

Costanti

$$\begin{aligned}
 R &= 8.314 \text{ J/mol} \cdot K \\
 \text{Coulomb constant } K &= 8.988 \times 10^9 \text{ Nm}^2\text{C}^{-2} \\
 k_B &= 1.381 \times 10^{-23} \text{ m}^2\text{kg} \cdot \text{s}^{-2}\text{K}^{-1} \\
 N_A &= 6.022 \times 10^{23} \\
 G &= 6.6741 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \\
 e &= 1.602 \times 10^{-19} \text{ C} \\
 \epsilon_0 &= 8.8542 \times 10^{-12} \text{ F} \cdot \text{m}^{-1} \\
 \mu_0 &= 4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}} \\
 c^2 &= \frac{1}{\epsilon_0 \mu_0} \\
 c &= 299.8 \times 10^6 \text{ m/s} \\
 e &= 1.602 \times 10^{-19} \text{ C} \quad m_p = 1.673 \times 10^{-27} \text{ kg} \\
 m_e &= 9.109 \times 10^{-31} \text{ kg}
 \end{aligned}$$

Units conversion

$$\begin{aligned}
 1 \text{ amu} &= 1,66 \times 10^{-27} \text{ kg} \\
 1 \text{ cal} &= 4.186 \text{ J} \\
 1 \text{ yd} &= 3 \text{ ft} \\
 1 \text{ m} &= 3.281 \text{ ft}
 \end{aligned}$$

Formule trigonometriche

$$\begin{aligned}
 \sin(\alpha) + \sin(\beta) &= 2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) \\
 \sin(\alpha) - \sin(\beta) &= 2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \\
 \cos(\alpha) + \cos(\beta) &= 2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) \\
 \cos(\alpha) - \cos(\beta) &= -2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \\
 \cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) \\
 \sin(2\alpha) &= 2 \sin(\alpha) \cos(\alpha) \\
 \tan(2\alpha) &= \frac{2 \tan(\alpha)}{1 - \tan^2(\alpha)} \\
 \cot(2\alpha) &= \frac{\cot^2(\alpha) - 1}{2 \cot(\alpha)} \\
 \cos(\alpha + \beta) &= \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta) \\
 \cos(\alpha - \beta) &= \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)
 \end{aligned}$$

Equazioni differenziali

$$\begin{aligned}
 \frac{df}{dx} &= A \xrightarrow{f(0)=B} f(x) = B + Ax \\
 \frac{df}{dx} &= -kf(x) \xrightarrow{f(0)=A} f(x) = Ae^{-kx} \\
 \frac{df}{dx} &= A + Bf(x) \xrightarrow{f(0)=C} f(x) = -\frac{A}{B} + \left(\frac{A}{B} + C\right)e^{Bx} \\
 \frac{d^2f}{dx^2} &= A \xrightarrow{f(0)=C, f'(0)=B} f(x) = C + Bx + \frac{1}{2}Ax^2 \\
 \frac{d^2f}{dx^2} &= -\omega^2 x \xrightarrow{f(0)=A, f'(0)=B} f(x) = A \cos \omega t + \frac{B}{\omega} \sin \omega t
 \end{aligned}$$

Cinematica

$$\begin{aligned}
 v_x &= \frac{dx}{dt}; a_x = \frac{dv_x}{dt} \\
 \Delta x &= v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\
 v_{fx}^2 - v_{ix}^2 &= 2a_x \Delta x \\
 &\text{moto oscillatorio} \\
 x(t) &= A \sin(\omega + \varphi), \quad \omega = \frac{2\pi}{T} \\
 v^2(x) &= \omega^2 (A^2 - x^2) \\
 &\text{moto rotatorio} \\
 \omega &= \frac{d\theta}{dt}; v = \omega R; \\
 \text{radial acceleration } a_r &= v\omega = \frac{v^2}{R} = \omega^2 R \\
 \text{accelerazione angolare } \alpha &= \frac{d\omega}{dt} = \frac{a_T}{R} \\
 \vec{v} &= \vec{\omega} \times \vec{r} \\
 \theta(t) &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\
 \omega_{fx}^2 - \omega_{ix}^2 &= 2\alpha \Delta \theta \\
 &\text{in generale ...} \\
 \vec{v} &= \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt} = \frac{dr}{dt} \hat{u}_r + r \frac{d\theta}{dt} \hat{u}_\theta = \vec{v}_r + \vec{v}_\theta \\
 \vec{a} &= \frac{d\vec{v}}{dt} = \frac{dr}{dt} \hat{u}_r + r \frac{d^2\theta}{dt^2} \hat{u}_\theta + 2 \frac{dr}{dt} \frac{d\theta}{dt} \hat{u}_\theta \\
 &\text{moti relativi inerziali} \\
 \vec{v}_{WT} &= \vec{v}_{WG} - \vec{v}_{TG} \\
 &\text{moti relativi non inerziali} \\
 \vec{a} &= \vec{a}' + \vec{a}_{O'} + \vec{\omega} \times \vec{\omega} \times \vec{r}' + \frac{d\vec{\omega}}{dt} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' \\
 \vec{F}_{centr} &= -m\vec{\omega} \times \vec{\omega} \times \vec{r}' \\
 \vec{F}_{coriolis} &= -2m\vec{\omega} \times \vec{v}'
 \end{aligned}$$

Dinamica del punto

$$\begin{aligned}
 &\text{quantità di moto } \vec{p} = m\vec{v} \\
 \sum \vec{F} &= m\vec{a} = \frac{d\vec{p}}{dt} \\
 \text{Forza gravitazionale } \vec{F} &= -G \frac{m_1 m_2}{r^2} \hat{r} \\
 \text{Forza d'attrito } \vec{F} &= \mu \vec{N} \\
 \text{Forza elastica } \vec{F} &= -kx \text{ where } k \text{ elastic constant} \\
 \text{Periodo del pendolo } T &= 2\pi \sqrt{\frac{L}{g}} \\
 \text{Momento angolare } \vec{L} &= \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \\
 \text{Momento di una forza } \vec{\tau} &= \vec{r} \times \vec{F} \\
 \frac{d\vec{L}}{dt} &= \vec{r} \times \sum \vec{F} = \vec{\tau} \\
 \text{Impulso } \vec{F} &= \frac{\Delta \vec{p}}{\Delta t}
 \end{aligned}$$

Energia

$$\begin{aligned}
 \text{Lavoro } W &= \vec{F} \cdot d\vec{s} \\
 \text{Potenza } \mathcal{P} &= \frac{dW}{dt} = \vec{F} \cdot \vec{v} \\
 \text{Energia cinetica } E_k &= \frac{1}{2} mv^2 = \frac{p^2}{2m} = \Delta W \\
 \text{Energia potenziale } U_{pot} &= -W \\
 \text{Conservazione dell'energia } \Delta K + \Delta U &= \Delta E_{mech} = W_{nc}, \\
 &\text{dove nc sta per non conservative} \\
 \text{Energia potenziale gravitazionale (campo ||)} U &= mgh \\
 \text{Energia potenziale gravitazionale } U &= -G \frac{mM}{r} \\
 \text{Energia potenziale elastica } U &= \frac{1}{2} kx^2
 \end{aligned}$$

Sistemi di punti

$$\begin{aligned}\text{centro di massa } \vec{r}_{CM} &= \frac{\sum m_i \vec{r}_i}{M} \\ \vec{v}_{CM} &= \frac{\vec{p}_{tot}}{M_{tot}} \\ \vec{a}_{CM} &= \frac{1}{M_{tot}} \sum \vec{F}^{(E)} = \frac{1}{M_{tot}} \vec{F}_{net}^{(E)}\end{aligned}$$

conservazione quantità di moto

$$\Delta \vec{p}_{tot} = 0$$

conservazione dell'energia

$$\Delta E_{tot} = 0$$

1a eq. cardinale

$$\vec{F}_{net}^{(E)} = \frac{d\vec{p}_{tot}}{dt}$$

2a eq. cardinale

$$\frac{d\vec{L}}{dt} = -\vec{v}_O \times m\vec{v}_{CM} + \vec{\tau}^{(E)}$$

1 Koenig

$$\vec{L} = \vec{L}' + \vec{L}_{CM}$$

2 Koenig

$$E_k = E'_k + E_{k,CM}$$

$$W^{(E)} + W^{(I)} = \Delta E_k$$

campo G ||

$$\vec{\tau} = \vec{r}_{CM} \times M_{tot} \vec{g}$$

$$U_G = M_{tot} g h_{CM}$$

- Anello $I = mR^2$

- Disco/cilindro $I = \frac{1}{2}mR^2$

- Sfera $I = \frac{2}{5}mR^2$

- Sfera cava $I = \frac{2}{3}mR^2$

- Asta $I = \frac{1}{12}mL^2$

Puro rotolamento

$$\vec{v}_{CM} = -\vec{\omega} \times \vec{r}$$

$$v_{CM} = \omega r$$

$$a_{CM} = \alpha r$$

$$\text{attrito volvente } \tau_v = hN$$

Corpo rigido

Densità $\rho = \frac{dm}{dV}$

$$\text{Centro di massa } \vec{r}_{CM} = \frac{1}{V} \int_V \rho \vec{r} dV$$

campo G ||

$$\text{in generale } \vec{F}_{||} = |\vec{F}| \hat{u}_{||}$$

$$\text{Forze peso } \vec{W} = mg\hat{z}$$

$$\text{Momento forza peso } \vec{\tau} = \vec{r}_{CM} \times mg\hat{z}$$

$$\text{Energia potenziale gravitazionale } E_p = mgz_{CM}$$

Quantità di moto

$$\vec{p}_{tot} = \vec{P} = m\vec{v}_{CM}$$

Momento angolare

$$\vec{L}_{CM} = \vec{r}_{CM} \times \vec{P}$$

$$\vec{L}_z = I_z \vec{\omega}$$

$$\frac{d\vec{L}_z}{dt} = I_z \frac{d\vec{\omega}}{dt} = I_z \vec{\alpha} = \vec{\tau}_z$$

$$\frac{d\vec{L}_\perp}{dt} = \vec{\tau}_\perp$$

$$E_k = \frac{L_z^2}{2I_z}$$

Momenti di inerzia

$$I = \sum_{i=1}^N m_i r_i^2 = Md^2 + I_{CM}$$