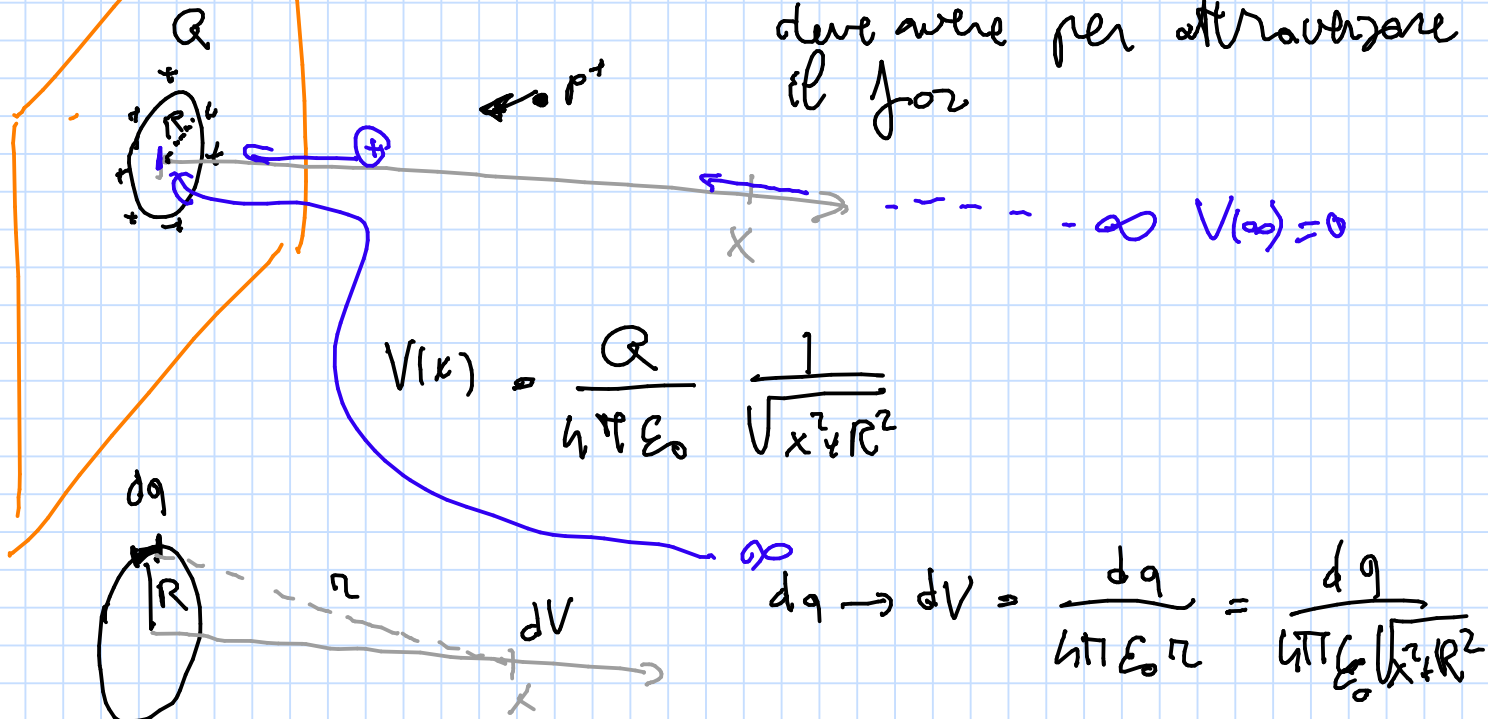


foro, carica Q

energia che particella con $q = e$
deve avere per attraversare
il foro



$$V = \underbrace{\int dV}_{\text{tutti i contributi}} = \frac{1}{4\pi\epsilon_0 \sqrt{x^2 + R^2}} \cdot \underbrace{\int dq}_Q$$

$$V(\infty) = 0 \rightarrow U_e(\infty) = 0$$

$$\begin{aligned} U_e(0) - U_e(\infty) &= e \cdot V(x=0) \\ &= e \frac{Q}{4\pi\epsilon_0 R} \end{aligned}$$

$$\begin{aligned} k_B &= 1.38 \cdot 10^{-23} \text{ J/K} \\ T &= 25^\circ \text{C} \\ k_B T &\approx 25.7 \text{ meV} \end{aligned}$$

$$\begin{aligned} Q &= 4e \\ R &= 1 \text{ mm} \end{aligned}$$

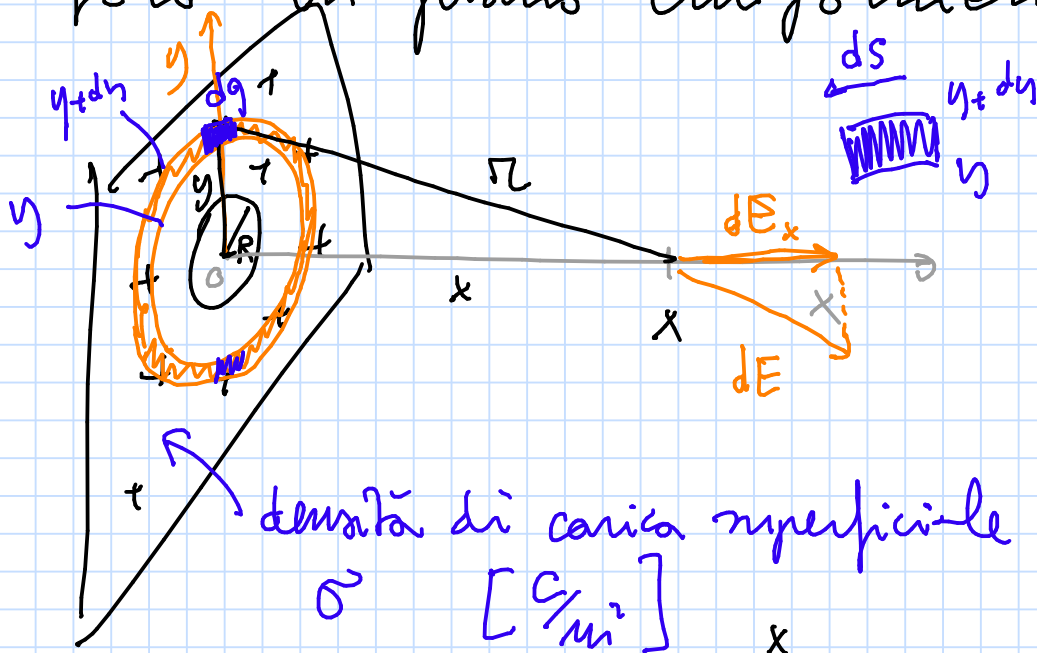
$$V(0) = \frac{Q}{4\pi\epsilon_0 R} = \frac{4 \cdot 1.6 \cdot 10^{-19} \text{ C}}{10^{-9} \text{ m}} \cdot 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} = 5.76 \text{ V}$$

$$\text{def. } 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

$$U_e = 5.76 \text{ eV}$$

energia che cambia quando
 $q = 1e$ passa 1V

foro in piano uniformemente carico



$$dq = \sigma \cdot dS$$

$$= \sigma dy ds$$

densità di carica superficiale
 σ $[C/m^2]$

$$dE_x(x) = \frac{dq}{4\pi\epsilon_0 r^2} \cos\theta$$

$$r = \sqrt{x^2 + R^2}$$

$$= \frac{\sigma dy ds}{4\pi\epsilon_0} \left(\frac{x}{r} \right)^{3/2}$$

integrare tutte le dq sulla striscia, raggio y

$$\int ds = 2\pi y$$

$$dE_x(x) = \frac{\sigma}{2\epsilon_0} \frac{y x dy}{(x^2 + y^2)^{3/2}}$$

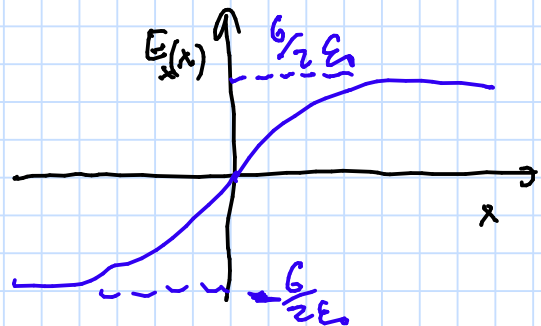
corconf.

$$E_x(x) = \int dE_x(x) = \int_{y=R}^{y=\infty} \frac{\sigma}{2\epsilon_0} \frac{x y dy}{(x^2 + y^2)^{3/2}}$$

$$\frac{d}{dy} \left[(x^2 + y^2)^{1/2} \right]$$

$$= - \frac{y}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\sigma}{2\epsilon_0} \frac{x}{(x^2 + R^2)^{1/2}}$$



$$\begin{aligned}
 V(0) - V(x) &= \int_0^x \vec{E} \cdot d\vec{x} \\
 (\text{ane}) & \\
 &= \int_0^x E_x(x') dx' \\
 &= \frac{6}{2\epsilon_0} \int_0^x \frac{x'}{(x'^2 + R^2)^{3/2}} dx' \\
 &= \frac{6}{2\epsilon_0} (x^2 + R^2)^{\frac{1}{2}} \Big|_0^x \\
 &= \frac{6}{2\epsilon_0} \left[(x^2 + R^2)^{\frac{1}{2}} - R \right]
 \end{aligned}$$

$$6 = \frac{8 \mu C}{m^2}$$

$$x = 3 \text{ cm}$$

$$R = 1 \text{ cm}$$

$$V(0) - V(x) = \frac{8 \cdot 10^{-9} \frac{C}{m^2}}{2 \cdot 8.85 \cdot 10^{-12}} \left[\sqrt{10} - 1 \right] 10^{-2} \text{ m}$$

$$= 9.77 \text{ V}$$

esercizio: regione V aumenta da $5V/mm$ in direzione $\hat{x}$

intro $q = 0.4 \text{ nC}$ nell'origine

(1) $\vec{E}_A$ in $A = (2, 2) \text{ cm}$

(2) accelerazione di $q_1 = 0.5 \text{ nC}$, $m = 2g$ in A

(3) V_B , $B = (2, 0) \text{ cm}$ dato che $V_A = 200V$

1. due componenti di $\vec{E}$:

$$E_1 = k \frac{q}{r^2} = k \frac{q}{x^2 + y^2} = \dots$$

$$= 4500 \text{ V/m}$$

$$E_{1x} = E_1 \cdot \cos 45^\circ = 3182 \text{ V/m}$$

$$E_{1y} = 3182 \text{ V/m}$$

$$E_{2x} = -\frac{\partial V}{\partial x} = -\frac{\Delta V}{\Delta x} = -\frac{5V}{1mm} = -5000 \text{ V/m}$$

somma:

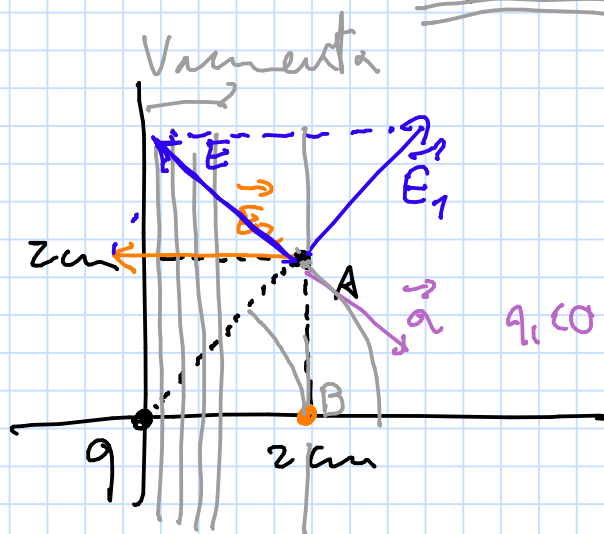
$$E_x = E_{1x} + E_{2x} = -1818 \text{ V/m}$$

$$E_y = E_{1y} = 3182 \text{ V/m}$$

modulo $E_A = \sqrt{1818^2 + 3182^2} \frac{V}{m} = 3665 \text{ V/m}$

(2) $\vec{F} = q_1 \vec{E}_A$

accel. $a = \left| \frac{q_1 \vec{E}}{m} \right| = \frac{0.5 \cdot 10^{-9} \text{ C} \cdot 3665 \text{ V/m}}{2 \cdot 10^{-3} \text{ kg}} = 0.92 \frac{mm}{s^2}$



consideriamo ΔV solo per q .

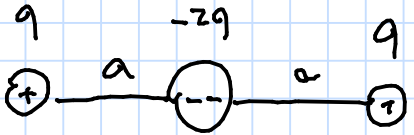
$$V_q(r_A) = \underbrace{k \frac{q}{r_A}} + V_{\text{cost}} = 200 \text{ V}$$

$$V_{\text{cost}} = 200 \text{ V} - k \frac{q}{r_A} = \dots = 72.7 \text{ V}$$

$$V_q(r_B) = k \frac{q}{r_B} + V_{\text{cost}} = 252.7 \text{ V}$$
$$> V_q(r_A)$$

Quadrupolo elettrico (q.e.)

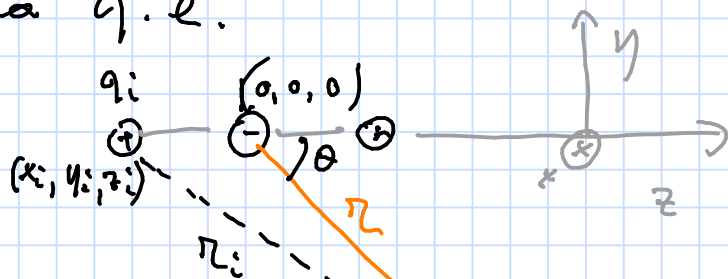
certe distribuzioni di carica sono neutre ($Q=0$)
e con momento di dipolo nullo ($\vec{p}=0$)

es)  $Q = 2q - 2q = 0$ $\vec{p} = 0$ q.e. lineare

es  $Q = 0$ $\vec{p} = 0$ q.e. bidimensionale.

potenziale V generato da q.e.

La espansione di $\frac{1}{r_i}$
con media $\frac{1}{r}$



$$\frac{1}{r_i} = \frac{1}{r} + \left(\partial_{x_i} \frac{1}{r_i} \right) x_i + \left(\partial_{y_i} \frac{1}{r_i} \right) y_i + \left(\partial_{z_i} \frac{1}{r_i} \right) z_i$$

$$+ \frac{1}{2} \left(\partial_{x_i}^2 \frac{1}{r_i} \right) x_i^2 + \frac{1}{2} \left(\partial_{y_i}^2 \frac{1}{r_i} \right) y_i^2 + \frac{1}{2} \left(\partial_{z_i}^2 \frac{1}{r_i} \right) z_i^2$$

$$+ \left(\partial_{x_i y_i}^2 \frac{1}{r_i} \right) x_i y_i + \left(\partial_{x_i z_i}^2 \frac{1}{r_i} \right) x_i z_i + \left(\partial_{y_i z_i}^2 \frac{1}{r_i} \right) y_i z_i$$

tutto noto con $r_i = r \Leftrightarrow x_i = y_i = z_i = 0$

esempio nel q.e. lineare $x_i = y_i = 0$

$$\frac{1}{r_i} = \frac{1}{r} + \left(\partial_{z_i} \frac{1}{r_i} \right) z_i + \frac{1}{2} \left(\partial_{z_i}^2 \frac{1}{r_i} \right) z_i^2 + \dots$$

$$\frac{1}{r_i} = \left(y^2 + (z - z_i)^2 \right)^{-\frac{1}{2}} \quad \text{pitagora}$$

$$\partial_{z_i} \left(\frac{1}{r_i} \right) = \partial_{z_i} \left(y^2 + (z_i - z)^2 \right)^{-\frac{1}{2}} = -\frac{1}{\sqrt{y^2 + (z_i - z)^2}} \Big|_{z_i=0} = -\frac{z}{r^3}$$

q.e. lineare

$$\frac{1}{r_i} = \frac{1}{r} + \frac{z}{r^3} z_i + \frac{1}{2} \left[\frac{1}{r^3} (3 \cos^2 \theta - 1) \right] z_i^2$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0} \left[Q/r + \frac{\cos\theta}{r^2} \sum_i q_i z_i + \frac{3 \cos^2 \theta - 1}{2 r^3} \sum_i q_i z_i^2 + \dots \right]$$

vogliamo solo $\frac{1}{r}, \theta$

P_q
momento di
quadrupolo
(q.e. lineare)

$$Q = \vec{p} = 0$$

$$V = \frac{1}{4\pi\epsilon_0} P_q \frac{3 \cos^2 \theta - 1}{r^3}$$

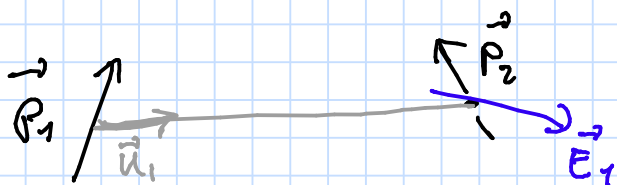
$$[V] = \frac{C}{m}, \quad \text{simmetria} \quad V(\theta, r) = V(\pi - \theta, r)$$

sull'asse z , campo $\vec{E} = - \frac{\partial V}{\partial z} \Big|_{\theta=0}$
 $r \gg a$

$$= \dots = \frac{3}{4} \frac{P_q}{4\pi\epsilon_0} \frac{1}{r^4}$$

interazioni tra dipoli

in generale $U_e = - \vec{p} \cdot \vec{E}$
(dipolo)



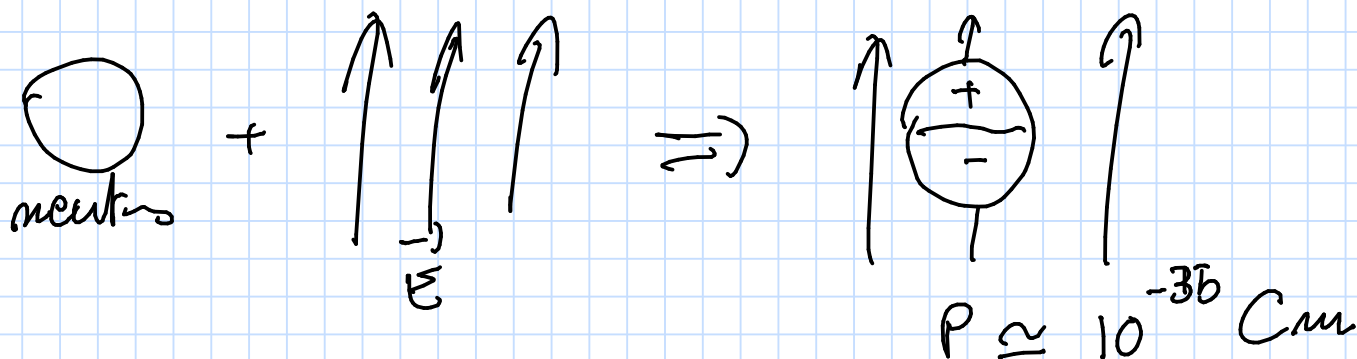
$$U_{e2} = - \vec{p}_2 \cdot \vec{E}_1$$

MNV
(2.62)
 $\vec{E}_1$

$$U_{e2} = \frac{1}{4\pi\epsilon_0 r^3} \left[\vec{p}_1 \cdot \vec{p}_2 - 3 (\vec{p}_1 \cdot \vec{u}_1) (\vec{p}_2 \cdot \vec{u}_1) \right]$$

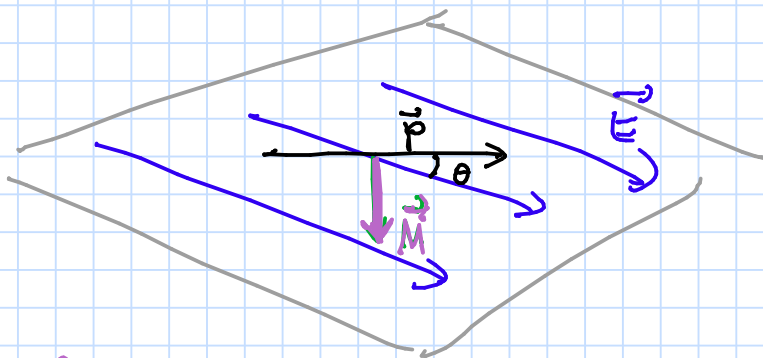
$$\vec{p}_{\text{H}_2\text{O}} = 6.31 \cdot 10^{-30} \text{ Cm}$$

molecolare



Piccole oscillazioni dei dipoli in $\vec{E}$

$$\vec{M} = \vec{p} \times \vec{E}$$



momento d'inerzia I

$$I \frac{d^2 \theta}{dt^2} = M = -pE \sin \theta$$

$$\approx -pE \theta$$

piccole oscill.

eq. oscill. armonica

$$\frac{d^2 \theta}{dt^2} + \frac{pE}{I} \theta = 0$$

$$\frac{d^2 \theta}{dt^2} + \omega^2 \theta = 0$$

pulsazione

$$\omega = \sqrt{\frac{pE}{I}}$$

soluzione

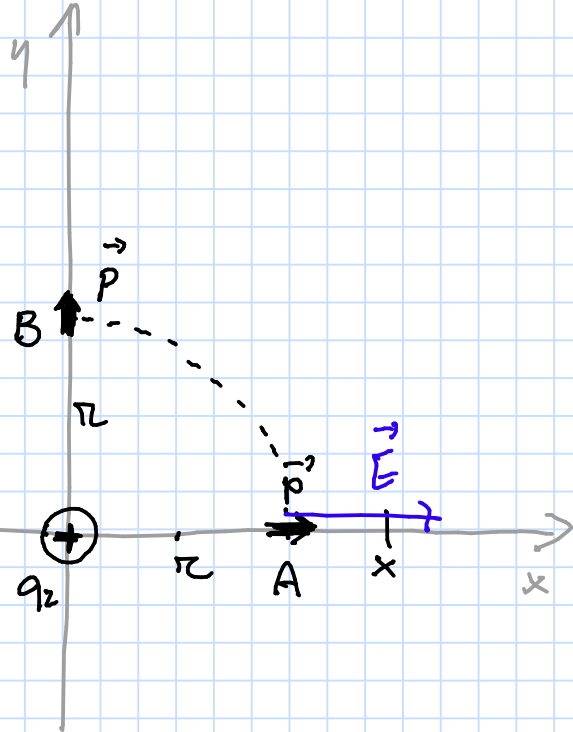
$$\theta(t) = \theta_0 \sin(\omega t + \phi)$$

$$q_1 = -2 \text{ nC}$$

$$q_2 = 10 \text{ nC}$$

$$r = 20 \text{ cm}$$

$$\vec{p}, p = 16 \text{ nC m}$$



1) V_e del dipolo (A) ?

2) forma di $\vec{p}$, orientat. fissa?

3) W del campo $\vec{E}$ generato da q_1 e q_2 per portare?

4) M_B ?

$$1) \vec{E}(x) = \vec{E}_1(x) + \vec{E}_2(x) = k \left[\frac{q_1}{(x+r)^2} + \frac{q_2}{x^2} \right] \hat{x}$$

$$\vec{E}(r) = k \frac{q_1}{(2r)^2} + k \frac{q_2}{r^2} = -112.5 \frac{\text{V}}{\text{m}} + 2250 \frac{\text{V}}{\text{m}} = 2137.5 \frac{\text{V}}{\text{m}}$$

$$V_e = -\vec{p} \cdot \vec{E} = -p E = -16 \cdot 10^{-9} \text{ C m} \cdot 2137.5 \frac{\text{V}}{\text{m}} = -34.2 \mu\text{J}$$

allineati

$V_e < 0$

$$2) V = V_1 + V_2$$

$$\vec{F}_e = -\vec{\nabla} V_e \Rightarrow F_e = -\frac{\partial V_e}{\partial x} = p \frac{\partial E(x)}{\partial x}$$

$$F_e(x) = p k \frac{d}{dx} \left[\frac{q_1}{(x+r)^2} + \frac{q_2}{x^2} \right]$$

$$= p k \left[-\frac{2q_1}{(x+r)^3} - \frac{2q_2}{x^3} \right] = -2 p k \left[\frac{q_1}{(x+r)^3} + \frac{q_2}{x^3} \right]$$

$$F_e(x=r) = -2 p k \left(\frac{q_1}{(2r)^3} + \frac{q_2}{r^3} \right) = -2 (16 \cdot 10^{-9} \text{ C m}) (9 \cdot 10^9) \left[-0.031 \cdot 10^{-6} + 1.25 \cdot 10^{-6} \right] = -351 \mu\text{N}$$

3. lavoro $A \rightarrow B$: W

in A :

$$E_1^A = k \frac{|q_1|}{(2r)^2} = 112.5 \frac{V}{m}$$

$\vec{E}_1^A \parallel -\vec{p} \Rightarrow$ max energia

$$U_{e1}^A = -\vec{p} \cdot \vec{E}_1^A = p E_1^A = 1.8 \mu J (\geq 0)$$

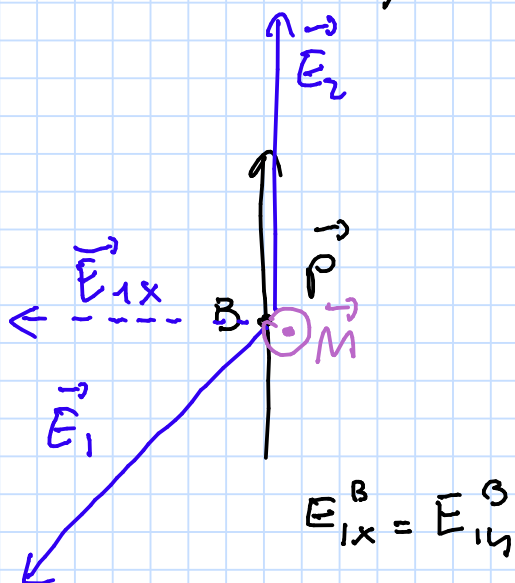
$E_1^B = k \frac{|q_1|}{(\sqrt{2}r)^2} \Rightarrow$ è interessa solo $E_{1y}^B \parallel \vec{p}_B$

$$E_{1y}^B = -\frac{\sqrt{2}}{2} k \frac{|q_1|}{2r^2} = -159.1 \frac{V}{m}$$

$$U_1^B = -p E_{1y}^B = -(1.6 \cdot 10^{-9} \text{ Cm}) (-159.1 \dots) = 254 \mu J (\geq 0)$$

$$W_{AB} = U_A - U_B = -0.75 \mu J$$

4. $\vec{M}_B$



$$\vec{M} = \vec{p} \times \vec{E}_B$$

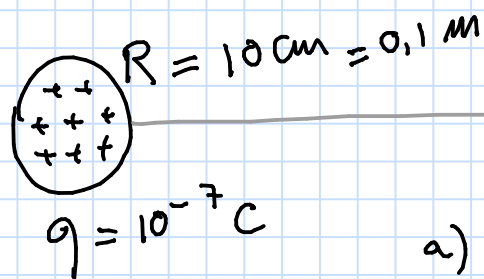
$$= \vec{p} \times \vec{E}_{1x}$$

$$= p E_{1x}^B \hat{z}$$



$$M = p E_{1x}^B = p E_{1y}^B = 2.55 \cdot 10^{-6} \text{ Nm}$$

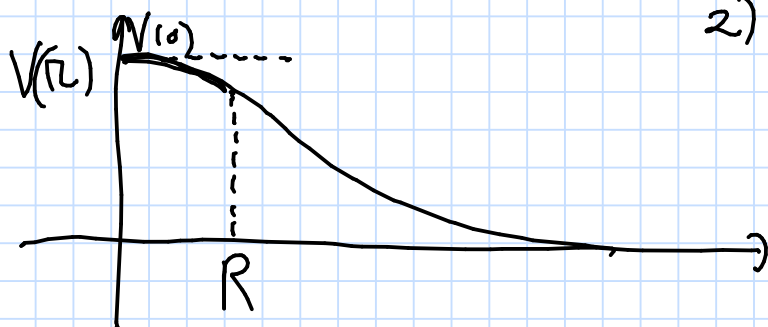
esercizio: protoni vs $q > 0$ (sfera uniform. carica)



a) π approssimando q

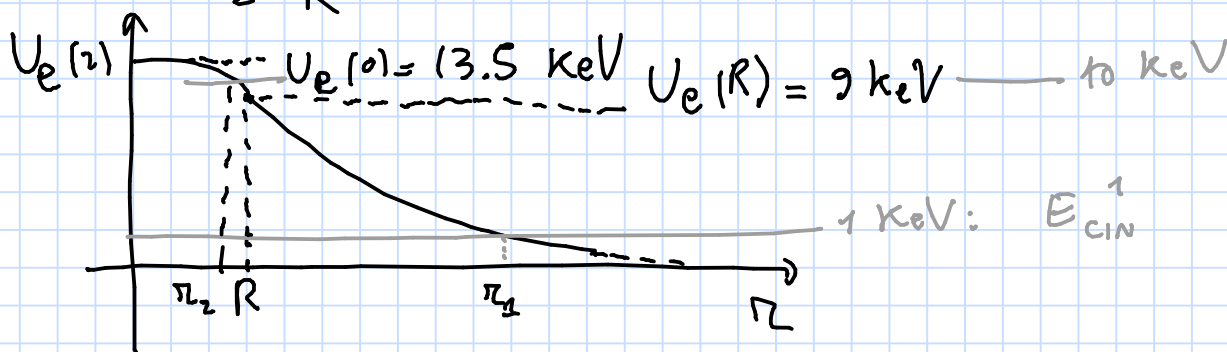
1) 1 keV

2) 10 keV



$$\left\{ \begin{array}{l} V(r) = \frac{kq}{r} \quad r \geq R \\ V(r) = \frac{kq}{R} \left(\frac{3}{2} - \frac{r^2}{2R^2} \right) \quad r < R \end{array} \right.$$

$$V(0) = \frac{3}{2} \frac{kq}{R} = 13.5 \text{ kV}$$



a) 1. $E_{cin}^1 = 1 \text{ keV} \longrightarrow U_e(r_1) = E_{cin}^1$

$$V_1 = V(r_1) = 1 \text{ kV} < V(R)$$

$$\frac{kq}{r_1} = V_1 \quad r_1 = \frac{kq}{V_1} = \frac{9 \cdot 10^9 \cdot 10^{-7} \text{ C}}{10^3} = 0.9 \text{ m}$$

$$r_1 > R$$

2. $E_{cin}^2 = 10 \text{ keV} \Rightarrow r_2 \text{ f.c. } V(r_2) = 10 \text{ kV} > V(R)$

$$V_2 = V(r_2) = V(R) \left(\frac{3}{2} - \frac{r_2^2}{2R^2} \right) \Rightarrow r_2 = R \sqrt{\frac{14}{18}} = 0.088 \text{ m}$$

b) $E_{cin}^{min} = 13.5 \text{ keV}$

c) $E_{cin} = 15 \text{ keV} \rightarrow U_{centro}?$

protoni
 p
antiprotoni
 $\bar{p}$ m_p
 $-e$