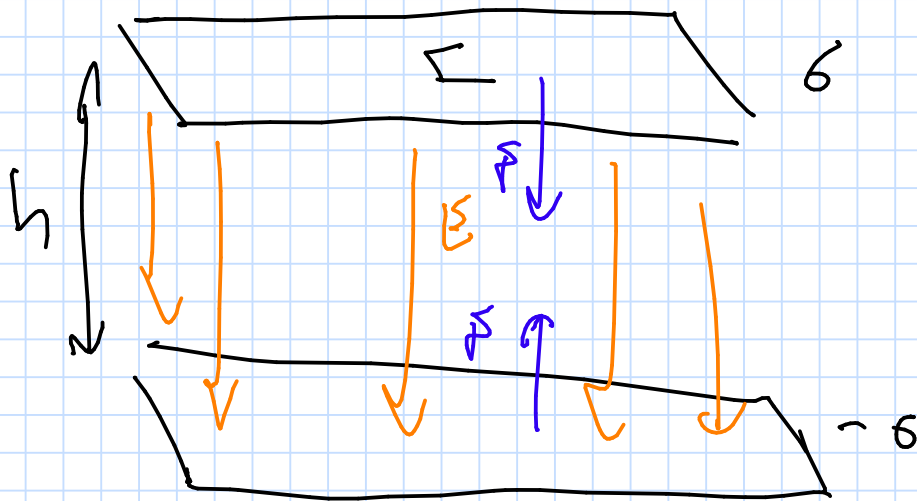


Pressione elettrostatica



carica
costante
(σ fissa)

$$E = \frac{\sigma}{\epsilon_0}$$

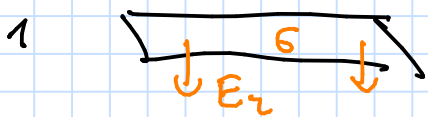
ene
$$U_e = \left(\frac{1}{2} \epsilon_0 E^2 \right) \cdot \Sigma h$$

$$= \frac{1}{2} \frac{\sigma^2}{\epsilon_0} \Sigma h$$

$$F = \left| -\frac{\partial U}{\partial h} \right| = \frac{1}{2} \frac{\sigma^2}{\epsilon_0} \cdot \Sigma$$

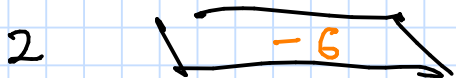
"pressione" elettrostatica
 $\Delta \vec{E}$ attira

in alternativa



E_z generata da $-\sigma$

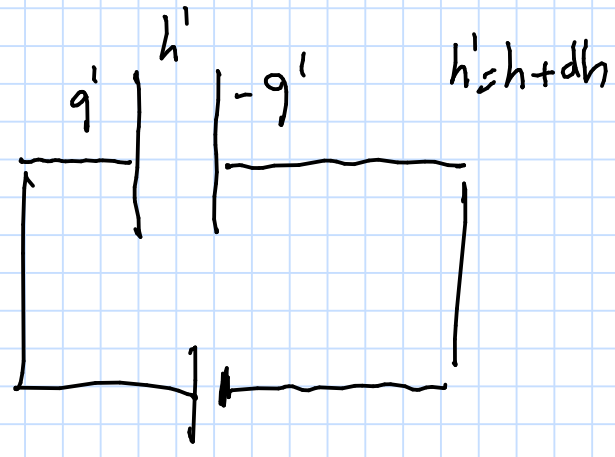
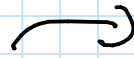
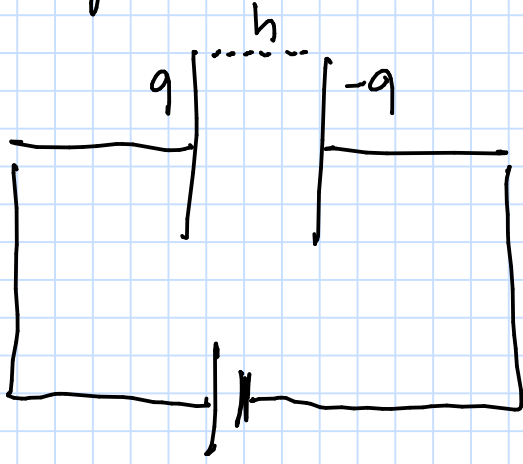
$$E_z = \left| \frac{-\sigma}{2\epsilon_0} \right| = \frac{\sigma}{2\epsilon_0}$$



tutta $q_1 = \sigma \Sigma$ è attratta
da E_z

$$F_{z1} = q_1 E_z = \frac{\sigma}{2\epsilon_0} (\Sigma \sigma) = \left(\frac{1}{2} \frac{\sigma^2}{\epsilon_0} \right) \Sigma$$

a potenziale costante (V fisso)



⚠ il sistema non è isolato $\Rightarrow U_{gen}$

nel condens. $U_e = \frac{1}{2} C V^2 \longrightarrow U'_e = \frac{1}{2} C' V^2$

c. piano $C = \frac{\sum \epsilon_0}{h}$

$$C' - C = dC = d\left(\frac{\epsilon_0 \sum}{h}\right) = -\frac{\epsilon_0 \sum}{h^2} dh$$

$$U'_e - U_e = \frac{1}{2} (C' - C) V^2 = -\frac{1}{2} \epsilon_0 \frac{\sum dh}{h^2} V^2 \quad \left| \begin{array}{l} E = \frac{V}{h} \end{array} \right.$$

$$= -\frac{1}{2} \epsilon_0 E^2 (dh V)$$

$dh < 0$ nel disegno $\Rightarrow U'_e > U_e$ nel condensatore

generatore:



La carica $q' - q$ è portata da $V=0$ a V

Lavoro fatto dal gen: $W^{gen} = (q' - q) V$

o $W^{gen} > 0$ corrisponde $U^{gen} < 0$

$$W^{gen} = - U^{gen} =$$

$$dW^{gen} = - dU^{gen} = dq V = (c' - c) V^2$$

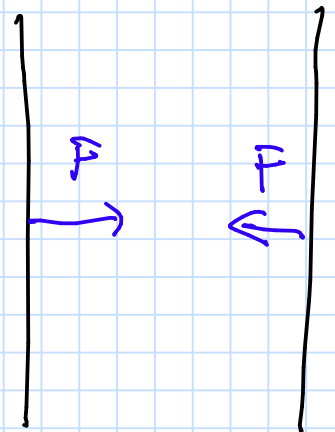
in totale

$$dU^{tot} = dU^{gen} + dU_e = -(c' - c) V^2 + \frac{1}{2} (c' - c) V^2 = - dU_e$$

$$dU^{tot} < 0$$

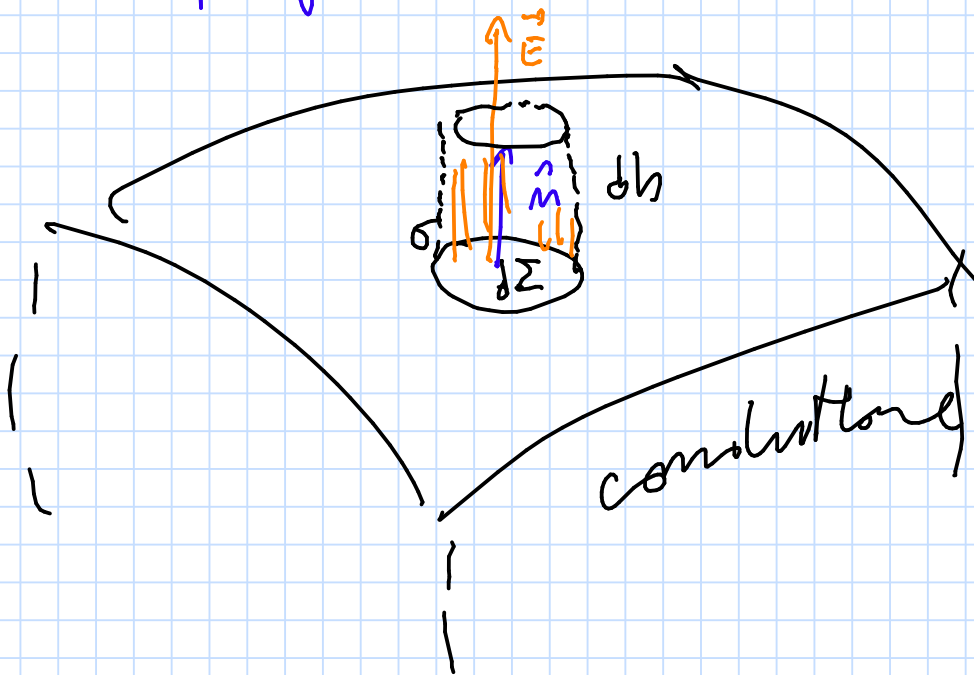
$$\left| \frac{dU^{tot}}{dh} \right| = \left(\frac{1}{2} \epsilon_0 E^2 \right) \Sigma = \left(\frac{1}{2} \frac{\sigma^2}{\epsilon_0} \right) \Sigma = P$$

p. elettr.

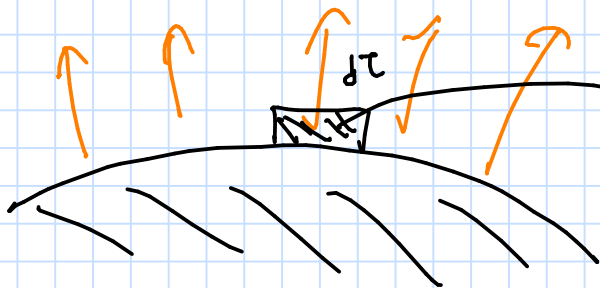


$\Rightarrow$ attrazione fra le piastre $dU^{tot} < 0$

su superficie di conduttore



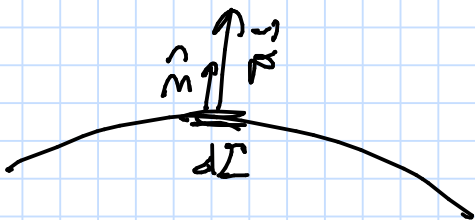
$$E = \frac{\sigma}{\epsilon_0}$$



ridotto U_e di
 $U_e \cdot d\tau$

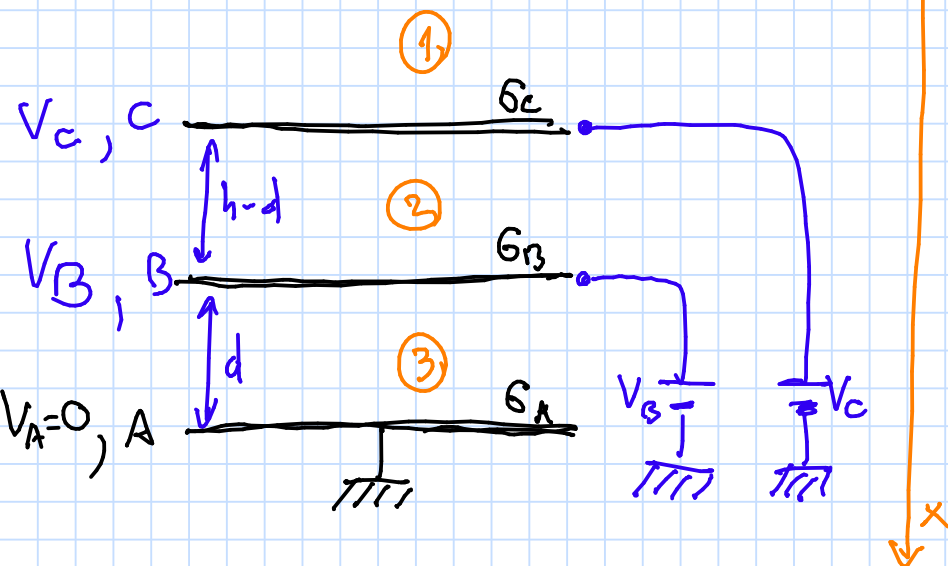
$$dU_e = - \underbrace{\left(\frac{1}{2} \epsilon_0 E^2 \right)}_{\text{densità } u_e} (d\Sigma dh)$$

$$dU_e < 0 \implies \vec{P} \parallel \hat{n}$$



$$\vec{P} = \left| \frac{dU_e}{dh} \right| \hat{n} = \underbrace{\left(\frac{1}{2} \epsilon_0 E^2 \right)}_{p. \text{ elettro.}} d\Sigma \hat{n}$$

M.N.V. 2.12



1) $\epsilon_A, \epsilon_B, \epsilon_C = ?$

2) V_C t.c. $P(B)=0$

① sistema neutro per avere $B=0$ in ①

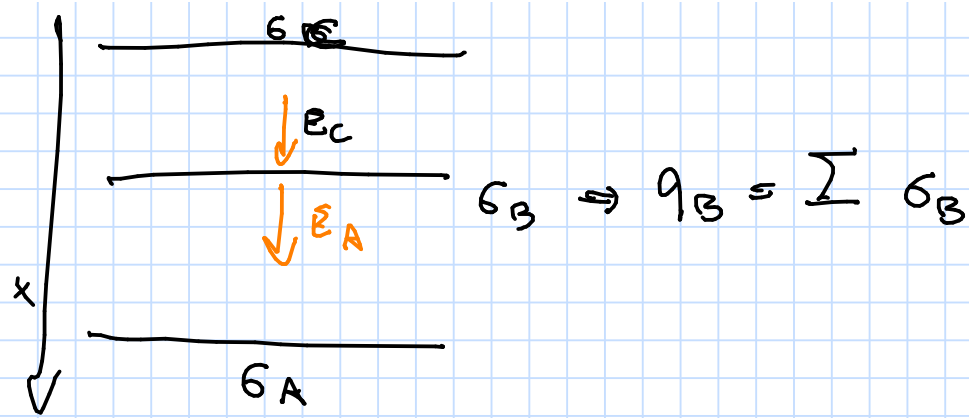
① $-\vec{E}_C - \vec{E}_B - \vec{E}_A = 0 \Rightarrow \frac{1}{2\epsilon_0} (\epsilon_C + \epsilon_B + \epsilon_A) \leq 0$

② $\vec{E}_C - \vec{E}_B - \vec{E}_A = \frac{V_C - V_B}{h-d} \Rightarrow \frac{1}{2\epsilon_0} (\epsilon_C - \epsilon_B - \epsilon_A) = \frac{V_C - V_B}{h-d}$

③ $\vec{E}_C + \vec{E}_B - \vec{E}_A = \frac{V_B}{d} \Rightarrow \frac{1}{2\epsilon_0} (\epsilon_C + \epsilon_B - \epsilon_A) = \frac{V_B}{d}$

... $\begin{cases} \frac{\epsilon_A}{\epsilon_0} = -\frac{V_B}{d} \\ \frac{\epsilon_B}{\epsilon_0} = -\frac{V_C - V_B}{h-d} + \frac{V_B}{d} \end{cases}$

$\frac{\epsilon_C}{\epsilon_0} = \frac{V_C - V_B}{h-d}$



dir. x: nel piano B, $E_c - E_A$

$$\begin{aligned}
 F = q_B E(B) &= q_B (E_c(B) - E_A(B)) \\
 &= \int \sigma_B \frac{1}{2\epsilon_0} (\sigma_c - \sigma_A) \\
 &= \frac{1}{2} \epsilon_0 \int \left(\frac{V_c - V_B}{h-d} + \frac{V_B}{d} \right) \left(\frac{V_B}{d} - \frac{V_c - V_B}{h-d} \right) \\
 &= \frac{1}{2} \left(\frac{V_B^2}{d^2} - \frac{(V_c - V_B)^2}{(h-d)^2} \right) \epsilon_0 \int
 \end{aligned}$$

se c'è anche gravità: $F_g = m g$

$$F^{tot} = F_g + F = 0 \quad \text{in equlib.}$$

risolvendo per V_c :

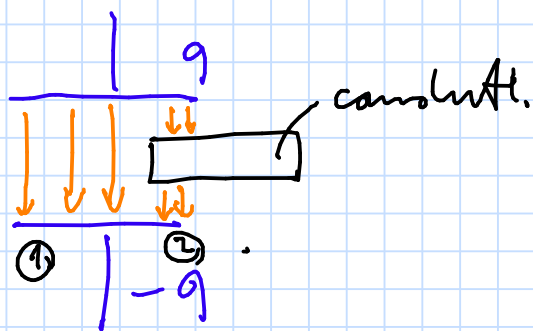
$$V_c = V_B + (h-d) \left(\frac{V_B^2}{d^2} + \frac{2mg}{\epsilon_0 \Sigma} \right)^{1/2}$$

$$\left\{ \begin{array}{l} MNV \\ V_B = V \\ V_c = V_0 \end{array} \right.$$

$$\sigma_c = 7.5 \frac{\mu C}{m^2} \rightarrow 23900 V$$

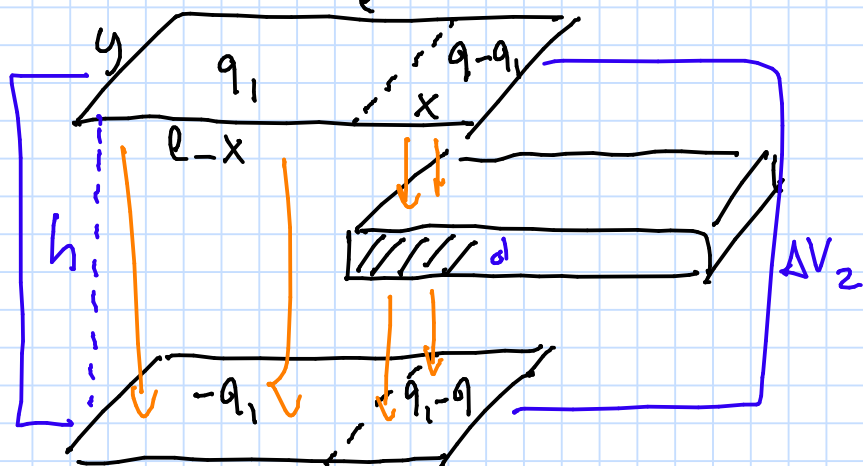
$$\sigma_B = -1.3 \frac{\mu C}{m^2} \xrightarrow{\downarrow mg} 3000 V$$

$$\sigma_A = -6.2 \frac{\mu C}{m^2} \rightarrow 0 V$$



$$U_e(x)$$

stema ΔV ①, ②



$$\Delta V_1 = E_1 \cdot h$$

$$\Delta V_2 = E_2 (h - d)$$

$$\Delta V_1 = \Delta V_2 \quad (*)$$

vincola q_1

$$E = \frac{\sigma}{\epsilon_0}$$

$$E_1 = \frac{q_1}{(l-x)y \epsilon_0}$$

$$E_2 = \frac{q - q_1}{x y \epsilon_0}$$

$$\frac{q_1}{\epsilon_0 (l-x)y} h = \frac{q - q_1}{\epsilon_0 x y} (h - d)$$

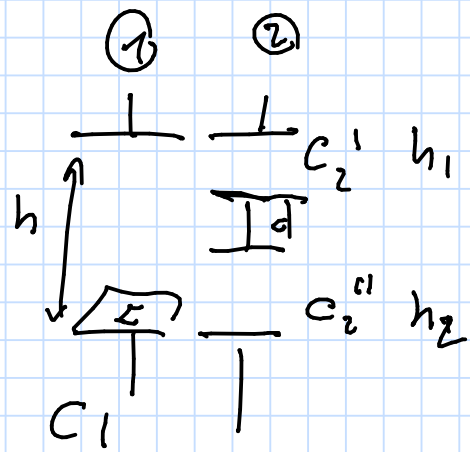
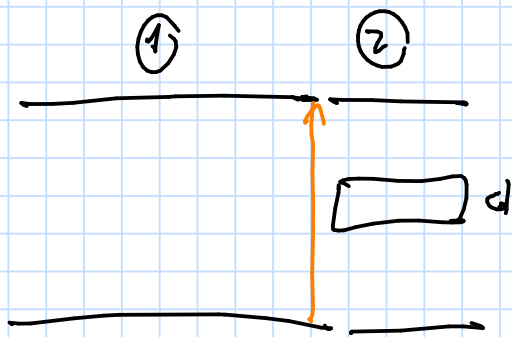
$$q_1(x) = \frac{(l-x)(h-d)}{(l-x)(h-d) + xh} q$$

$U_e(x)$: decrease con x

$$1^\circ \text{ modo} \quad U_e^1 \cdot \tau^{(1)} + U_e^2 \cdot \tau^{(2)}$$

$$\frac{\epsilon_0}{2} E_1^2 \cdot h(l-x)y + \frac{\epsilon_0}{2} E_2^2 \cdot x y (h-d)$$

2° modo



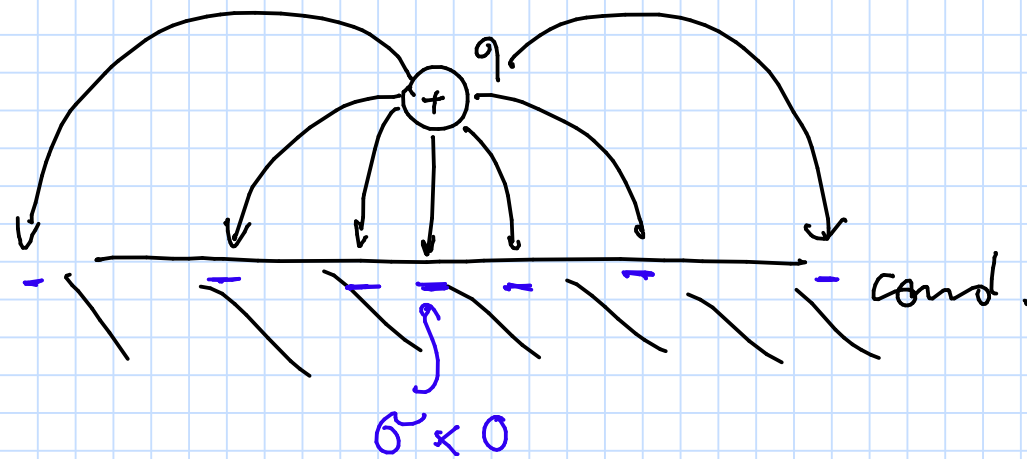
$$C_1 = \frac{\epsilon_0 \epsilon (l-x)y}{h}$$

$$h_1 + h_2 = h - d$$

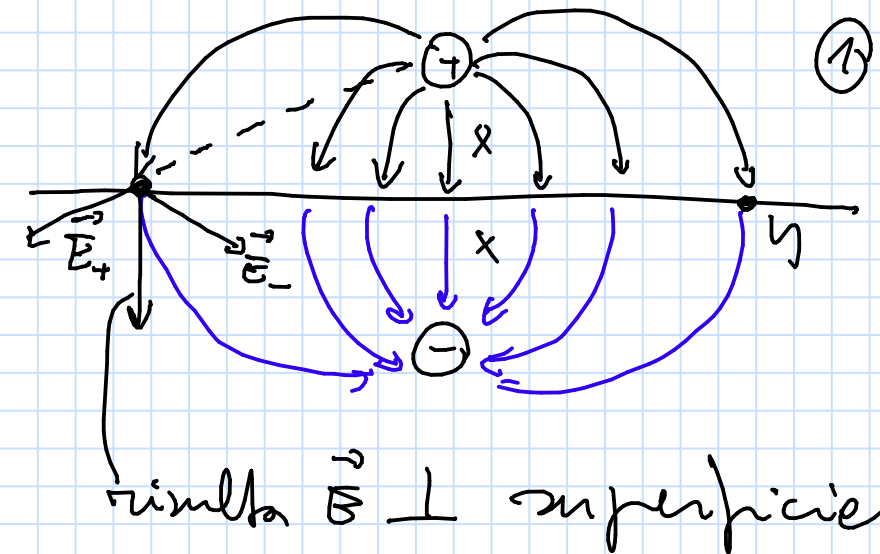
$$C_2 = \frac{1}{\frac{1}{C_2'} + \frac{1}{C_2''}} = \frac{\epsilon_0 \epsilon y}{h-d}$$

$$U_e(x) = \frac{1}{2} \frac{q_1^2}{C_1} + \frac{1}{2} \frac{(q - q_1)^2}{C_2}$$

Carica immagine



σ tale che
 $\vec{E} \perp \text{superf.}$



① nella zona ①
 stesso $\vec{E}$ del
 caso con
 conduttore

dato $\vec{E}$ in sup. come somma $\vec{E} = \vec{E}_+ + \vec{E}_-$
 Coulomb $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$ in sup. conduttrice

ricaviamo $\sigma = \epsilon_0 E = - \frac{q \times}{2\pi(x^2 + y^2)^{3/2}}$

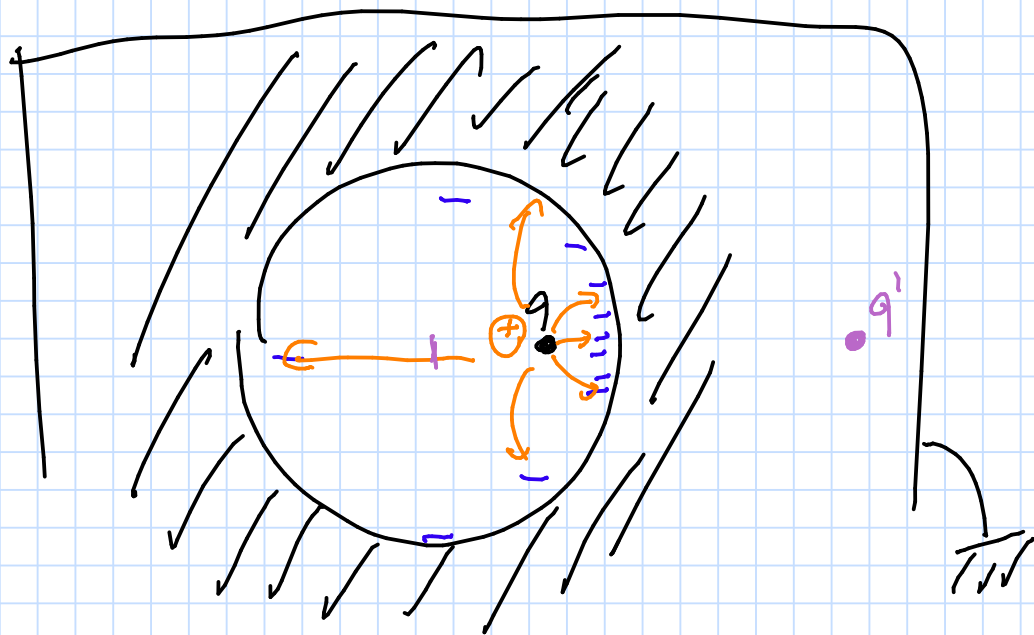
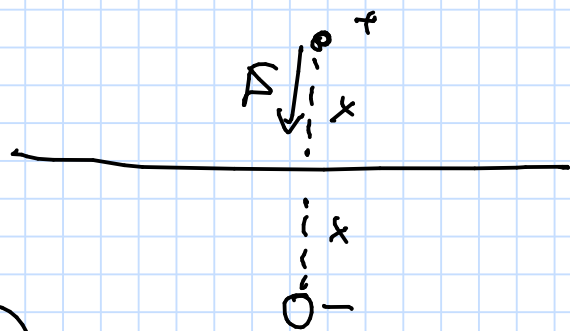
$$\int_{\text{piano}} \sigma = -q$$

carica immagine genera stesso effetto
 di distrib $-\sigma$ in piano

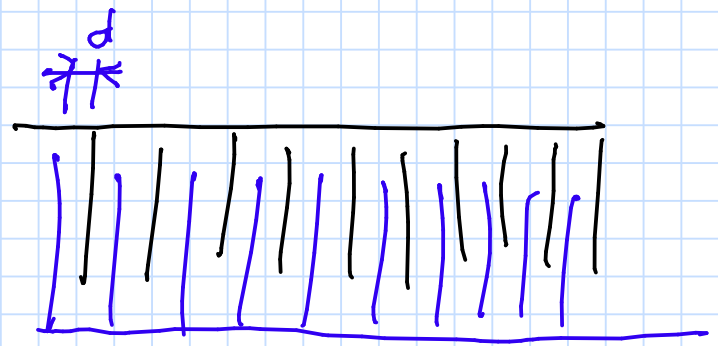
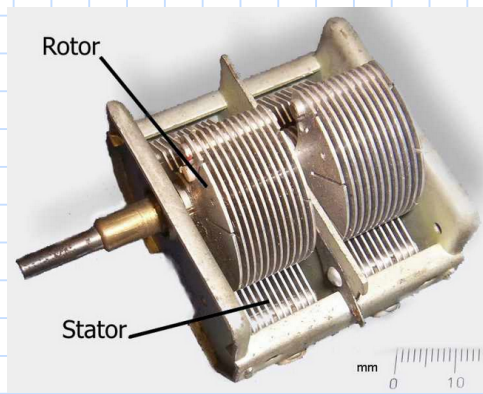
F attrattiva $q / \text{pian} =$

$P \quad q / -q$

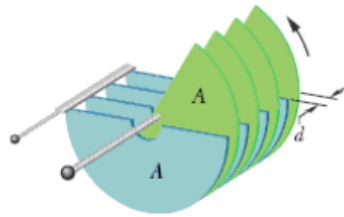
$$F = \frac{q^2}{4\pi\epsilon_0 (2x)^2}$$



per
esercizio

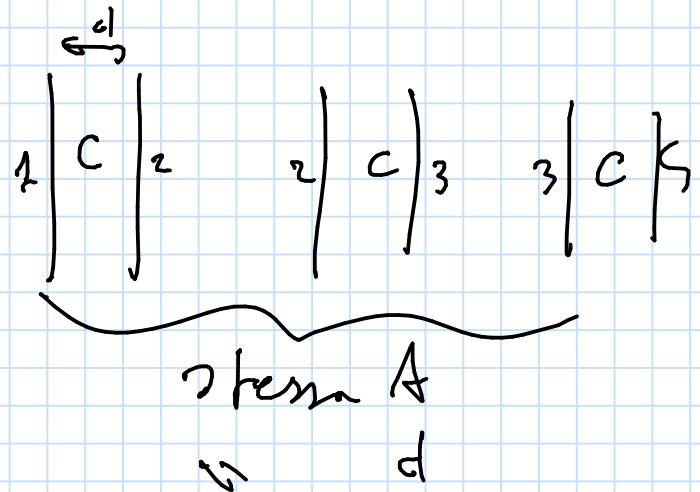
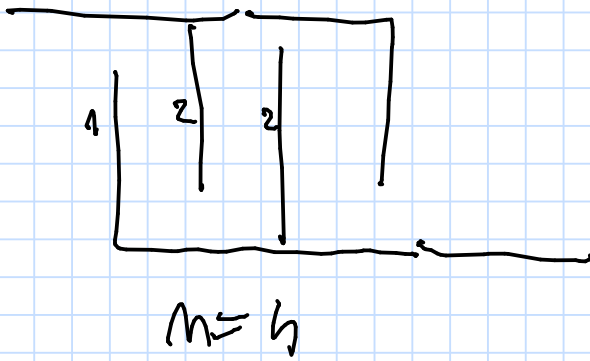


The figure shows a variable "air gap" capacitor of the type used in manually tuned radios. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of $n = 8$ plates of alternating polarity, each plate having area $A = 1.53 \text{ cm}^2$ and separated from adjacent plates by distance $d = 3.50 \text{ mm}$. What is the maximum capacitance of the device?



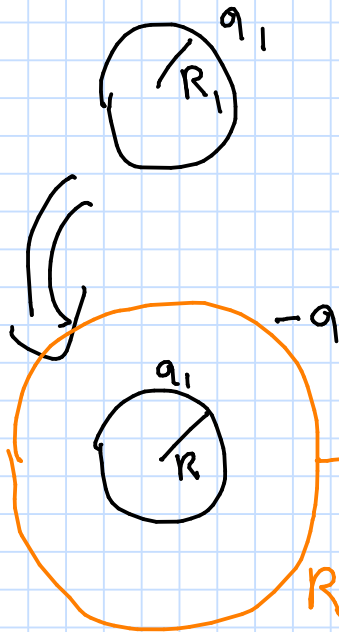
n armature

$$n/2 + n/2$$



$$C_{\text{tot}} = (n-1)C = (n-1) \frac{A \epsilon_0}{d}$$

sfera concentriche + sfera esterna costruttiva



$$U_e = \frac{1}{2} \frac{q_1^2}{C_{\text{gen}}} = \frac{1}{8\pi\epsilon_0 R_1} q_1^2$$

come cambia U_e ?

$$V_2 = V_3$$

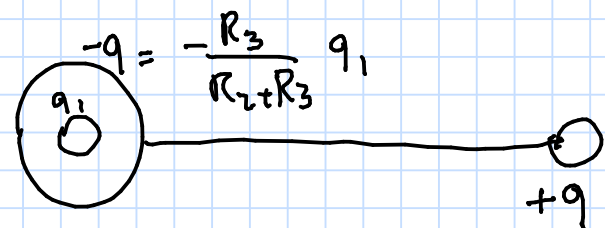
$$R_3 \text{ \& } R_2 \text{ neutri : } q_2 = -q \Rightarrow q_3 = +q$$

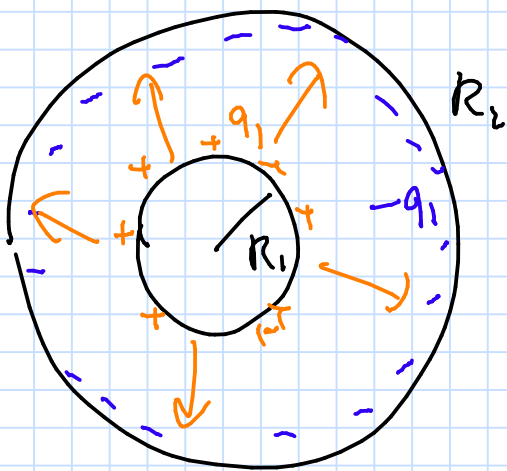
$$V_2 = \frac{q_1 - q}{4\pi\epsilon_0 R_2} = V_3 = \frac{q}{4\pi\epsilon_0 R_3}$$

|
unic
costruttive

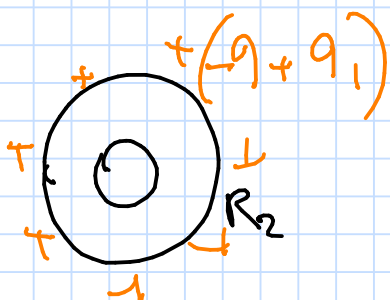
$$R_3 q_1 - R_3 q = R_2 q$$

$$q = \frac{R_3}{R_2 + R_3} q_1$$





$$U_e^{(1)} = \frac{q_1^2}{2 C_{R_1 R_2}}$$



$$U_e^{(2)} = \frac{(q_1 - q)^2}{2 C_{R_2}}$$

$$C_{R_2} = 4\pi\epsilon_0 R_2$$

$$R_3 \odot q$$

$$U_e^{(3)} = \frac{1}{2} \frac{q^2}{C_{R_3}}$$

$$U_e^{TOT} = U_e^{(1)} + U_e^{(2)} + U_e^{(3)} + \cancel{U_{int}}$$

$$U_{int} \sim \frac{1}{d}$$

$$U_{int} = \frac{(-q + q_1) q}{4\pi\epsilon_0 d}$$

