2t^*/\tau}\right]$$



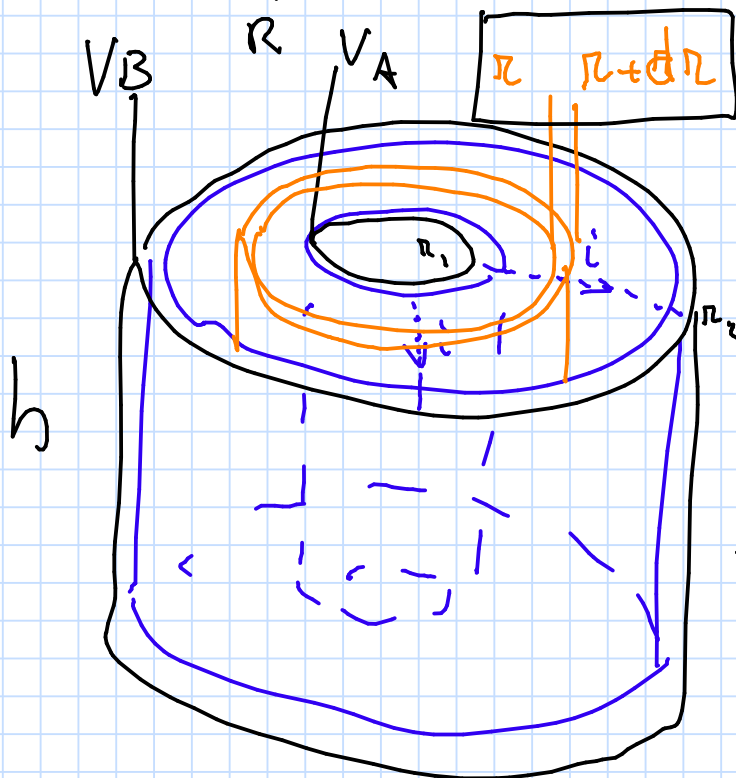
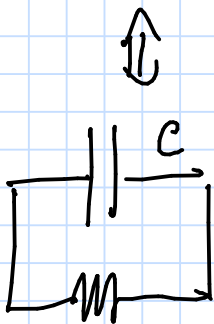
$$R = \rho \frac{h}{\Sigma}$$

$$C = \kappa \epsilon_0 \frac{\Sigma}{h}$$

$$Rc = \rho \epsilon_0 \kappa$$

$$= \rho \epsilon = \tau$$

vale in generale?



Resistenza?

$$R = \int_{r_1}^{r_2} dR(r) = \int_{r_1}^{r_2} \frac{\rho}{\Sigma(r)} dr$$

$$\Sigma(r) = 2\pi r h$$

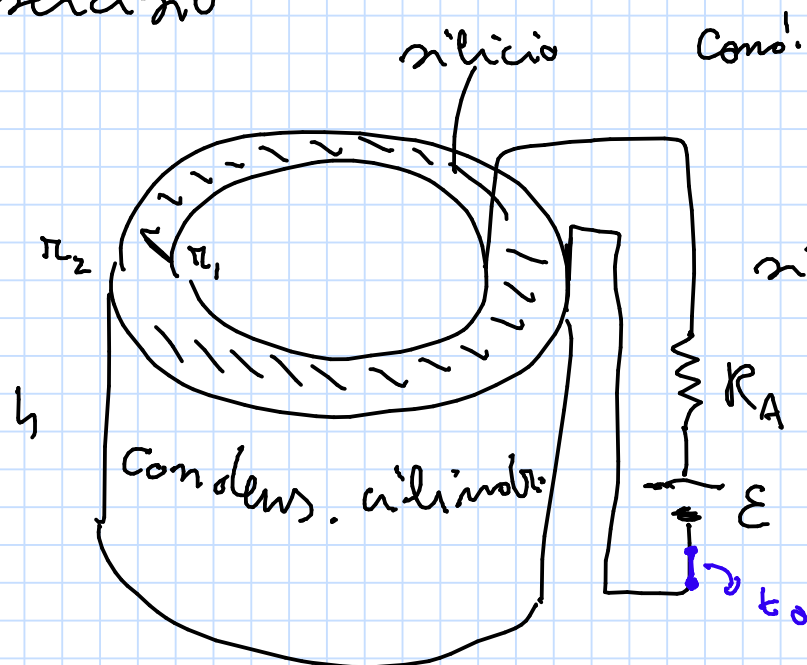
$$= \int_{r_1}^{r_2} \frac{\rho}{2\pi h} \frac{1}{r} dr$$

$$= \frac{\rho}{2\pi h} \log \frac{r_2}{r_1}$$

$$C_{cil} = \kappa \frac{2\pi \epsilon_0 h}{\log(r_2/r_1)}$$

$$C_{cil} R_{cil} = \rho \epsilon_0 \kappa = \rho \epsilon$$

esercizio



Cond. cil.

$$r_1 = 25 \text{ cm}$$

$$r_2 = 27 \text{ cm}$$

$$h = 1.4 \text{ m}$$

silicio: conduttività
 $\sigma = 2.52 \cdot 10^{-4} (\Omega \text{ m})^{-1}$

$$K = 11.7$$

$$R_A = 14 \Omega$$

$$E = 5 \text{ V}$$

in quanto tempo si raggiunge $\tilde{\Delta V} = 0.2 \text{ V}$ nel C. cilindrico, se a $t=0$ si apre l'interruttore e quindi scollegiamo il gen.

$$C_{\text{cil}} = C = K \frac{2\pi \epsilon_0 h}{\ln(r_2/r_1)} = 11.8 \text{ nF}$$

interno di cond. = resistenza cilindrica

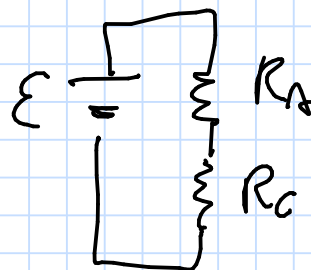
$$R_C = \int_{r_1}^{r_2} dR(r) = \dots = \rho \epsilon_0 K \frac{1}{C} = 34.7 \Omega$$

$$\text{con } \rho = \frac{1}{\sigma} = 3968 \Omega \text{ m}$$

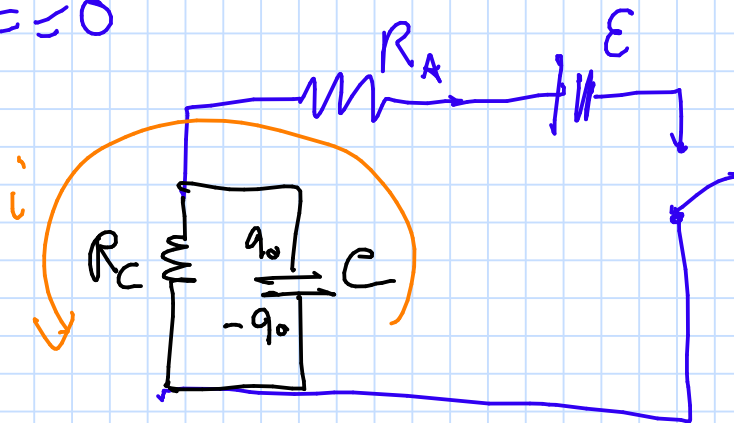
$$\tau = R_C C = \epsilon \rho = 0.41 \mu\text{s}$$

prima di $t=0$, Ohm,

$$\Delta V_0 = \frac{R_C}{R_C + R_A} E = 3.56 \text{ V}$$



$$t=0$$



i con i
non costante

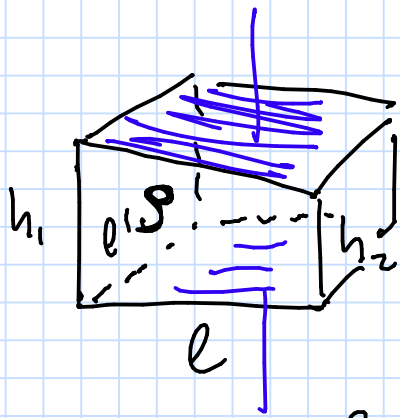
$$V(t) = \Delta V_0 e^{-t/\tau}$$

$$V(\tilde{t}) = \tilde{\Delta V} = 0.2V$$

$$\frac{\tilde{\Delta V}}{\Delta V_0} = e^{-\tilde{t}/\tau} \Rightarrow \tilde{t} = \left(\log \frac{\Delta V_0}{\tilde{\Delta V}} \right) \tau =$$

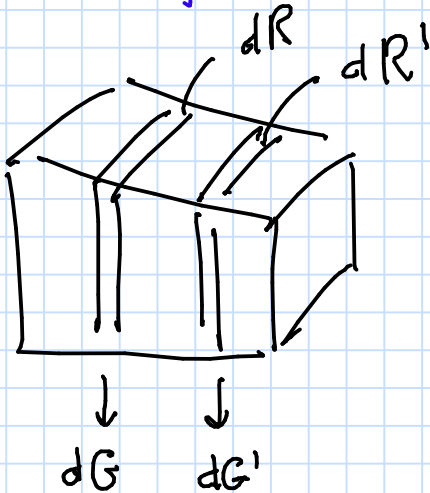
$$\log 17.81 \tau$$

$$= 1.18 \mu s$$



materiale: resistività ρ

$$h_1 \approx h_2$$



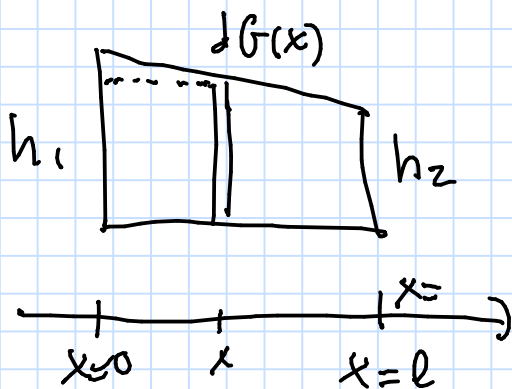
ricorda che è l'inverso delle resist. che si sommano se sono in parallelo

$$G = \frac{1}{R}$$

conduttanza
Siemens = $\frac{1}{\Omega}$

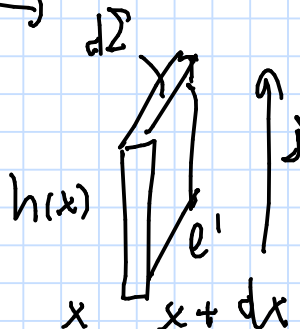
$$S = \frac{1}{\Omega}$$

$$G = \int dG$$



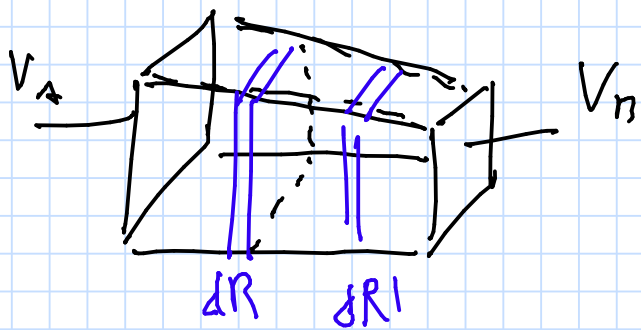
$$h(x) = h_1 + \frac{x}{l} (h_2 - h_1)$$

$$dG(x) = \sigma \frac{dx}{h(x)}$$



$$G = \int_{x=0}^{x=l} \sigma \frac{dx}{h_1 + \frac{x}{l} (h_2 - h_1)} = \dots$$

S



in serie, $dR, dR_1, \dots$

$$R = \int dR = \dots \text{ conto simile}$$

$$dR = \rho \frac{dx}{\bar{I}(x)}$$



esempio 6.1: $\bar{v}$ media degli elettroni in un metallo, considerandolo gas ideale

Equipartizione: ogni g.d.l. quadratico ha energia $\frac{1}{2} k_B T$

elettrone: libero, $E_k = \frac{1}{2} m_e \bar{v}^2 = \frac{3}{2} k_B T$
 v_x, v_y, v_z

$$m = 9.1 \cdot 10^{-31} \text{ kg}$$

$$k_B T \approx 4 \cdot 10^{-21} \text{ J} \approx 0.025 \text{ eV}$$

$$\frac{3}{2} k_B T \approx 6 \cdot 10^{-21} \text{ J} \approx 0.038 \text{ eV} = E_k$$

$$\bar{v} = \sqrt{\frac{2 E_k}{m_e}} \approx 1.16 \cdot 10^5 \text{ m/s}$$

velocità di deriva $v_d \ll \bar{v}$



$$l = 4 \text{ mm}$$

$$i = 8 \text{ A}$$

$$v_d = ?$$

dens. di corrente $j = \frac{i}{S} = \frac{8 \text{ A}}{4 \cdot 10^{-6} \text{ m}^2} = 2 \cdot 10^6 \text{ A/m}^2$

n° di e^- per m^3

$$n = 8.49 \cdot 10^{28} \text{ elettroni/m}^3$$

$$j = n (-e) v_d$$

$$\lambda_d = \frac{h}{m_e v} = 1.47 \cdot 10^{-4} \text{ m} = 0.147 \text{ mm}$$

$$\lambda_d \approx 10^{-10} \text{ m}$$