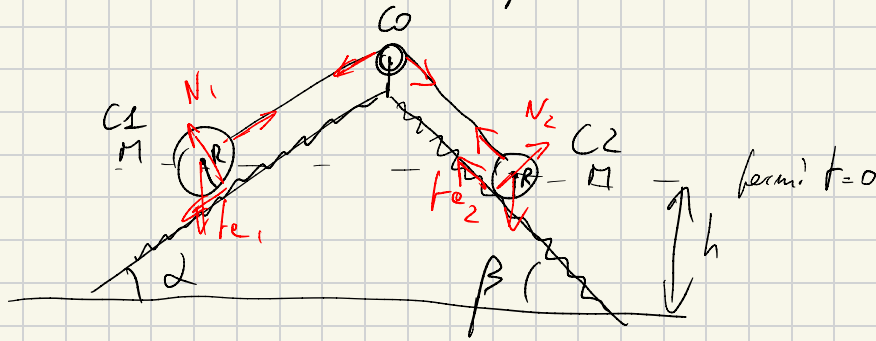




1) $T, f_e, \alpha, \varphi_{cm}$

$$\begin{aligned}
 2M \varphi_{cm} &= M g \sin \beta - M g \sin \alpha - (f_1 N_1) - (f_2 N_2) \\
 \begin{cases} m \varphi_{cm} = T - m g \sin \alpha - f_1 \\ m \varphi_{cm} = m g \sin \beta - T - f_2 \end{cases}
 \end{aligned}$$

$$N_1 = m g \cos \alpha$$

$$N_2 = m g \cos \beta$$

$$R \alpha = \varphi_{cm}$$

$$R f_1 = I \alpha = \frac{I}{R} \varphi_{cm}$$

$$R f_2 = I \alpha$$

$$I = \frac{1}{2} m r^2$$

$$2M \varphi_{cm} = M g \sin \beta - M g \sin \alpha - M \varphi_{cm}$$

$$\varphi_{cm} = \frac{\sin \beta - \sin \alpha}{3} g$$

$$\alpha = \frac{\sin \beta - \sin \alpha}{3R} g$$

$$f_c = I \frac{\alpha}{R} = \frac{1}{2} \pi R^2 \frac{\sin \beta - \sin \alpha}{3R} g$$

$$= \frac{1}{6} \pi (\sin \beta - \sin \alpha) g$$

$$T = m a_{cm} + f_c + m g \sin \alpha$$

$$= m \frac{\sin \beta - \sin \alpha}{3} g + m \sin \alpha g + \frac{1}{6} \pi (\sin \beta - \sin \alpha) g$$

$$= \frac{1}{2} \pi (\sin \beta + \sin \alpha) g$$

$$f_{c2} \leq \mu_s m g \cos \beta \quad \text{obzto linke}$$

$$f_{c1} \leq \mu_s m g \cos \alpha$$

$$\frac{1}{6} (\sin \beta - \sin \alpha) < \mu_s \cos \beta$$

$$Mgh - Mgh(h-R) \frac{\mu_2}{\mu_B} = Mv_{\text{cm}}^2 + \frac{1}{2} I \omega^2$$

$\downarrow$
 $\frac{v_{\text{cm}}^2}{R^2}$

$$h_1 = h + (h-R) \frac{\mu_2}{\mu_B}$$

$$Mgh(h-R) - Mgh(h-R) \frac{\mu_2}{\mu_B} = \frac{3}{2} Mv_{\text{cm}}^2$$

$$\frac{2}{3} g(h-R) \left(1 - \frac{\mu_2}{\mu_B}\right) = v_{\text{cm}}^2$$

$$h_2 = h_1 + \frac{1}{\cancel{Mg}} \frac{1}{\cancel{2}} \cancel{M} \frac{2}{3} g(h-R) \left(1 - \frac{\mu_2}{\mu_B}\right)$$

$$= h + (h-R) \frac{\mu_2}{\mu_B} + \frac{1}{3} (h-R) \left(1 - \frac{\mu_2}{\mu_B}\right)$$

Esercizio 1

1) Un magazzino è lungo 20.0 yd, largo 10.0 yd e alto 15 ft. Quale è il suo volume in SI?

$$1 \text{ yd} = 3 \text{ ft}$$

$$1 \text{ m} = 3,281 \text{ ft}$$

$$L = 20.0 \text{ yd} \frac{3 \text{ ft}}{1 \text{ yd}} \frac{1 \text{ m}}{3,281 \text{ ft}}$$

$$W = 10 \text{ yd} \frac{3 \text{ ft}}{1 \text{ yd}} \frac{1 \text{ m}}{3,281 \text{ ft}}$$

$$H = 15 \text{ ft} \frac{1 \text{ m}}{3,281 \text{ ft}}$$

$$V = \left\{ \left[3 \times (10.0 + 20.0) \right] + 15 \right\} \times \frac{1}{3,281} = 264 \text{ m}^3$$

2) Il sangue nell'aorta di un uomo può raggiungere una velocità di 35.0 cm/s

a) ft/s ? $v = 0,35 \text{ m} \frac{3,281 \text{ ft}}{1 \text{ m}} / 1 \text{ s} = 1.15 \text{ ft/s}$

b) km/h ? $v = 0,35 \text{ m} \frac{1 \text{ km}}{1000 \text{ m}} / \left(1 \text{ s} \frac{1 \text{ h}}{3600 \text{ s}} \right) = 1.26 \text{ km/h}$

3) In fisica $\exists$ 3 cost fondamentali:

- G cost gravitazionale $G = 6.62 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
- h cost Planck $h = 6.63 \cdot 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}}$
- c velocità della luce nel vuoto $c = 3,00 \cdot 10^8 \text{ m/s}$

Mostra che $L_p = \sqrt{\frac{h G}{c^3}}$ ha le dimensioni di una lunghezza

Quale è il suo ordine di grandezza?

$$h \cdot G \sim 36 \cdot 10^{-23} \frac{\text{m}^5}{\text{s}^3}$$

$$\Rightarrow \frac{h \cdot G}{c^3} \sim 4 \cdot 10^{-1} \frac{\text{m}^5}{\text{s}^3} \frac{\text{s}^3}{\text{m}^3}$$

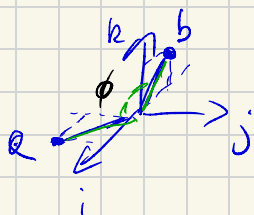
$$L_p = \sqrt{\frac{h \cdot G}{c^3}} \sim \sqrt{40 \cdot 10^{-2}} \text{ m} = 6 \cdot 10^{-1} \text{ m}$$

4) orologi nucleari sbagliano di 0,01 ps/giorno
 Se un orologio di questo tipo fosse in funzione
 dal Big Bang quale sarebbe adesso l'errore?
 Età dell'Universo $T = 13.8 \times 10^9$ anni

$$T = 13.8 \times 10^9 \text{ anni} \times \frac{365 \text{ gg}}{1 \text{ anno}} = 5.04 \times 10^{12} \text{ gg}$$

$$\begin{aligned} e_T = e \times T &= 0.01 \times 10^{-12} \frac{\text{s}}{\text{gg}} \times 5.04 \times 10^{12} \text{ gg} \\ &= 0,05 \text{ s} \end{aligned}$$

5) Quale è l'angolo compreso fra
 $a = (3.0i, -4.0j)$ $b = (-2.0i, 3.0k)$



$$\cos \phi = \frac{a \cdot b}{|a||b|} = \frac{a_i b_i + a_j b_j + a_k b_k}{\sqrt{a_i^2 + a_j^2 + a_k^2} \sqrt{b_i^2 + b_j^2 + b_k^2}} = \frac{-6}{5 + \sqrt{13}}$$

$$\phi = 108^\circ$$

c) Due vettori sono dati da $\vec{a} = 3.0\vec{i} + 5.0\vec{j}$ e $\vec{b} = 2.0\vec{i} + 4.0\vec{j}$. Si trovano

a) $\vec{a} \times \vec{b}$

b) $\vec{a} \cdot \vec{b}$

c) la proiezione di $\vec{a}$ lungo $\vec{b}$

a)

$$\vec{a} \times \vec{b} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.0 & 5.0 & 0 \\ 2.0 & 4.0 & 0 \end{vmatrix} = \begin{vmatrix} 3.0 & 0 \\ 2.0 & 0 \end{vmatrix} \hat{i} + \begin{vmatrix} 5.0 & 0 \\ 4.0 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 3.0 & 5.0 \\ 2.0 & 4.0 \end{vmatrix} \hat{k}$$
$$= (12.0 - 10.0) \hat{k} = 2.0 \hat{k}$$

b)

$$\vec{a} \cdot \vec{b} = a_i b_i + a_j b_j + a_k b_k = 6.0 + 20.0 = 26.0$$

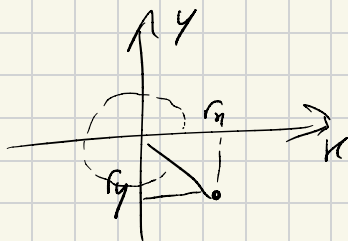
c)

$$a_b = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{26.0}{\sqrt{20.0}} \approx 5.8$$

7) Dato le componenti $r_x = 1.62 \text{ m}$ $r_y = -1.15 \text{ m}$
calcola $|\vec{r}|$ e ϕ

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.62)^2 + (-1.15)^2} = 2.03 [\text{m}]$$

$$\phi = \arctan \frac{r_y}{r_x} = \arctan \frac{-1.15}{1.62} = -35^\circ$$



8) Sapendo che $\vec{A} + \vec{B} = \vec{0}$, che cosa si può dire

a) di $|\vec{A}|$ e $|\vec{B}|$?

b) delle direzioni di $\vec{A}$ e $\vec{B}$?

$$\vec{A} = -\vec{B} \quad \Rightarrow \quad |\vec{A}| = |\vec{B}|$$

$\Rightarrow$ direzioni di $\vec{A}$ e $\vec{B}$ sono opposte

$$g) \quad \vec{A} = (1,2 \text{ m}, 0 \text{ m})$$

$$\vec{B} = (-3,4 \text{ m}, 0 \text{ m})$$

$$|\vec{A}| \geq |\vec{B}| ?$$

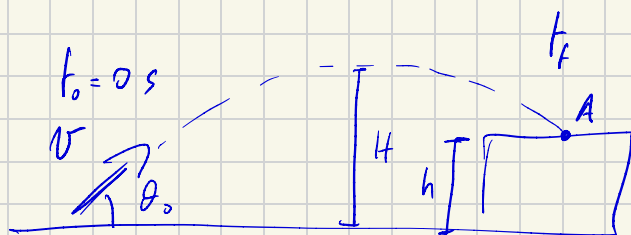
$$a) \quad 3,4 > 1,2$$

$$b) \quad B_x < 0$$

$$c) \quad A_x > 0$$

$$d) \quad |\vec{A}| = \sqrt{A_x^2 + A_y^2} = 1,2 \quad < \quad |\vec{B}| = \sqrt{B_x^2 + B_y^2} = 3,4$$

1)



$$v = 42 \text{ m/s}$$

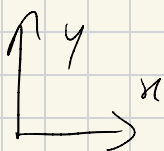
$$\theta_0 = 60.0^\circ$$

$$t_f = 5.5 \text{ s}$$

a) h ?

b) v_f ?

c) H ?



$$e) \quad y = v \sin \theta_0 t - \frac{1}{2} g t^2$$

$$h = y(t_f) = 51 \text{ m}$$

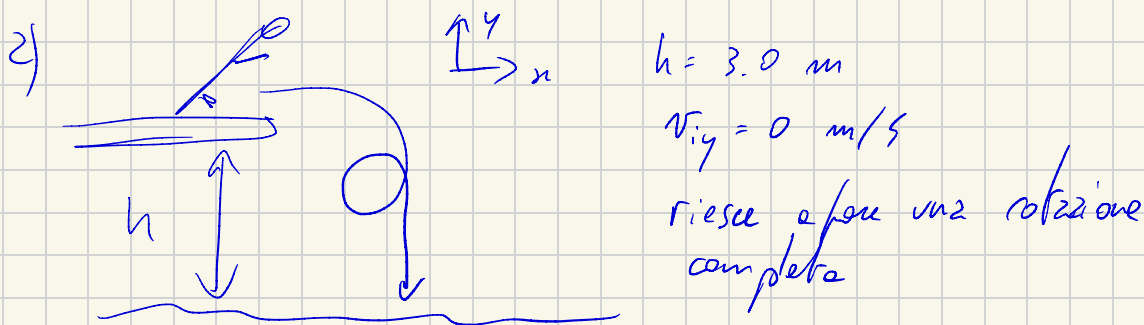
$$b) \quad v_{xf} = v_{xi} = v \cos \theta_0 = 21.0 \text{ m/s}$$

$$v_{yf} = v_{yi} - g t_f = v \sin \theta_0 - g t_f = -17.5 \text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = 27.4 \text{ m/s}$$

$$c) \quad v_{yf}(t_{\max}) = 0 \quad t_{\max} = \frac{v \sin \theta_0}{g}$$

$$H = \frac{(v \sin \theta_0)^2}{g} - \frac{1}{2} g \left(\frac{v \sin \theta_0}{g} \right)^2 = \frac{1}{2} \frac{(v \sin \theta_0)^2}{g} = 47.8 \text{ m}$$



a) $\langle \omega \rangle$?

b) se $v_{iy} = 2.1 \text{ m/s} \Rightarrow$ quante rotazioni?
 se $\langle \omega \rangle = \text{cost.}$

a) Il tempo del tuffo è

$$-h = -\frac{1}{2}gt^2 \quad t = \sqrt{\frac{2h}{g}}$$

$$\langle \omega \rangle = \frac{2\pi}{t} = \pi \sqrt{\frac{2g}{h}} \sim 8.0 \text{ rad/s}$$

b) Il tempo del tuffo è

$$-h = v_{iy}t - \frac{1}{2}gt^2$$

$$gt^2 - 2v_{iy}t - 2h = 0$$

$$t = \frac{v_{iy} \pm \sqrt{v_{iy}^2 + 2gh}}{g} = 1.04 \text{ s}$$

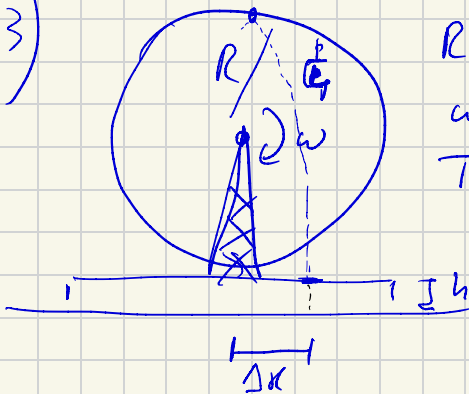
Solteggj i t kempo delle solte

$$v_{yt} = 0 = v_{0i} - g t \Rightarrow t_{\max} = \frac{v_{0i}}{g} = 0.21 \text{ s}$$

$$t_{\text{down}} = t - t_{\max} = 0.815 \text{ s}$$

$$\Rightarrow \text{giri} = \frac{\omega t_{\text{down}}}{2\pi} = 1.31 \text{ rotazioni}$$

3)



$$R = 5.0 \text{ m}$$

$$h = 1.75 \text{ m}$$

$$\omega = \text{cost}$$

$$T = 32 \text{ s}$$

$$a) v_T?$$

$$b) a_c?$$

$$c) \Delta x?$$

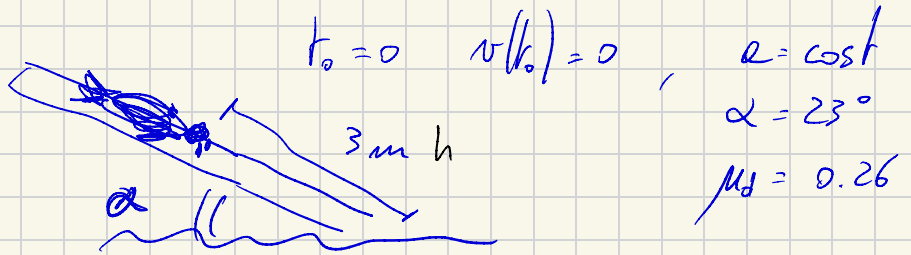
$$a) v_T = \omega R \quad \omega = \frac{2\pi}{T}$$

$$b) a_c = \frac{v_T^2}{R} = \omega^2 R$$

$$c) H = 2R + h \quad \bar{v}_0 = \bar{v}_T \Rightarrow t = \sqrt{\frac{2H}{g}}$$

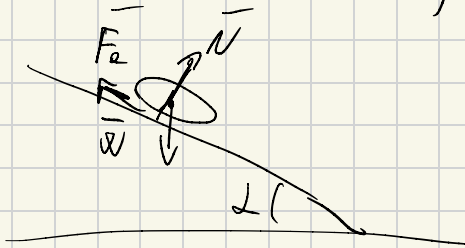
$$\Delta x = v_T t$$

4) Un leone marino



a) t per arrivare in acqua?

b) se $\mu_d = 0 \Rightarrow t$ per arrivare in acqua?



$$a) \quad m a = m g \sin \alpha - \mu_d m g \cos \alpha$$

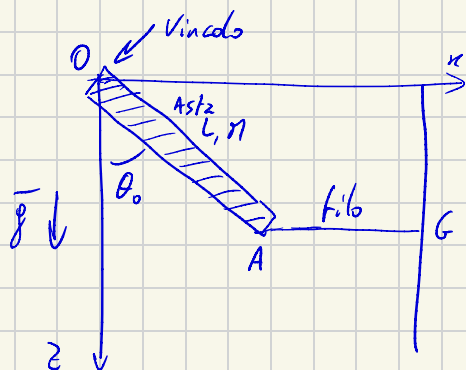
$$a = g (\sin \alpha - \mu_d \cos \alpha)$$

$$h = \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2h}{a}}$$

$$b) \quad m a = m g \sin \alpha$$

12)



f. lo ideal

Se o suporte é fixo

a) T ? $\vec{R}_0(\theta_0)$?

b) x e $t=0$ f. lo se desprende quando $\theta_0 = \theta_c$?

1 E_k ?

2 $\vec{R}_0$?

3 $\vec{L}_0$?

c) se in O movimento pendulo θ_c v_{max} quando $\theta = \theta_c$?

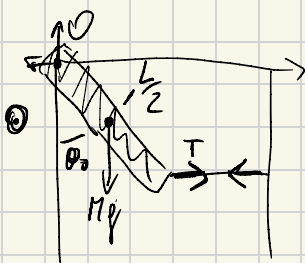
a)

$$\begin{cases} R_{0x} - T = 0 \\ R_{0z} - mg = 0 \\ mg \frac{L}{2} \sin \theta_0 = T L \cos \theta_0 \end{cases}$$

$$\Rightarrow \vec{T} = \frac{1}{2} mg \tan \theta_0 \hat{x}$$

$$|\vec{R}_0| = \sqrt{mg^2 + \frac{1}{4} m^2 g^2 \tan^2 \theta_0}$$

$$\phi = \arctan \left(\frac{mg}{\frac{1}{2} mg \tan \theta_0} \right)$$



$$I_0 = \frac{1}{3} mL^2$$

b)

$$\Delta E_g = mg \left(\frac{L}{2} \cos \theta_0 - \frac{L}{2} \right)$$

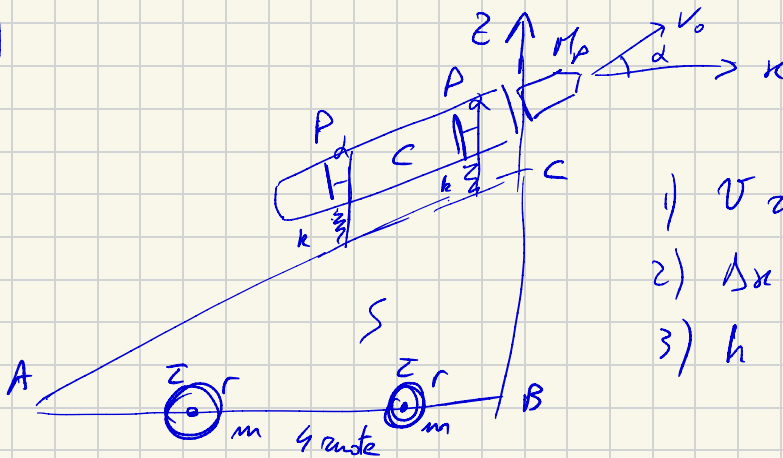
$$E_k = \frac{1}{2} I \omega^2 = \frac{mgL}{2} (1 - \cos \theta_0)$$

$$I \omega^2 R = R_0 - mg$$

$$L = I \omega$$

c) $E_k = \frac{1}{2} I \omega^2 = \Delta E_g - z_e \theta_0$

5)



- 1) v rincubo
- 2) Δx molle
- 3) h rincubo

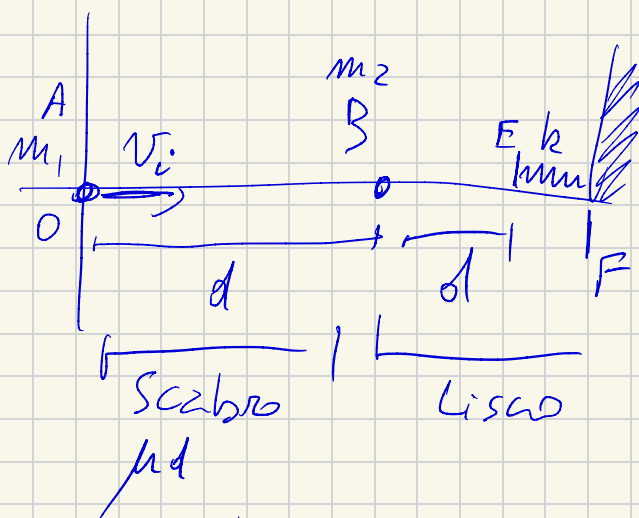
1) $(4m + M_s + M_c) v_x = -M_p v_0 \cos \alpha$

$(4m + M_s + M_c) v_y = -M_p v_0 \sin \alpha$

$\frac{1}{2} M_c v_x^2 = 4x \left(\frac{1}{2} k (\Delta x \cos \alpha)^2 \right) - M_y (\Delta x \cos \alpha)$

$\cancel{E_{kf}} - E_{ki} = \left(E_{gi} - E_{pf} \right) + 4x \left(\cancel{E_{ci}} - E_{cf} \right)$
 $-\frac{1}{2} M_c v_x^2 \quad M_y \Delta x \cos \alpha \quad -\frac{1}{2} k (\Delta x \cos \alpha)^2$

6)



$$m_2 = 2m_1$$

1) v_i in cui B tocca E

2) $\Delta x_{\max}$ molla

3) A urto B una seconda volta?

$$v_0^2 - v_i^2 = 2ad$$

$$= -2\mu_d g d$$

$$f_a = m_1 a = -\mu_d m_1 g$$

$$m_1 v_0 = m_1 v_A + m_2 v_B$$

$$m_1 v_0^2 = m_1 v_A^2 + m_2 v_B^2$$

}

$$v_A = v_0 - \frac{m_2}{m_1} v_B$$

$$m_1 v_0^2 = m_1 v_0^2 + m_1 \frac{m_2^2}{m_1^2} v_B^2 - 2m_1 \frac{m_2}{m_1} v_0 v_B + m_2 v_B^2$$

$$0 = m_2 v_B^2 - 2m_1 v_0 v_B + m_1 v_B^2 = 2m_1 v_B^2 + m_1 v_B^2 - 2m_1 v_0 v_B$$

$$v_B = \frac{2}{3} v_0$$

$$v_A = v_0 - 2v_B = -\frac{1}{3} v_0$$

2)

$$E_n^2 = \frac{1}{2} h \nu_{MAX}^2$$

3)

δ_i