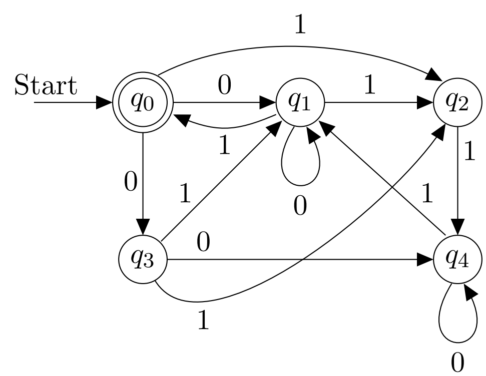
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

July 6th, 2026

1. **[6 points]** Consider the FA A whose transition function is graphically represented as follows



Consider the algorithm for transforming a FA into a regular expression, based on state elimination. Apply the algorithm to eliminate state q_2 from A, and display the resulting automaton A'. Furthermore, eliminate state q_3 from A', and display the resulting automaton A''. If you simplify any of the resulting regular expressions, **add a clear explanation**.

2. **[6 points]** Consider the alphabet $\Sigma = \{a, b\}$ and the following languages defined over Σ

$$\begin{aligned} L_1 &= \{ba^n ba^n \mid n \geq 1\}; \\ L_2 &= \{ba^n a^n b \mid n \geq 1\}; \\ L_3 &= \{ba^n \mid n \geq 1\} \cdot \{ba^n \mid n \geq 1\}. \end{aligned}$$

For each of the above languages, state whether it belongs to REG, to CFL$\smallsetminus$REG, or else whether it is outside of CFL. Provide a mathematical proof for all of your answers.

3. **[5 points]** With reference to push-down automata (PDA), answer the following questions.

   (a) Provide the definition of language accepted by final state and language accepted by empty stack.

   (b) Prove that if P_N is a PDA accepting the language $N(P_N)$ by empty stack, then there exists a PDA P_F accepting the language $L(P_F)$ by final state such that $L(P_F) = N(P_N)$.

(please turn to the next page)

4. **[7 points]** Assess whether the following statements are **true** or **false**, providing clear explanations for all your answers.

   (a) There exist languages L_1 and L_2 both in CFL$\smallsetminus$REG and with $L_1 \neq L_2$, such that $L_1 \cap L_2$ is not in CFL.

   (b) There exist languages L_1 and L_2 both in CFL$\smallsetminus$REG and with $L_1 \neq L_2$, such that $L_1 \cap L_2$ is in CFL$\smallsetminus$REG.

   (c) Let $\mathcal{P}$ be the class of languages that can be recognized in polynomial time by a TM. There exist languages L_1 and L_2 both in $\mathcal{P}$ such that $L_1 \cdot L_2$ is not in $\mathcal{P}$.

   (d) Let $\mathcal{NP}$ be the class of languages that can be recognized in polynomial time by a nondeterministic TM. There exist languages L_1 and L_2 both in $\mathcal{NP}$ such that $L_1 \cdot L_2$ is not in $\mathcal{NP}$.

5. **[6 points]** In relation to the theory of undecidability, answer the following questions. All the TMs introduced below are defined over the input alphabet $\Sigma = \{0, 1\}$. For a language L, we write $|L|$ to denote the number of strings in L, with $|L| = +\infty$ if L is not finite.

   Consider the following property of the RE languages

$$\mathcal{P} = \{L \mid L \in \text{RE},\ |L| < 5\}$$

   and define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Use Rice's theorem to prove that $L_{\mathcal{P}}$ is not in REC.

   (b) Prove that $L_{\mathcal{P}}$ is not in RE.

6. **[3 points]** In relation to the theory of intractability, answer the following questions.

   (a) Define the notion of independent set in a graph and define the independent set (IS) decision problem.

   (b) Prove that IS is in $\mathcal{NP}$, the class of languages that can be recognized in polynomial time by a nondeterministic TM.