Università degli Studi di Padova

Department of Industrial Engineering

Design of Experiments
Lesson #7

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Recap

ANOVA

- what are the factors impacting on the response?
- what are the treatments that induce statistically different response?
- individual value plot
- interaction plot
- residuals tests

RESPONSE SURFACE

- what is the functional relation among factors and response?
- how factors impact on the response?
- response surface models through regression
- easy visualization: surface and contour plots
- residuals tests
- uncertainty of parameters & y estimation
- optimization & confirmation tests

Example: bottle filling

Bottle filling process

- A soft drink producer is interested in understanding the sources of variability of the bottle filling height and eventually optimize it
  - the filling machine theoretically fills each bottle to the correct target height
  - in practice, variation around the target is experienced
  - three variables can be controlled during the filling process:
    - percent carbonation of the soft drink
    - operating pressure of the filler
    - number of bottles produced per minute, i.e.: the line speed

- Objective: obtaining uniform filling heights in the bottles produced by the manufacturing process avoiding to:
  - loose product
  - have costumer complaints

Experimental campaign

- An experimental campaign is carried out considering the abovementioned **factors**:
  - 3 levels for carbonation: 10, 12, and 14%
  - 2 levels for pressure: 25 and 30 psi
  - 2 levels for line speed: 200 and 250 bpm (bottles per minute)
  - two replicates of a factorial design in these three factors
  - 24 runs taken in random order
- The **response variable** observed is the *average deviation from the target fill height* observed in a production run of bottles at each set of conditions

ANOVA model

- The percentage of carbonation, operating pressure, and line speed significantly affect the fill volume

- The carbonation-pressure interaction *F* ratio has a *p*-value of 0.0558, indicating some interaction between these factors

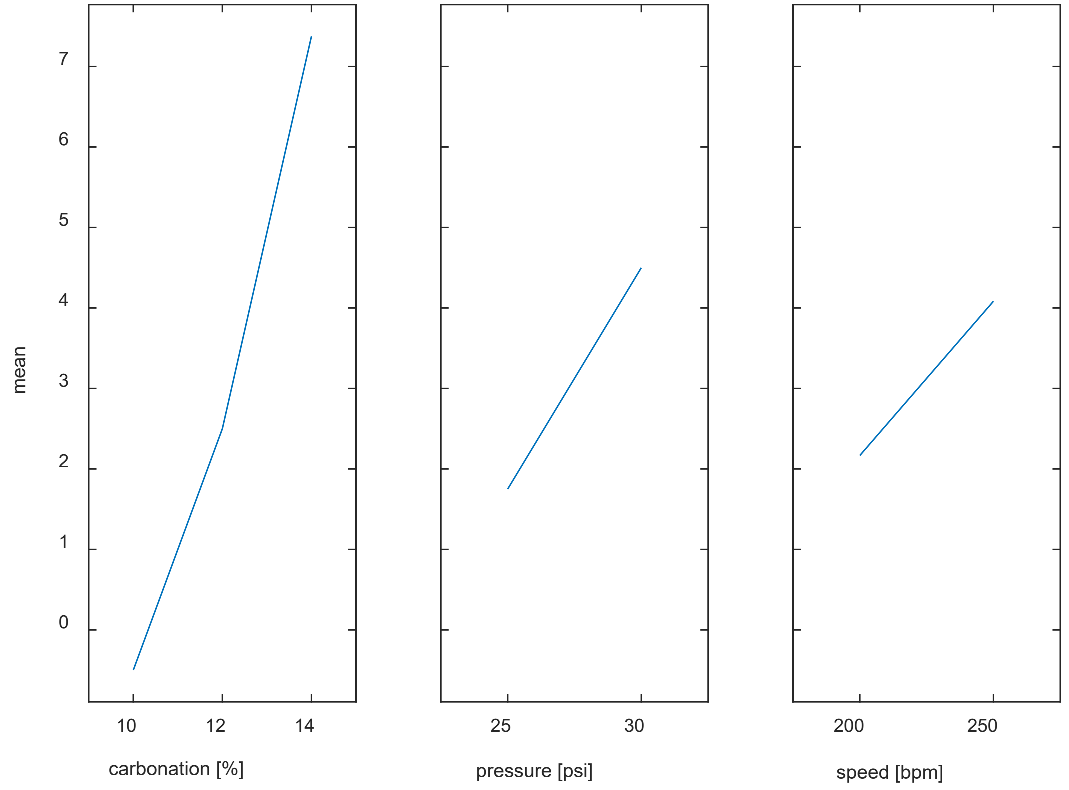
Analysis of Variance

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
X1	252.75	2	126.375	178.41	0
X2	45.375	1	45.375	64.06	0
X3	22.042	1	22.042	31.12	0.0001
X1*X2	5.25	2	2.625	3.71	0.0558
X1*X3	0.583	2	0.292	0.41	0.6715
X2*X3	1.042	1	1.042	1.47	0.2486
X1*X2*X3	1.083	2	0.542	0.76	0.4869
Error	8.5	12	0.708		
Total	336.625	23			

Constrained (Type III) sums of squares.

Main effects

- All three variables have *positive* main effects:
  - increasing the variable moves the average deviation from the fill target upward
    - increasing all the variables determine an increase of the filling level
- The most important effect is the one of soft drink carbonation:
  - the higher the content of CO_2, the higher the filling volume



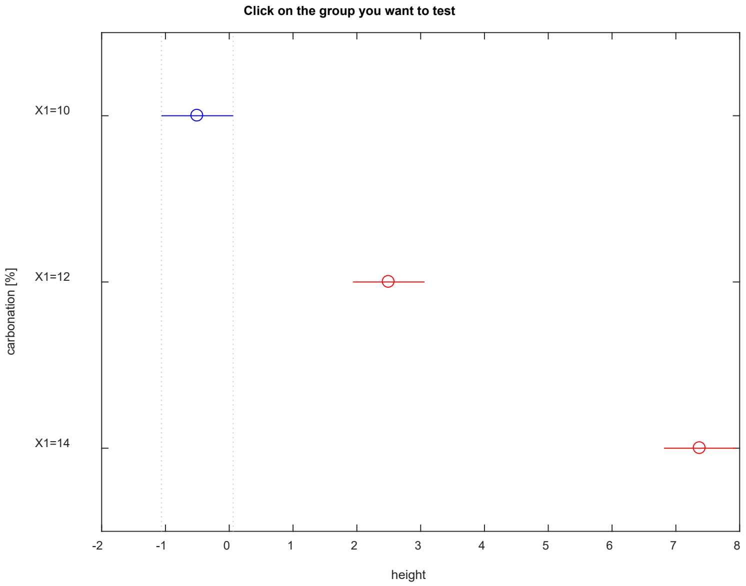
Main effect plot outcomes discussion

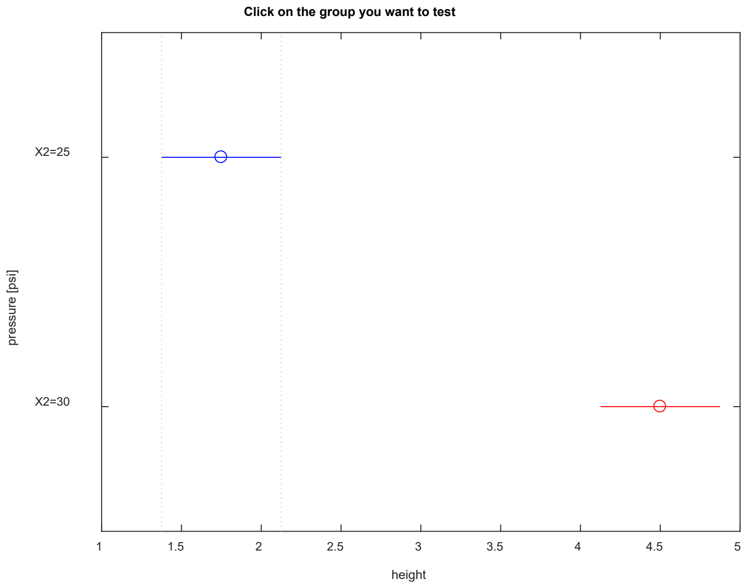
Ranking of factors importance	Factors effects	Linearity of factors effects
• What is the most important factor determining response variability? • What is the least important? • And what is the ranking of importance?	• Does response increase with the factor? • Does response decrease with the factor?	• Is the variability of the response linear with the factor variation? • Is the variability of the response non-linear with the factor variation?

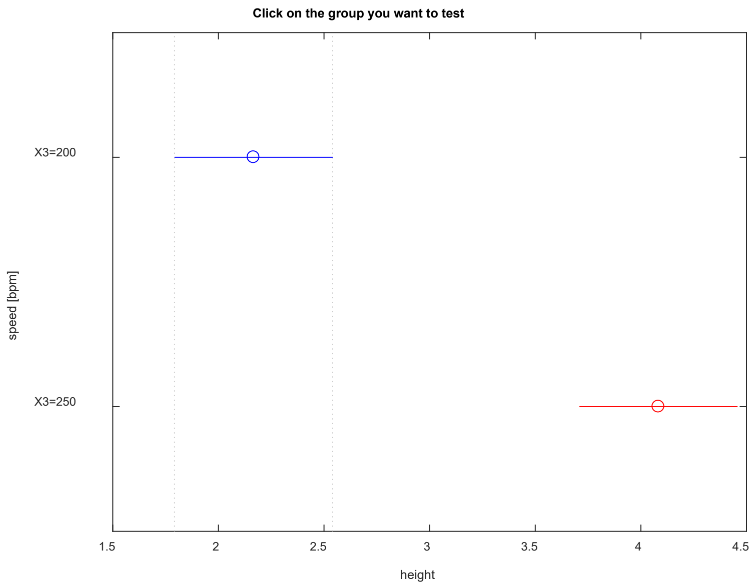
Conjectures on the statistical significance of the effects cannot be done

Mean effects

- Mean effects are well distinguishable one another and low variability of the means with respect to different levels



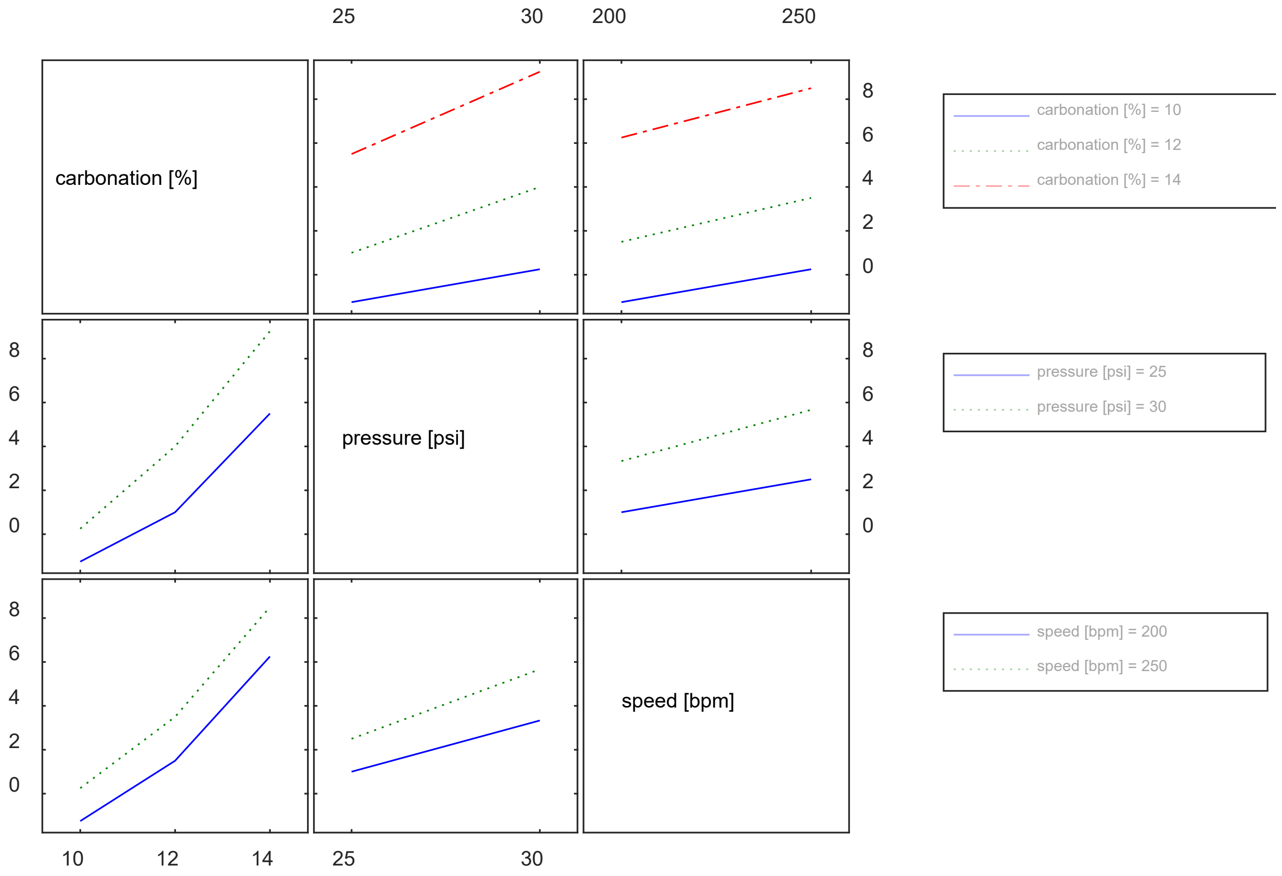




Interaction effects

- The interactions between carbonation, pressure and line speed are fairly small
    - lines are almost parallel

- Conjectures on the statistical significance of the interactions cannot be done



Practical results and conclusions form ANOVA

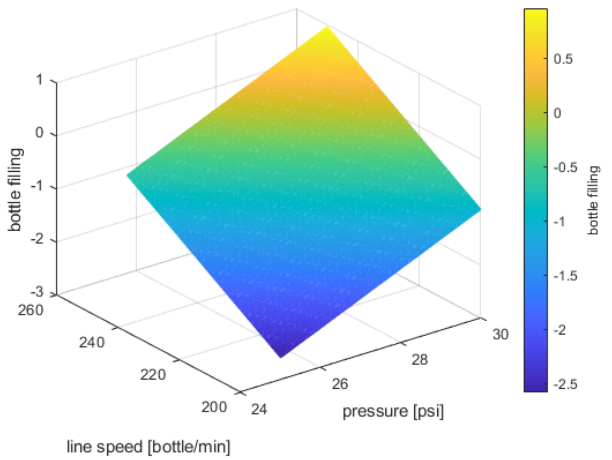
- The ANOVA model suggests that:
  - the carbonation of the soft drink should be maintained at the minimum level to guarantee the proper filling volume of the bottles
  - being the speed the factor with the lower impact increase it at the maximum to increase the production and the revenue

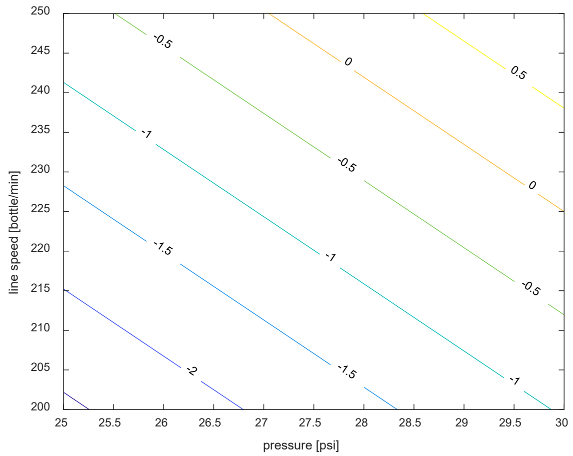
- Can we improve these results?

Response surface representation

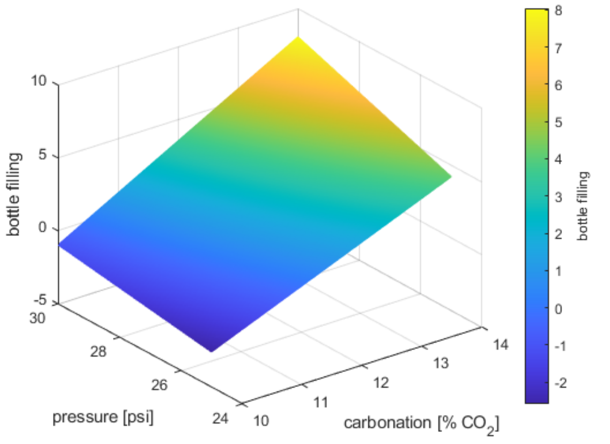
- The response surface regression model can be represented through **surfaces** and **contour plots**

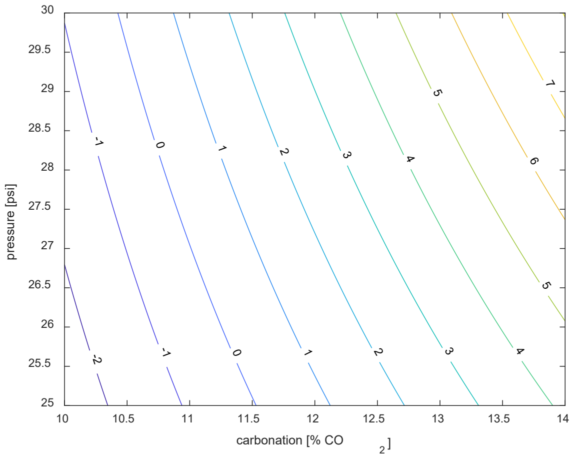
at constant minimum carbonation





at constant minimum line speed



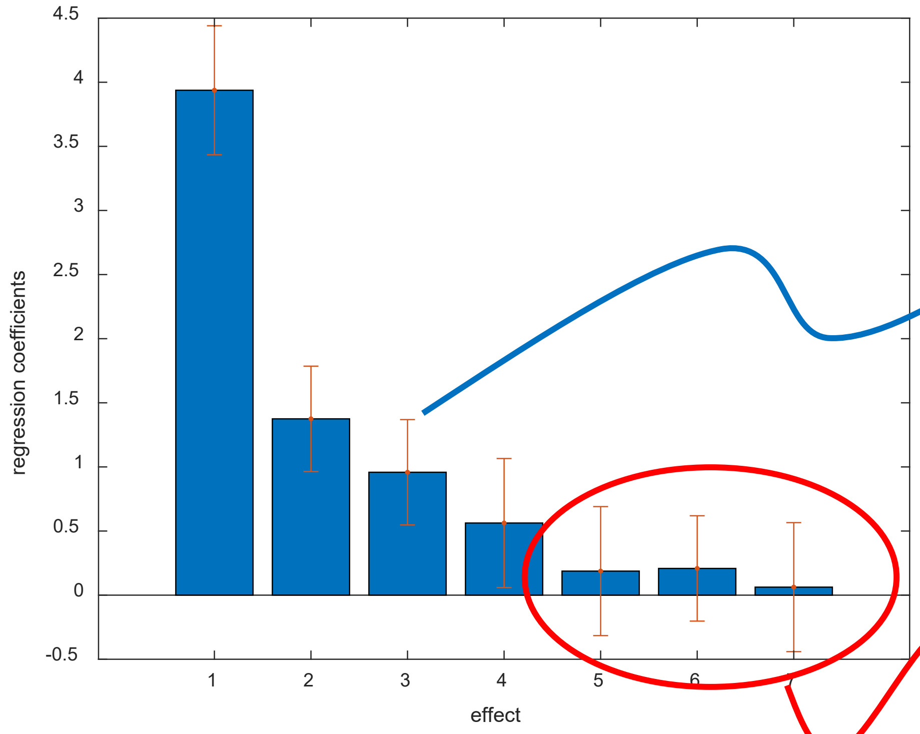


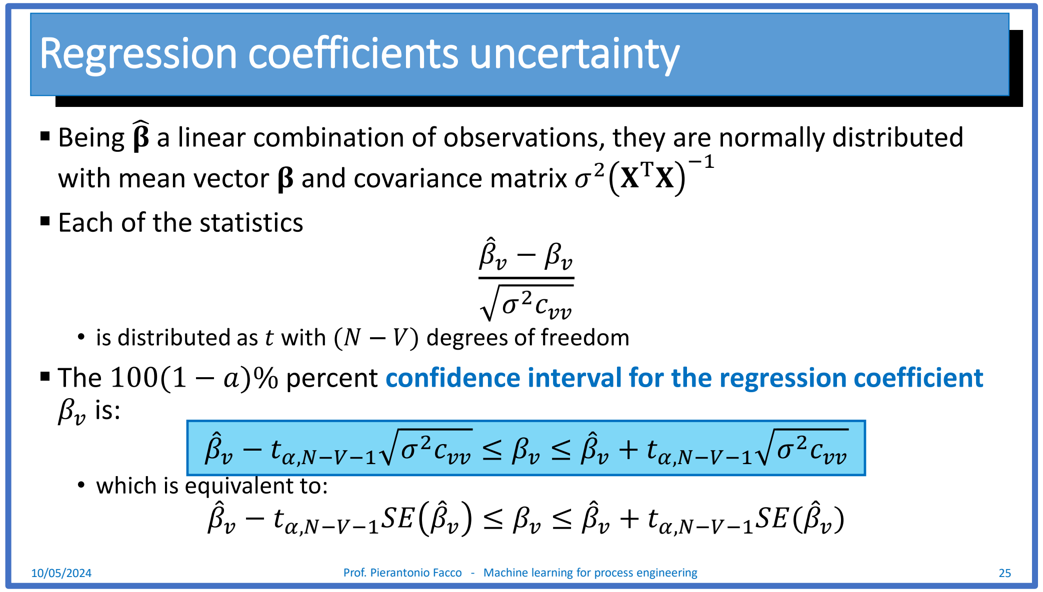
First tentative response surface model

- The complete response surface model is:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3 + \beta_7 x_1 x_2 x_3$$

- $R^2 > 0.95$





Regression coefficients uncertainty

- Being $\widehat{\boldsymbol{\beta}}$ a linear combination of observations, they are normally distributed with mean vector $\boldsymbol{\beta}$ and covariance matrix $\sigma^2(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}$
- Each of the statistics

$$\frac{\hat{\beta}_v - \beta_v}{\sqrt{\sigma^2 c_{vv}}}$$

  - is distributed as t with $(N - V)$ degrees of freedom
- The $100(1 - a)\%$ percent **confidence interval for the regression coefficient** β_v is:

$$\hat{\beta}_v - t_{\alpha,N-V-1}\sqrt{\sigma^2 c_{vv}} \leq \beta_v \leq \hat{\beta}_v + t_{\alpha,N-V-1}\sqrt{\sigma^2 c_{vv}}$$

  - which is equivalent to:

$$\hat{\beta}_v - t_{\alpha,N-V-1}SE(\hat{\beta}_v) \leq \beta_v \leq \hat{\beta}_v + t_{\alpha,N-V-1}SE(\hat{\beta}_v)$$

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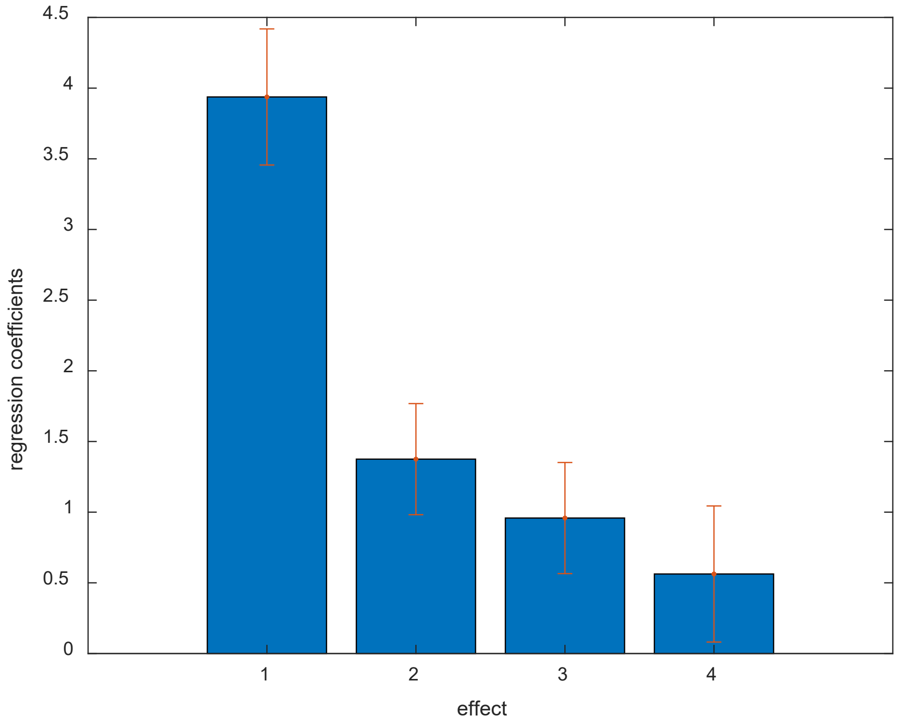
high uncertainty on these regression coefficients: the respective effects could be discarded

Updated response surface

- The updated version of the response surface model considers only the valuable effects with sufficiently low uncertainty:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2$$

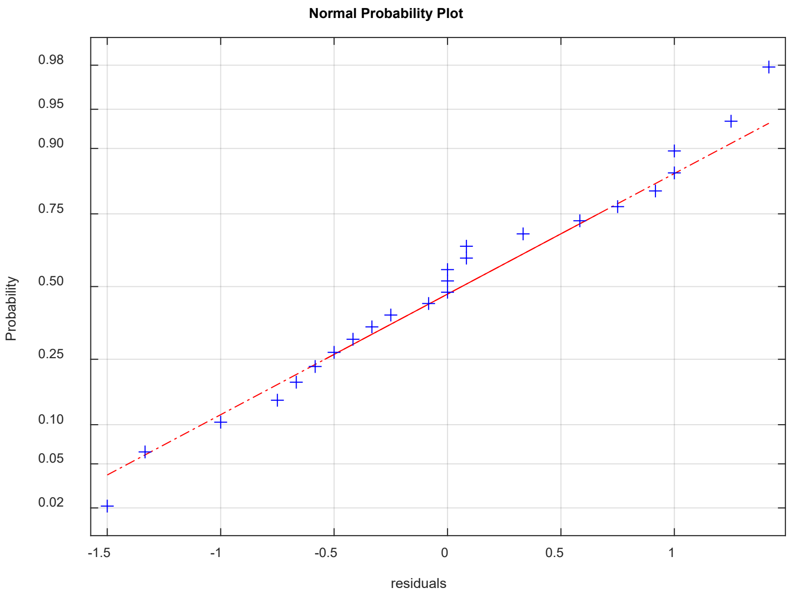


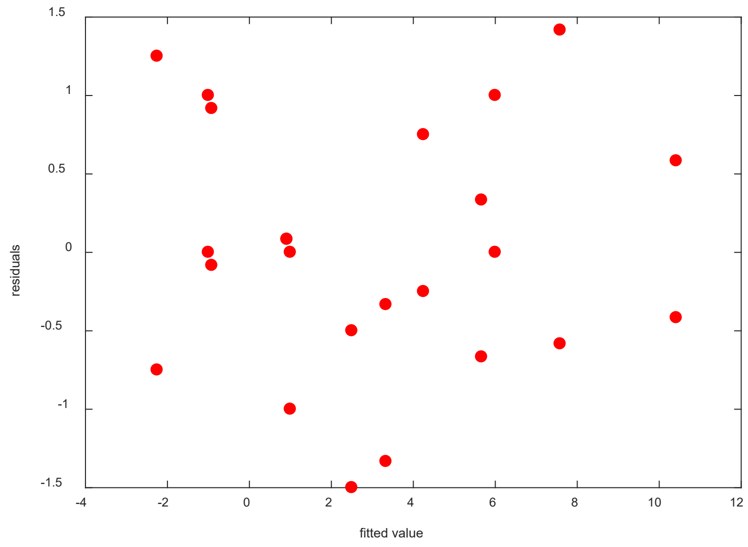
The ranking of the effect importance and the type of effect are clearly shown here

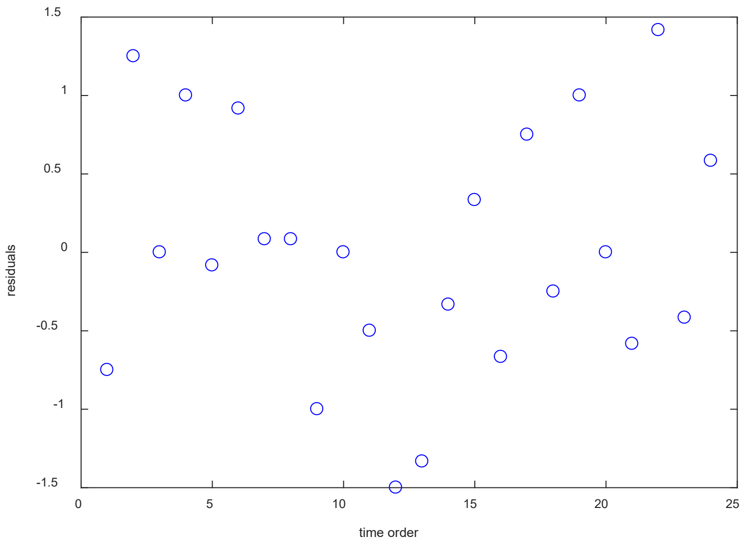
- ***CO_2 is the most impacting factor***
- ***a marginal interaction among CO_2 and pressure is present***

Residuals analysis

- Regular

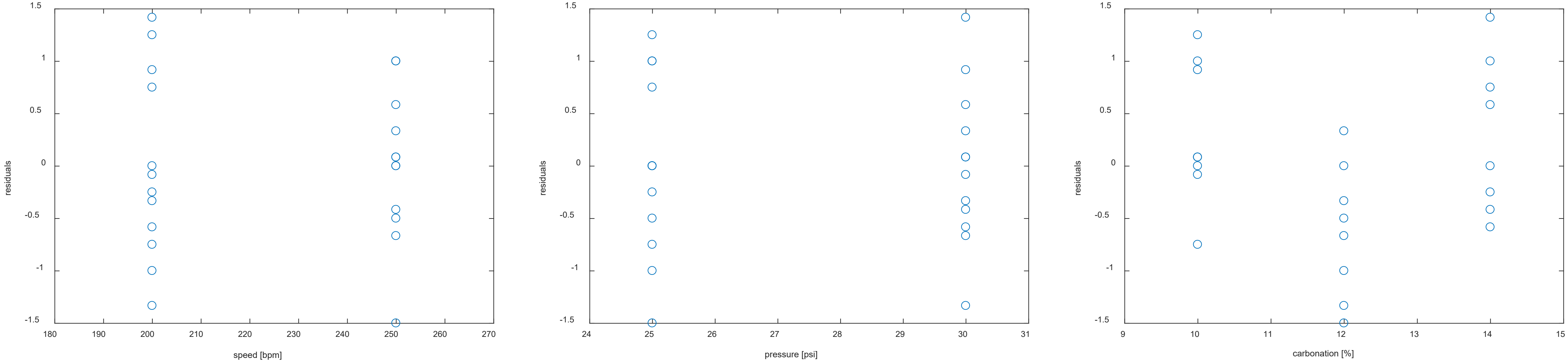






Residuals vs. variables

- Regular

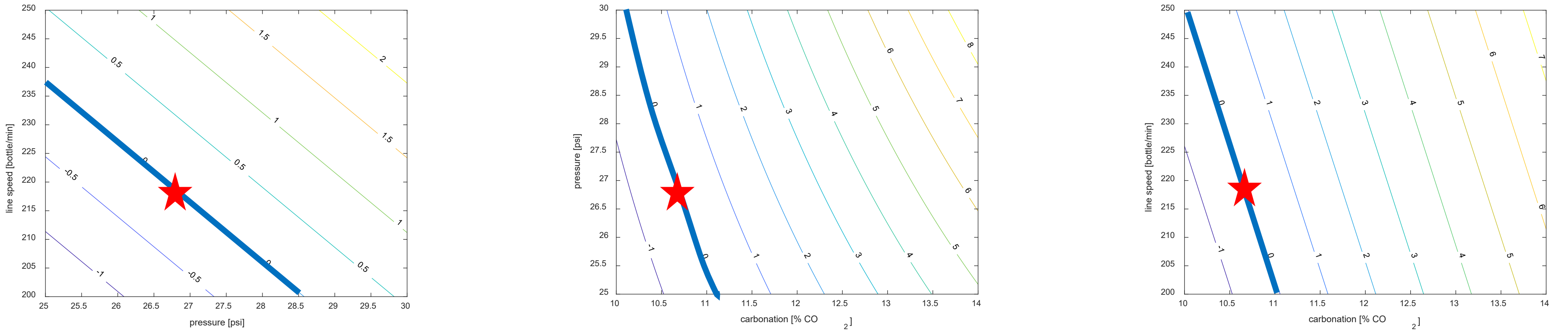


Finding optimal operating conditions

- The response surface approach gives us an extra value: **optimal operating conditions** can be found from the model!
- **Objective**: ensuring the optimal bottle filling:
  - no waste the soda
  - no clients' complaints
- The objective is that the bottle filling is on the target: $\hat{y} = 0$
- To find the solution both *good sense* and automatic computer software may be utilized:
  - suggested Matlab® commands:
    - `fzero`
    - `fmincon`
- For sure low levels of carbonation, low pressures and low line speed guarantee to avoid wasting drink:
  - however, we want to avoid also costumer complaints
  - we have to take into account that the bottle filling has a much wider sensitivity to carbonation then to other factors

Unconstrained optimal operating conditions

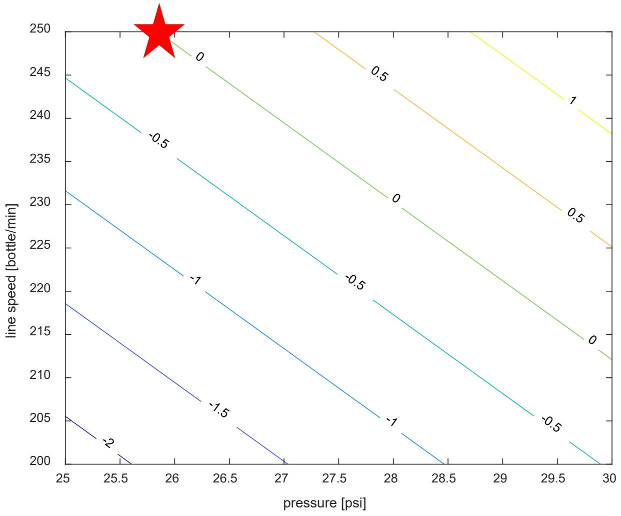
- The **optimal operating conditions** can be found applying blindly the automatic procedures in the software
- The suggested optimum to ensure $\hat{y} = 0$ is:
  - CO_2 carbonation = 10.7%
  - filling pressure = 26.8 psi
  - line speed = 218 bottle/min → ***decreasing the productivity is a major issue!***

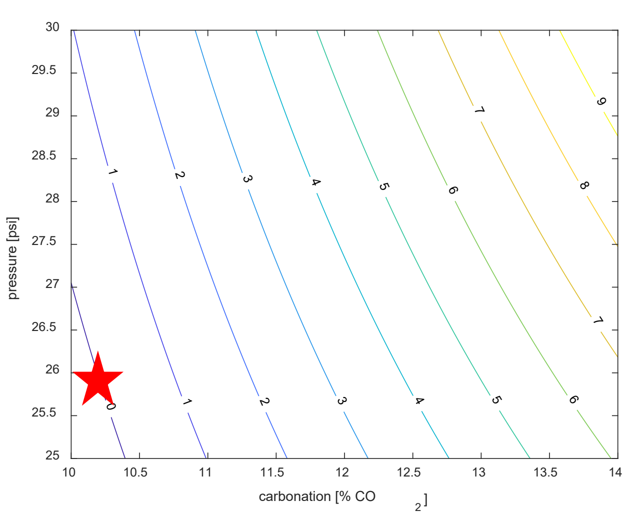


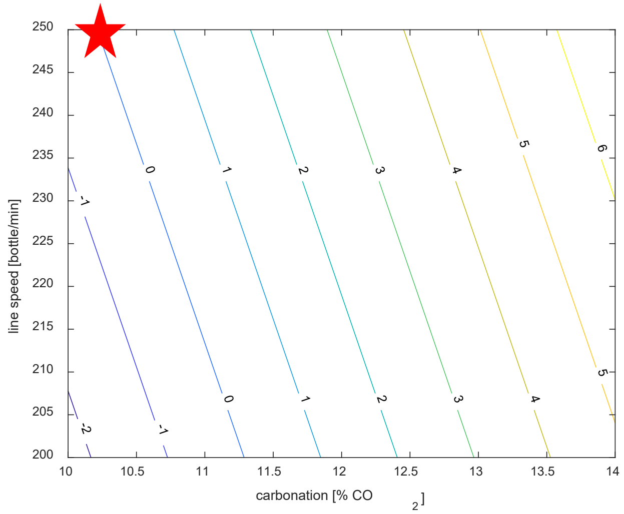
good point: we have good sense and we realize that a lot of equivalent conditions can guarantee the target response

Constrained optimal operating conditions

- We can add a constraint to the **optimal operating conditions** in such a way as to ensure the **maximum productivity** with line speed at 250 bottle/min:
- The suggested optimal process operating conditions to ensure the target $\hat{y} = 0$:
  - CO_2 carbonation = 10.2%
  - filling pressure = 25.9 psi
  - line speed = 250 bottle/min

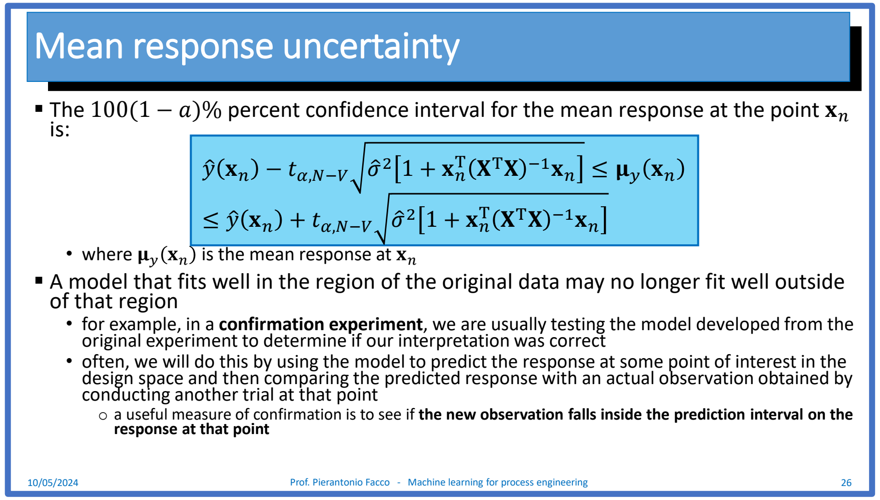






Estimation uncertainty and confirmation experiments

- The response estimation of the regression model is subject, as well as regression coefficients, to uncertainty



Mean response uncertainty

- The $100(1-\alpha)\%$ percent confidence interval for the mean response at the point $\mathbf{x}_n$ is:

$$\hat{y}(\mathbf{x}_n) - t_{\alpha,N-V}\sqrt{\hat{\sigma}^2\left[1+\mathbf{x}_n^{\mathrm{T}}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{x}_n\right]} \leq \boldsymbol{\mu}_y(\mathbf{x}_n) \leq \hat{y}(\mathbf{x}_n) + t_{\alpha,N-V}\sqrt{\hat{\sigma}^2\left[1+\mathbf{x}_n^{\mathrm{T}}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{x}_n\right]}$$

  - where $\boldsymbol{\mu}_y(\mathbf{x}_n)$ is the mean response at $\mathbf{x}_n$
- A model that fits well in the region of the original data may no longer fit well outside of that region
  - for example, in a **confirmation experiment**, we are usually testing the model developed from the original experiment to determine if our interpretation was correct
  - often, we will do this by using the model to predict the response at some point of interest in the design space and then comparing the predicted response with an actual observation obtained by conducting another trial at that point
    - a useful measure of confirmation is to see if **the new observation falls inside the prediction interval on the response at that point**

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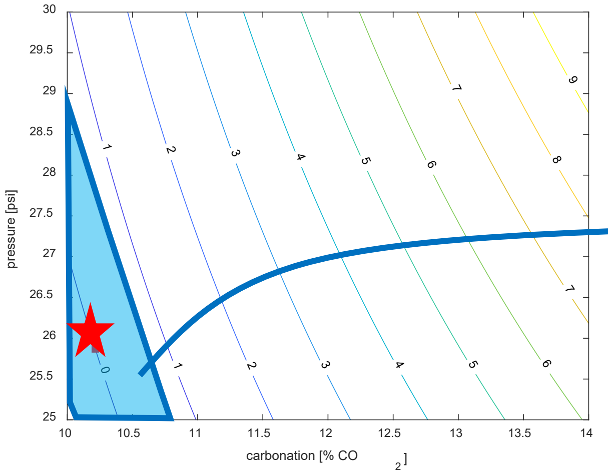


The outcome y of a confirmation experiment should not necessarily stay on the $y = 0$ contour line (whose selected point, the red star, is one of the possible optimal solutions), but it could be shifted of 0.8 due to model uncertainty

- The **uncertainty** associated to the response in the optimum operating point is, with a confidence level of $\alpha = 0.95$:

$$\hat{y} = 0 \pm 0.8$$

- Since **confirmation experiments** replicated at the suggested optimum [10.2% CO_2, 25.9 psi, 250 b/min] give an average bottle filling of $y = +0.2$
  - this means that our model is **fully reliable**!

DoE code for the bottle filling example (1/5)

```
clc
close all
clear all
load('…where the file is…\bottle_filling.mat')

% visual inspection
boxplot(Y);xlabel('y variable');ylabel('bottle filling height');

%% preliminary information
figure;maineffectsplot(Y,X,'varnames',{'CO_2 [%]','pressure [psi]','line speed [b/min]'})
figure;interactionplot(Y,X,'varnames',{'CO_2 [%]','pressure [psi]','line speed [b/min]'})

% n-way ANOVA
[p,tbl,stats]=anovan(Y,X,'model','full','varnames',{'CO_2 [%]','pressure [psi]','line speed [b/min]'});
figure;multcompare(stats,'dimension',1);
figure;multcompare(stats,'dimension',2);
figure;multcompare(stats,'dimension',3);
```

DoE code for the bottle filling example (2/5)

```
%% response surface modelling
XX=[ones(size(Xc,1),1) Xc(:,1) Xc(:,2) Xc(:,3) Xc(:,1).*Xc(:,2) Xc(:,1).*Xc(:,3) Xc(:,2).*Xc(:,3) Xc(:,1).*Xc(:,2).*Xc(:,3)];
[b,bint,r,rint,stats1]=regress(Y,XX);

figure;bar(b(2:end,1));hold on;errorbar(b(2:end,1),(bint(2:end,2)-bint(2:end,1))./2,'.');
xlabel('effect');ylabel('regression coefficients');

% updated model
XX=[ones(size(Xc,1),1) Xc Xc(:,1).*Xc(:,2)];
[b,bint,r,rint,stats2]=regress(Y,XX);

figure;bar(b(2:end,1));hold on;errorbar(b(2:end,1),(bint(2:end,2)-bint(2:end,1))./2,'.');
xlabel('effect');ylabel('regression coefficients');

% analysis of the residuals
figure;normplot(r);box on
figure;plot(r./std(r));xlabel('experiment');ylabel('standardized residuals');box on
figure;scatter(Y,r);ylabel('residuals');xlabel('y');box on
figure;scatter([ones(size(Xc,1),1) Xc Xc(:,1).*Xc(:,2)]*b,r);ylabel('residuals');xlabel('fitted y');box on
```

DoE code for the bottle filling example (3/5)

```
% display response surface and contour plot
f=@(b,x)b(1)+b(2)*x(:,1)+b(3)*x(:,2)+b(4)*x(:,3)+b(5)*x(:,1)*x(:,2);


response=zeros(201,201,201);
for n=-1:0.01:1
    for m=-1:0.01:1
        for k=-1:0.01:1
            response(round(100*n+101),round(100*m+101),round(100*k+101))=f(b,[n,m,k]);
        end
    end
end

rs(:,:)=response(1,:,:);
figure; mesh(25:5/200:30,200:50/200:250,rs'); xlabel('pressure [psi]');ylabel('line speed [bottle/min]');zlabel('bottle filling’);
c = colorbar;c.Label.String = 'bottle filling';
figure; contour(25:5/200:30,200:50/200:250,rs','ShowText','on'); xlabel('pressure [psi]');ylabel('line speed [bottle/min]');
rs(:,:)=response(:,:,1);
figure; mesh(10:4/200:14,25:5/200:30,rs'); xlabel('carbonation [% CO_2]');ylabel('pressure [psi]');zlabel('bottle filling’);
c = colorbar;c.Label.String = 'bottle filling';
figure; contour(10:4/200:14,25:5/200:30,rs','ShowText','on'); xlabel('carbonation [% CO_2]');ylabel('pressure [psi]');
rs(:,:)=response(:,1,:);
figure; mesh(10:4/200:14,200:50/200:250,rs'); xlabel('carbonation [% CO_2]');ylabel('line speed [bottle/min]');zlabel('bottle filling');c = colorbar;c.Label.String = 'bottle filling';
figure; contour(10:4/200:14,200:50/200:250,rs','ShowText','on'); xlabel('carbonation [% CO_2]');ylabel('line speed [bottle/min]’);
```

DoE code for the bottle filling example (4/5)

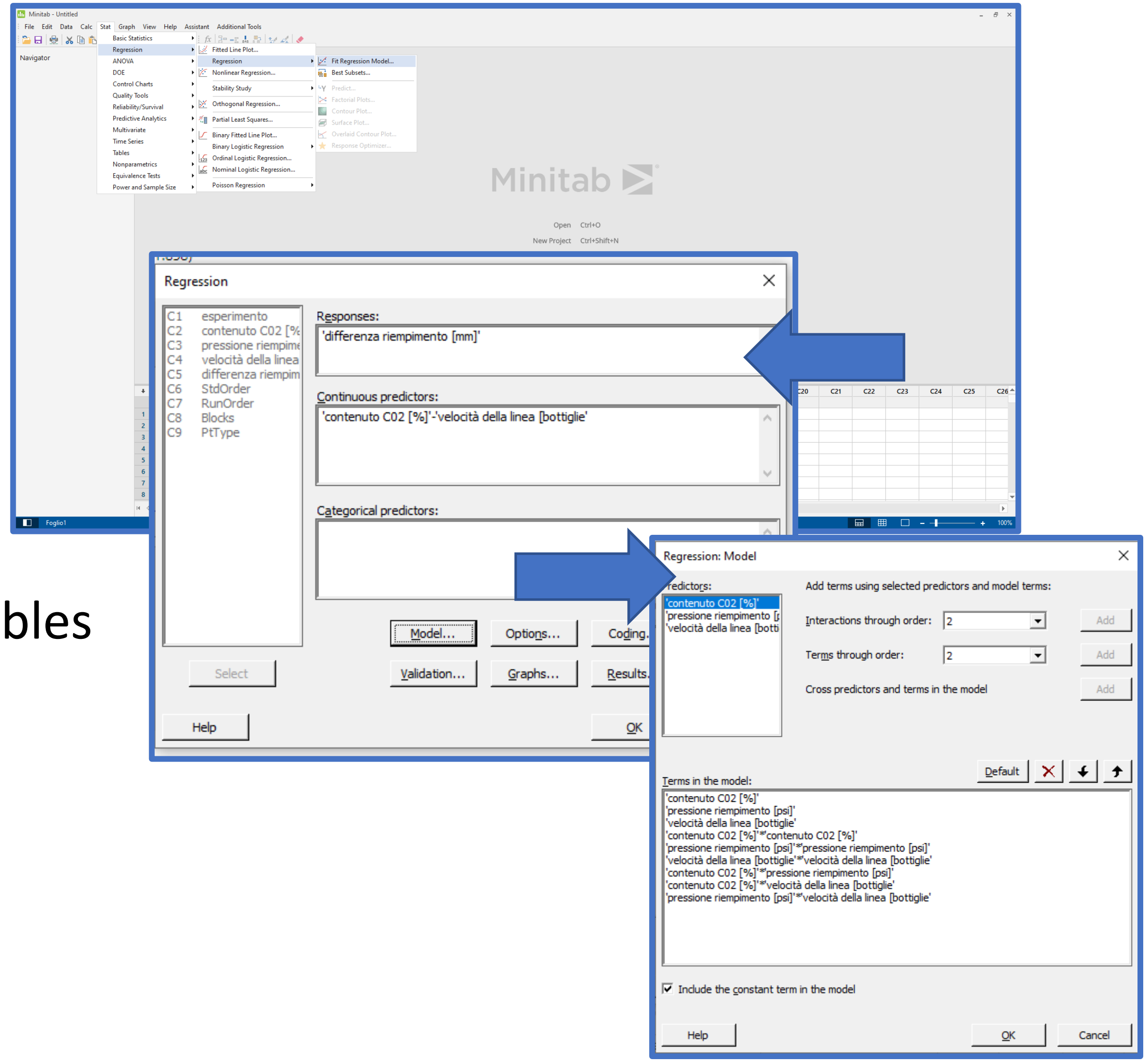
```
% optimum search
ff=@(x,b)b(1)+b(2)*x(1)+b(3)*x(2)+b(4)*x(3)+b(5)*x(1)*x(2);
fun=@(x)abs(ff(x,b));
x0=[0 0 0];
lb=[-1 -1 -1];
ub=[1 1 1];
optimum1=fmincon(fun,x0,[],[],[],[],lb,ub);
display('The optimal operatingh point is');
ff(optimum1,b)
display('at conditions:’);
optimum1

rs(:,:)=response(round(100*(optimum1(1)+1)+1),:,:);
figure; contour(25:5/200:30,200:50/200:250,rs','ShowText','on'); xlabel('pressure [psi]’);
ylabel('line speed [bottle/min]');hold on; scatter(5/2*(1+optimum1(2))
+25,50/2*(1+optimum1(3))+200,'sr','filled');hold off
rs(:,:)=response(:,:,round(100*(optimum1(3)+1)+1));
figure; contour(10:4/200:14,25:5/200:30,rs','ShowText','on'); xlabel('carbonation [% CO_2]’);
ylabel('pressure [psi]'); hold on;
scatter(4/2*(1+optimum1(1))+10,5/2*(1+optimum1(2))+25,'sr','filled');hold off
rs(:,:)=response(:,round(100*(optimum1(2)+1)+1),:);
figure; contour(10:4/200:14,200:50/200:250,rs','ShowText','on'); xlabel('carbonation [% CO_2]');ylabel('line speed [bottle/min]');hold on;
scatter(4/2*(1+optimum1(1))+10,50/2*(1+optimum1(3))+200,'sr','filled');hold off
```

DoE code for the bottle filling example (5/5)

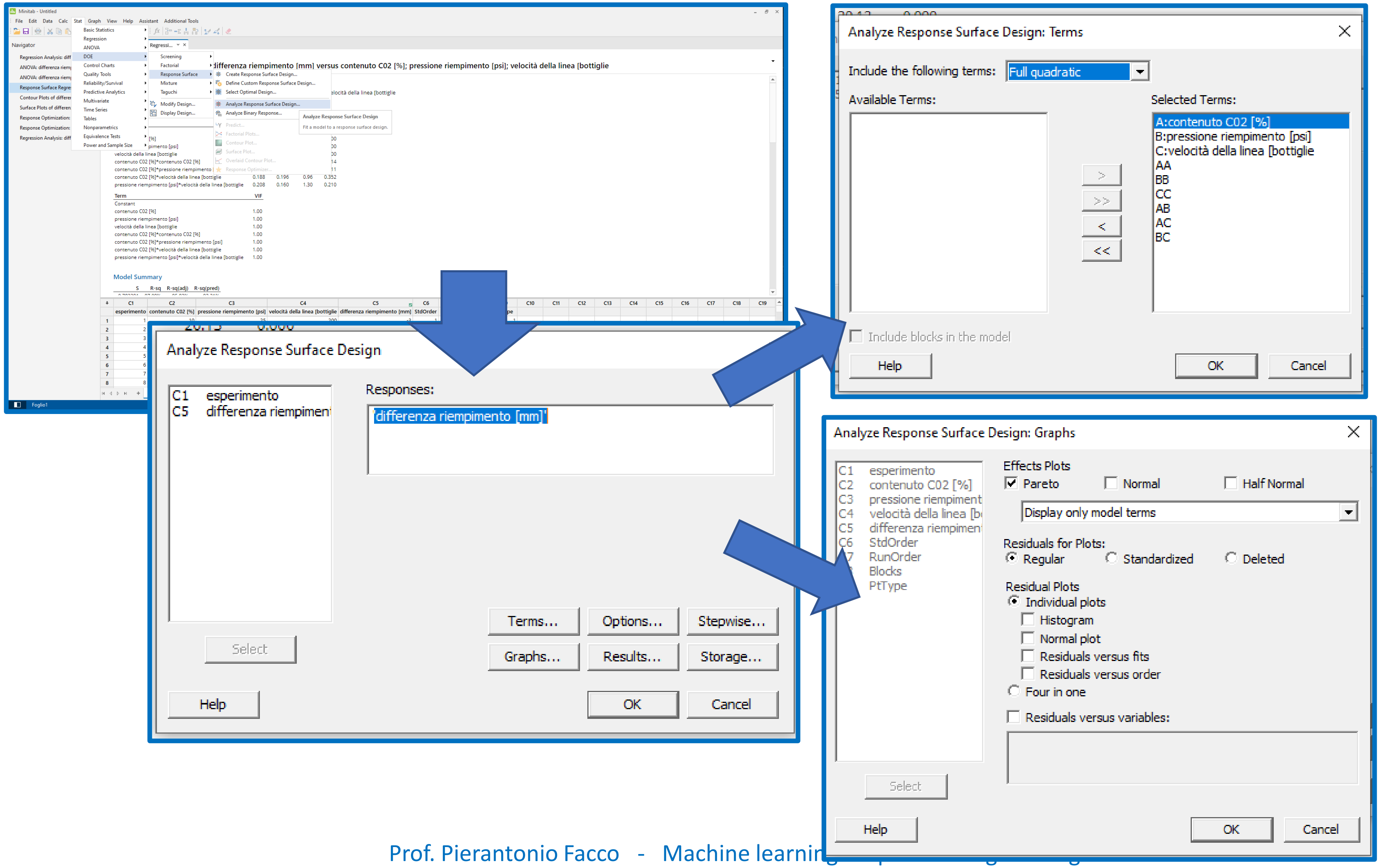
```
% constrained optimum search!
x0=[0 0 0];
lb=[-1 -1 -1];
ub=[1 1 1];
optimum2=fmincon(fun,x0,[],[],[0 0 1],[250],lb,ub);
display('The optimal operating point subject to the constaint of maximum productivity is:');
ff(optimum2,b)
display('at conditions:');
optimum2


rs(:,:)=response(round(100*(optimum2(1)+1)+1),:,:);
figure; contour(25:5/200:30,200:50/200:250,rs','ShowText','on'); xlabel('pressure [psi]');ylabel('line speed [bottle/min]');hold on;
scatter(5/2*(1+optimum2(2))+25,50/2*(1+optimum2(3))+200,'sr','filled');hold off
rs(:,:)=response(:,:,round(100*(optimum2(3)+1)+1));
figure; contour(10:4/200:14,25:5/200:30,rs','ShowText','on'); xlabel('carbonation [% CO_2]');ylabel('pressure [psi]'); hold on;
scatter(4/2*(1+optimum2(1))+10,5/2*(1+optimum2(2))+25,'sr','filled');hold off
rs(:,:)=response(:,round(100*(optimum2(2)+1)+1),:);
figure; contour(10:4/200:14,200:50/200:250,rs','ShowText','on'); xlabel('carbonation [% CO_2]');ylabel('line speed [bottle/min]');
hold on; scatter(4/2*(1+optimum2(1))+10,50/2*(1+optimum2(3))+200,'sr','filled');hold off

% prediction uncertainty
display('The uncertainty associated to the optimum operating point is:');


tinv(0.975,size(XX,1)-size(XX,2))*realsqrt(sum(r.^2)/(size(XX,1)-size(XX,2))*[1 optimum2 optimum2(1,1)*optimum2(1,2)]*pinv(XX'*XX)*[1 optimum2 optimum2(1,1)*optimum2(1,2)]')
```

Implementation in Minitab®

- Select:
  - **Stat**
  - **DOE**
  - **Response surface**
  - **Analyze response surface Design**
- Click OK
- Select as **Continuous factors** the variables:
  - ***carbonation***
  - ***pressure***
  - ***velocity***
- Select **High/Low**, verify the summary of the variables and click OK
- Select **Filling height** as **Responses**
- Select the appropriate **Terms**
- Select the appropriate **Graphs**
- Click OK



Response superface implementation

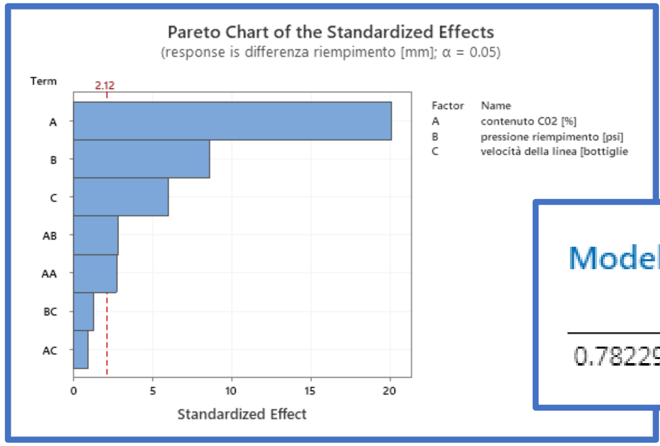


Response surface results

- Summary of the model outcome

Regression Equation in Uncoded Units

differenza riempimento [mm] = 56.7 - 7.59 contenuto C02 [%]
- 1.550 pressione riempimento [psi]
- 0.0983 velocità della linea [bottiglie
+ 0.2344 contenuto C02 [%]*contenuto C
+ 0.1125 contenuto C02 [%]*pressione ri
+ 0.00375 contenuto C02 [%]*velocità de
+ 0.00333 pressione riempimento [psi]*v
tiglie



Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.500	0.277	9.04	0.000
contenuto C02 [%]	3.937	0.196	20.13	0.000
pressione riempimento [psi]	1.375	0.160	8.61	0.000
velocità della linea [bottiglie	0.958	0.160	6.00	0.000
contenuto C02 [%]*contenuto C02 [%]	0.938	0.339	2.77	0.014
contenuto C02 [%]*pressione riempimento [psi]	0.563	0.196	2.88	0.011
contenuto C02 [%]*velocità della linea [bottiglie	0.188	0.196	0.96	0.352
pressione riempimento [psi]*velocità della linea [bottiglie	0.208	0.160	1.30	0.210

Term	VIF
Constant	
contenuto C02 [%]	1.00
…nto [psi]	1.00
…ottiglie	1.00
…ontenuto C02 [%]	1.00
…ressione riempimento [psi]	1.00
…elocità della linea [bottiglie	1.00
pressione riempimento [psi]*velocità della linea [bottiglie	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.782291	97.09%	95.82%	93.21%

Regression model

- **Accurate regression model**: the determination coefficient is high
  - CO_2 has the highest coefficient, meaning that it is the most influential factor n the response
  - all the parameters are positive, meaning that increasing the factors, the response increases

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.500	0.277	9.04	0.000
contenuto C02 [%]	3.937	0.196	20.13	0.000
pressione riempimento [psi]	1.375	0.160	8.61	0.000
velocità della linea [bottiglie	0.958	0.160	6.00	0.000
contenuto C02 [%]*contenuto C02 [%]	0.938	0.339	2.77	0.014
contenuto C02 [%]*pressione riempimento [psi]	0.563	0.196	2.88	0.011
contenuto C02 [%]*velocità della linea [bottiglie	0.188	0.196	0.96	0.352
pressione riempimento [psi]*velocità della linea [bottiglie	0.208	0.160	1.30	0.210

Term	VIF
Constant	
contenuto C02 [%]	1.00
pressione riempimento [psi]	1.00
velocità della linea [bottiglie	1.00
contenuto C02 [%]*contenuto C02 [%]	1.00
contenuto C02 [%]*pressione riempimento [psi]	1.00
contenuto C02 [%]*velocità della linea [bottiglie	1.00
pressione riempimento [psi]*velocità della linea [bottiglie	1.00

Regression Equation in Uncoded Units

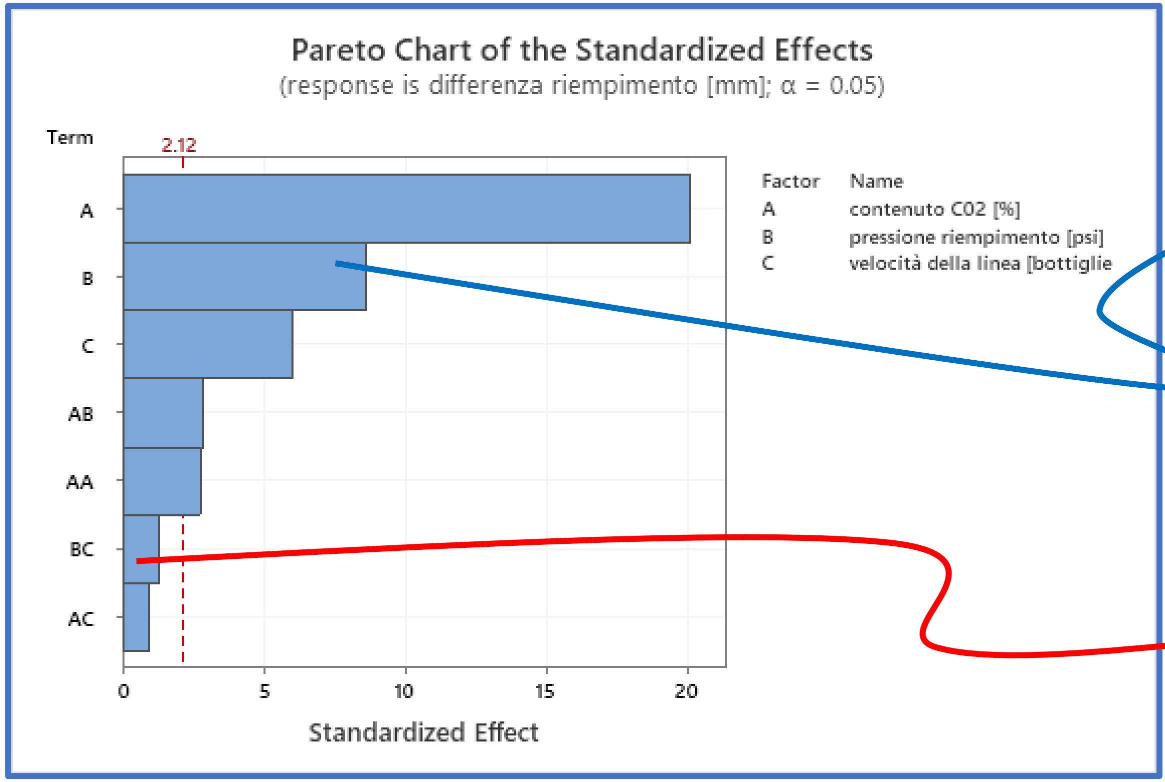
differenza riempimento [mm] = 56.7 - 7.59 contenuto C02 [%]
- 1.550 pressione riempimento [psi]
- 0.0983 velocità della linea [bottiglie
+ 0.2344 contenuto C02 [%]*contenuto C02 [%]
+ 0.1125 contenuto C02 [%]*pressione riempimento [psi]
+ 0.00375 contenuto C02 [%]*velocità della linea [bottiglie
+ 0.00333 pressione riempimento [psi]*velocità della linea [bot
tiglie

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.782291	97.09%	95.82%	93.21%

Factors importance

- The Pareto diagram shows what are the most important and the statistically meaningful factors



most influential factors

factors not significantly influencing the response

Regression coefficients

- The most important factors are those whose absolute value is larger and whose p-value is lower (<0.05)

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	2.500	0.277	9.04	0.000
contenuto C02 [%]	3.937	0.196	20.13	0.000
pressione riempimento [psi]	1.375	0.160	8.61	0.000
velocità della linea [bottiglie	0.958	0.160	6.00	0.000
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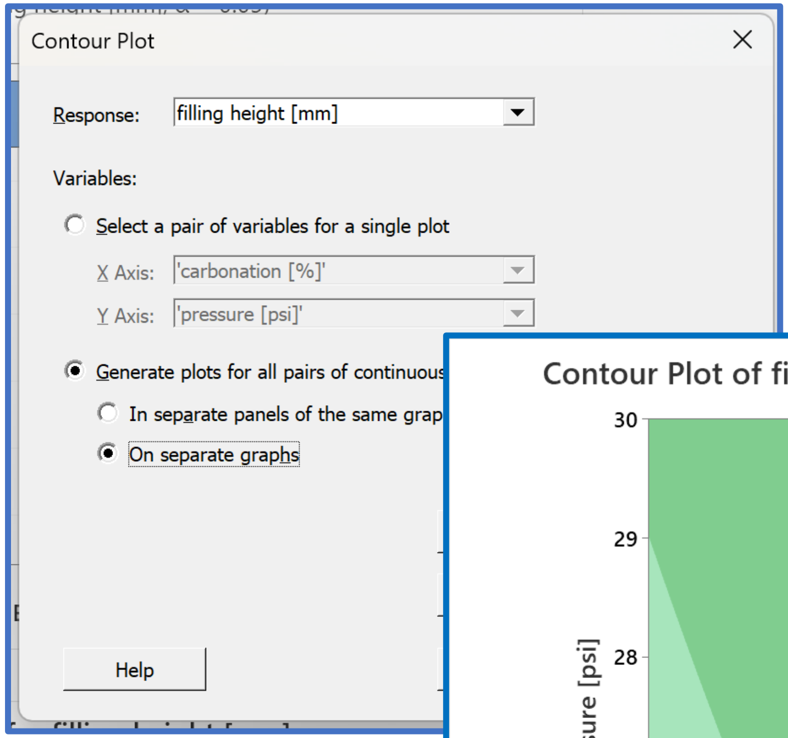
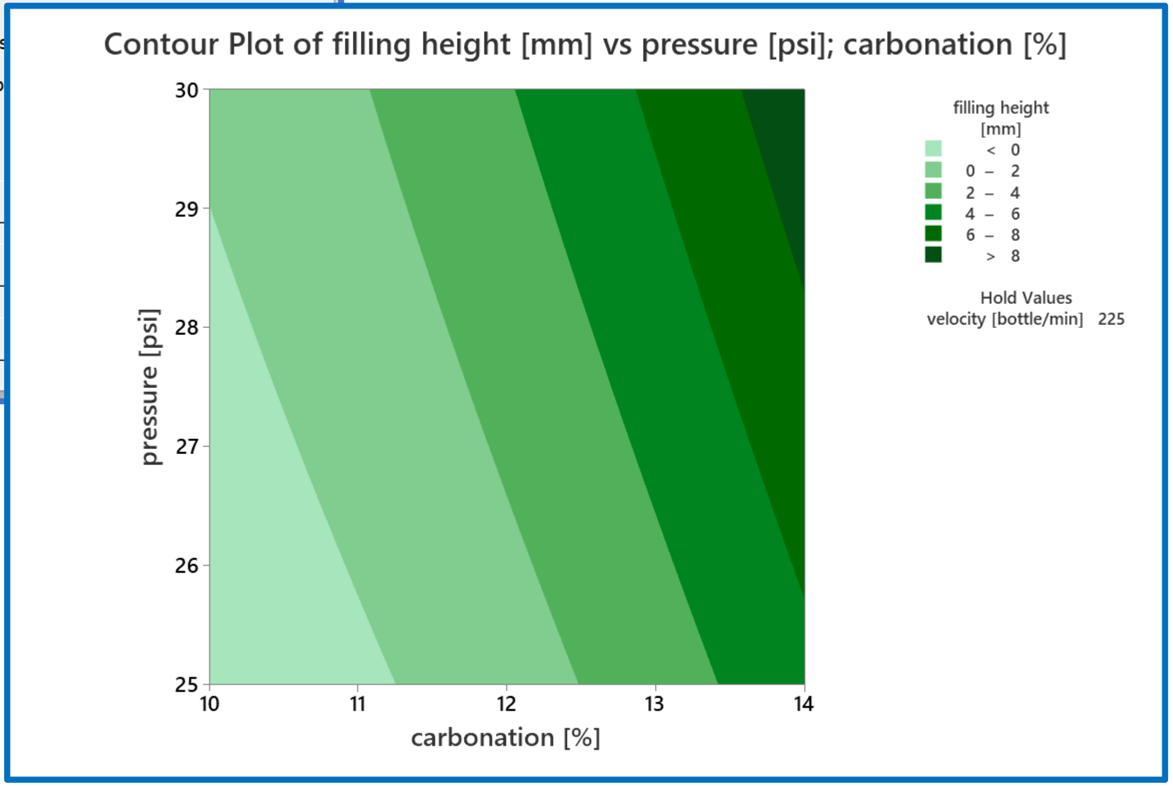
Term	VIF
Constant	
contenuto C02 [%]	1.00
pressione riempimento [psi]	1.00
velocità della linea [bottiglie	1.00
contenuto C02 [%]*contenuto C02 [%]	1.00
contenuto C02 [%]*pressione riempimento [psi]	1.00
contenuto C02 [%]*velocità della linea [bottiglie	1.00
pressione riempimento [psi]*velocità della linea [bottiglie	1.00

these coefficients refer to coded variables in the range -1, 0, 1, to give the same importance to the factors despite the dfferences in the measurement units

Response surface visualization as a contour plot

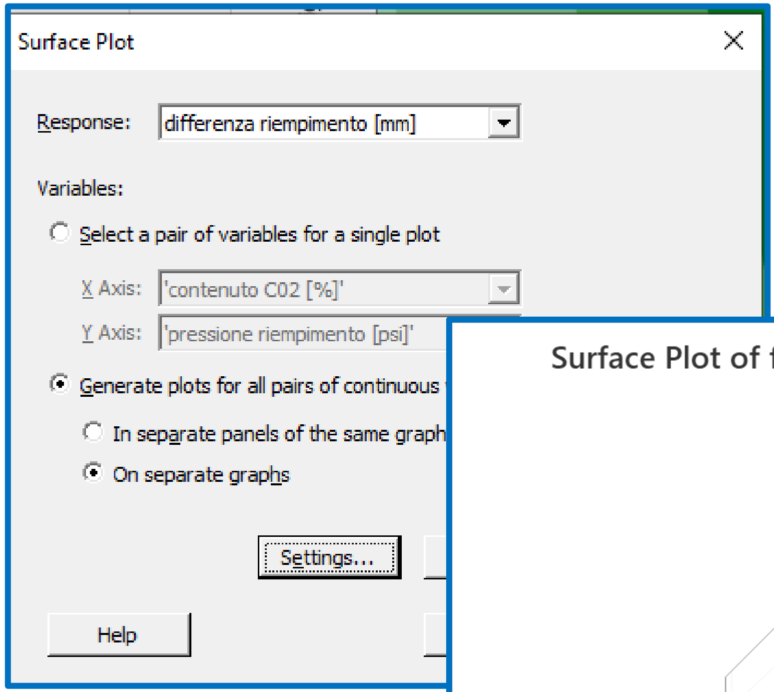
- Select:
  - **Stat**
  - **DOE**
  - **Response surface**
  - **Contour plot**
- Select the appropriate options
- Click OK

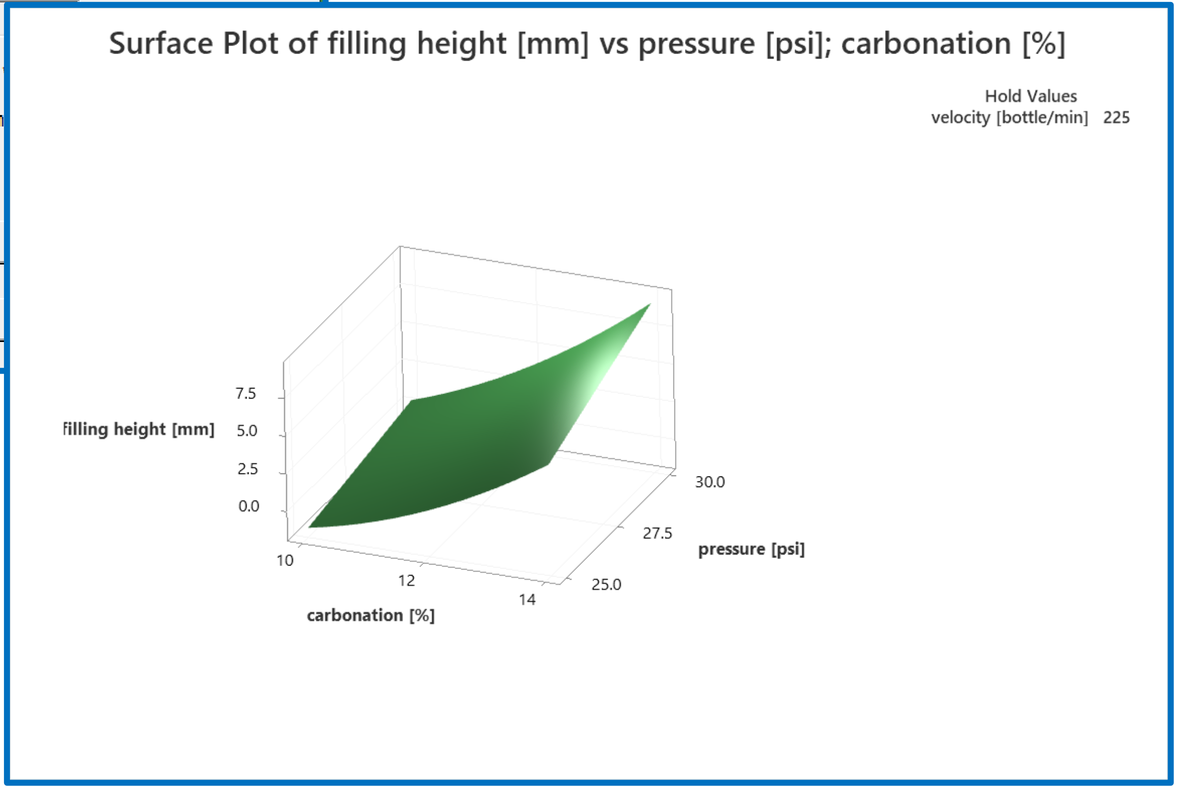




Response surface visualization as a surface

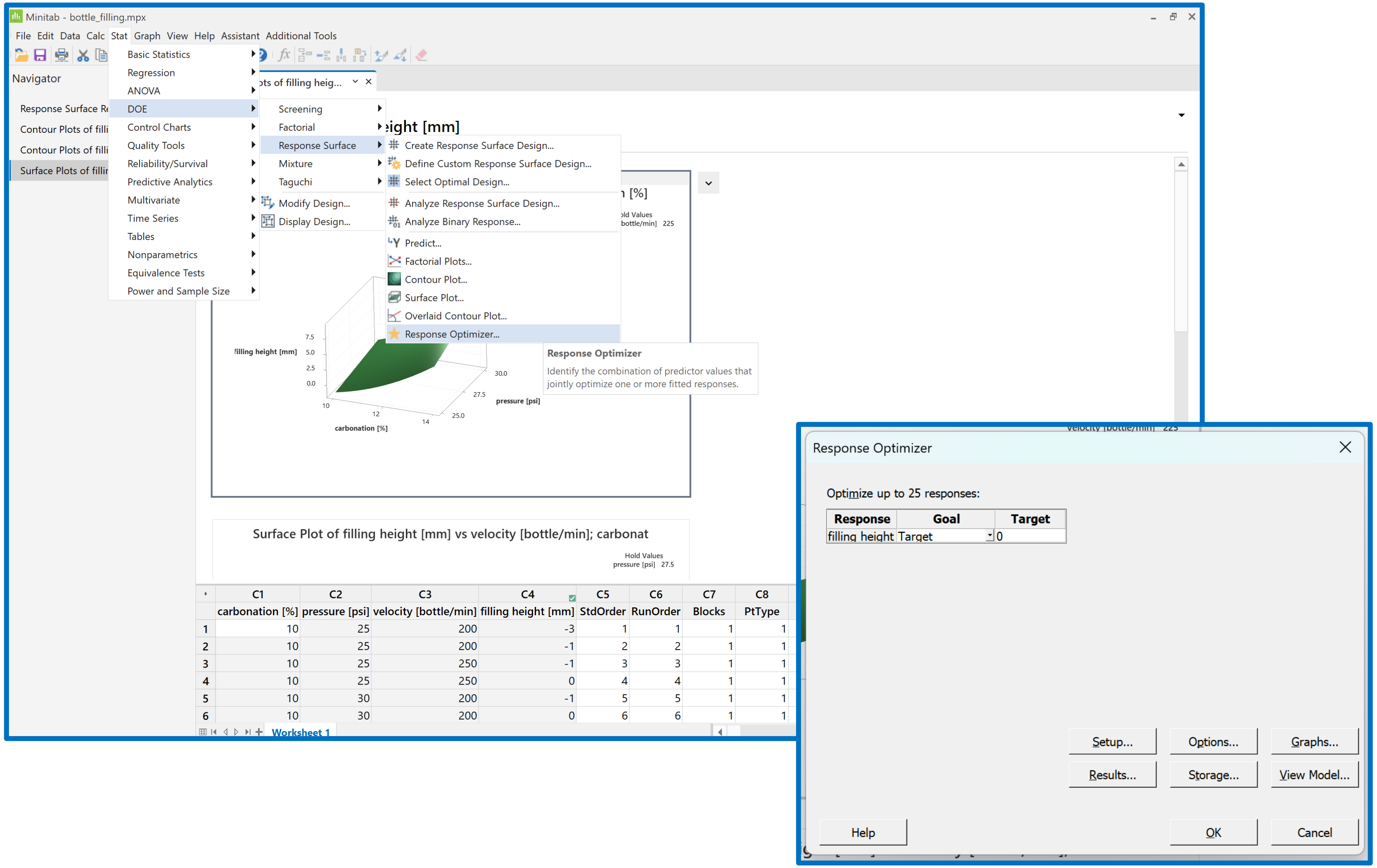
- Select:
  - **Stat**
  - **DOE**
  - **Response surface**
  - **Surface plot**
- Select the appropriate options
- Click OK





Find the optimal point

- Select:
  - **Stat**
  - **DOE**
  - **Response surface**
  - **Response optimizer**
- Select **Target = 0**
- Click OK twice



Optimization outcome

- Suggested optimal point:
  - CO_2 content = 11.8%
  - pressure = 25 psi
  - velocity = 200 bott./min

Multiple Response Prediction

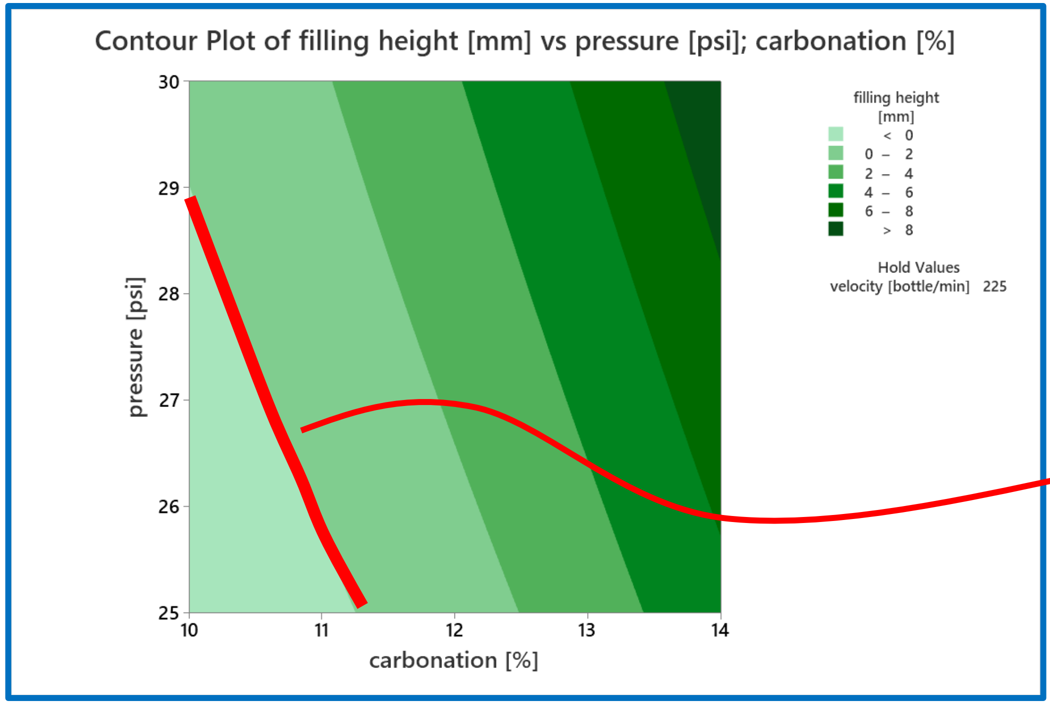
Variable	Setting
carbonation [%]	11.756
pressure [psi]	25
velocity [bottle/min]	200

Response	Fit	SE Fit	95% CI	95% PI
filling height [mm]	0.000	0.390	(-0.828; 0.828)	(-1.853; 1.853)

this is not the maximum productivity!

… judge the results in a critical manner!

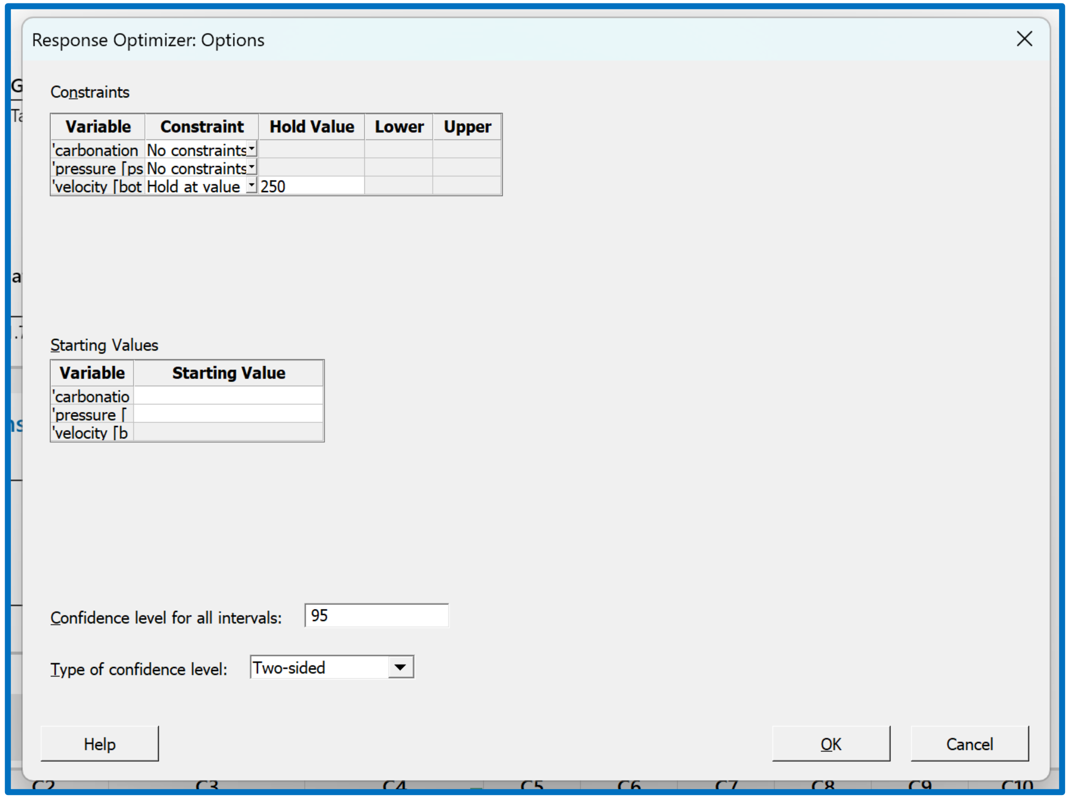
- **Many combinations of the factors guarantee to obtain the target**
- The filling is **not very sensitive to the line velocity**
- **One can ensure the maximum productivity, namely the maximum line velocity and the target at the same time!**



multiple combinations of the factors may warrant the target

Risultato di una ottimizzazione vincolata

- Select:
  - **Stat**
  - **DOE**
  - **Response surface**
  - **Response optimizer**
- Select **Target = 0**
- Select:
  - **Options**
  - **Hold at avalue = 250** for the velocity of the line
- Click OK twice



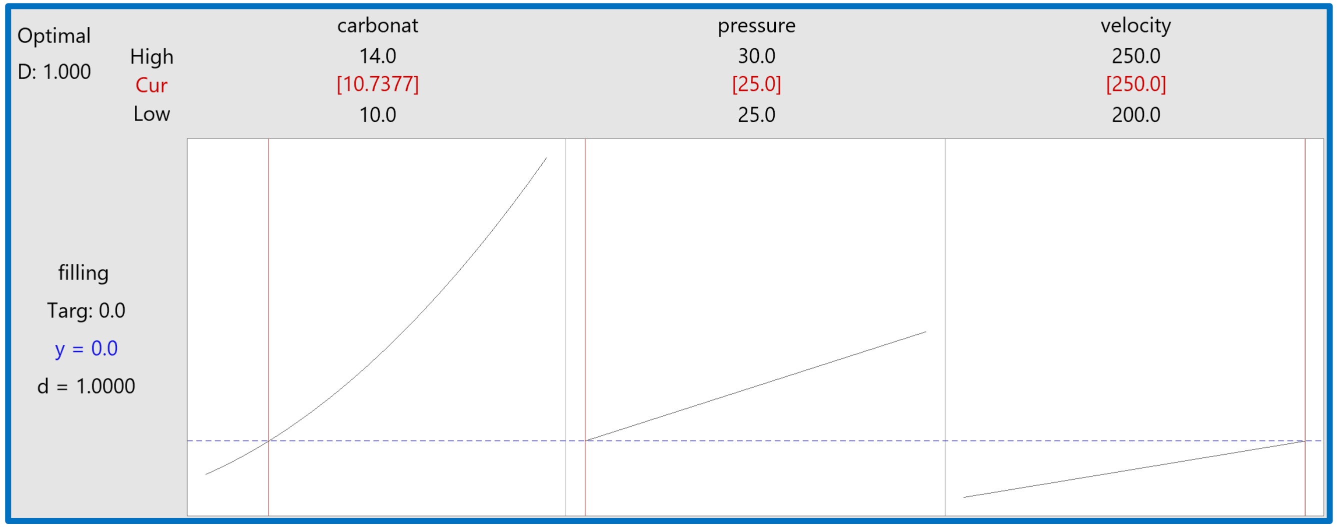
Optimal process conditions

- The optimal point is:
  - carbonation = 10.7%
  - pressure = 25 psi
  - line velocity = 250 bott./min

Multiple Response Prediction

Variable	Setting
carbonation [%]	10.7377
pressure [psi]	25
velocity [bottle/min]	250

Response	Fit	SE Fit	95% CI	95% PI
filling height [mm]	0.000	0.395	(-0.837; 0.837)	(-1.858; 1.858)



Example: cutting tool

Cutting machine case study

- The effective life of a cutting tool installed in a numerically controlled machine is thought to be affected by:
  - the **cutting speed**
  - the **tool angle**
- **Objective**: understanding the relation among **cutting machine life** and the two factors
- Experimental campaign: 3^2 full factorial design
  - cutting speed: 125, 150, 175 in/min
  - cutting angle: 15°, 20°, 25°
  - replicates per experiment: 2
  - total of 18 experiments

N-way ANOVA

- Interaction is of paramount importance:
  - only 1 of the degrees of freedom in the interaction is exploited
  - further interaction terms can be computed

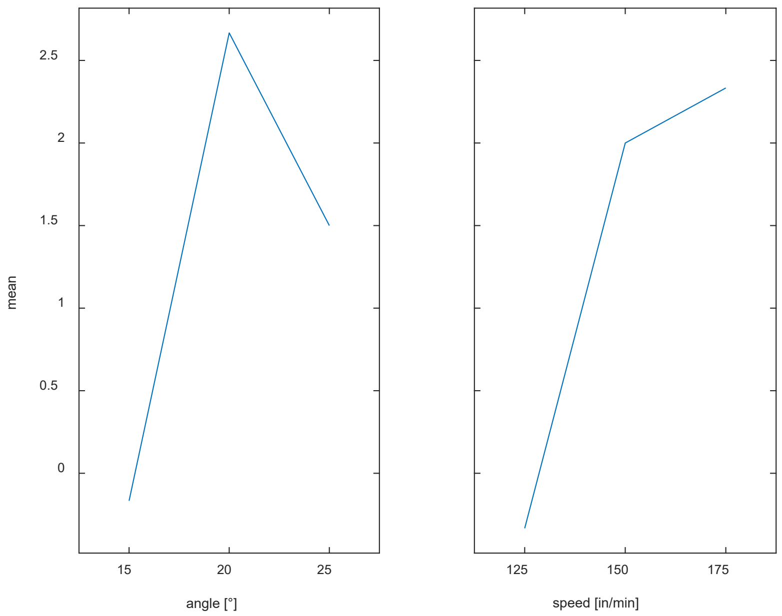
Analysis of Variance

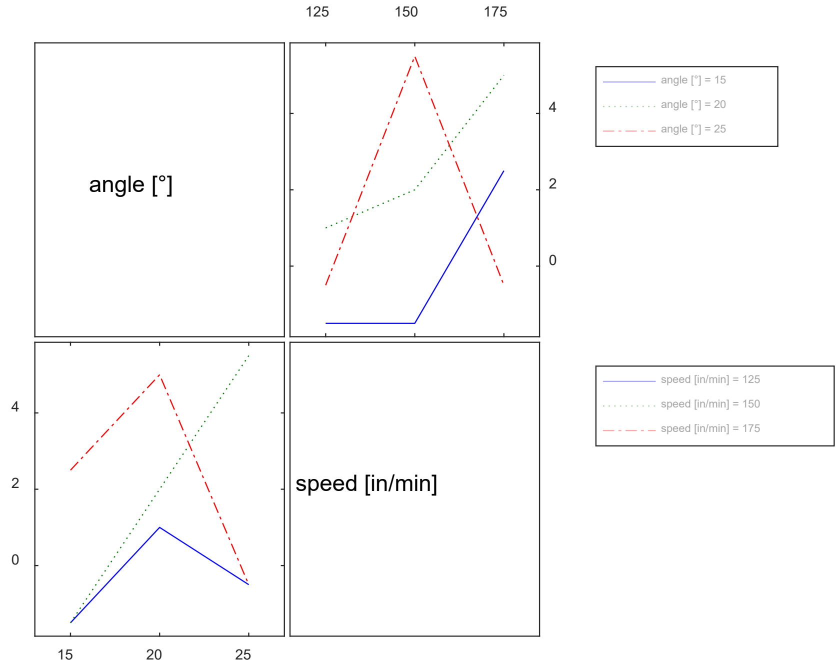
Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
X1	24.3333	2	12.1667	8.42	0.0087
X2	25.3333	2	12.6667	8.77	0.0077
X1*X2	61.3333	4	15.3333	10.62	0.0018
Error	13	9	1.4444		
Total	124	17			

Constrained (Type III) sums of squares.

Main effects and interactions

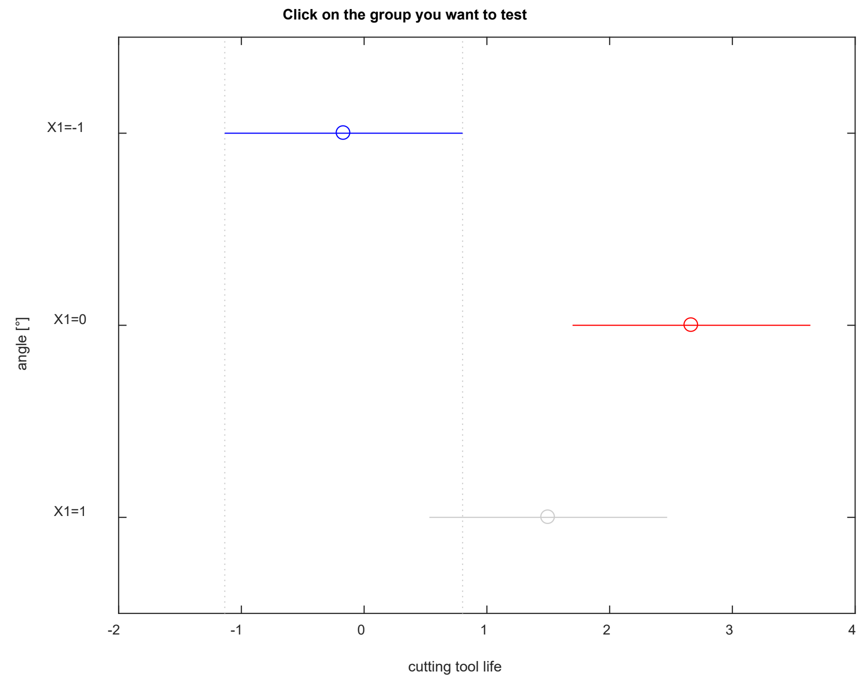
- Main effect of the factor is large, but the interaction have a larger effect
  - speed seems to increase the life of the cutting tool
  - extreme angles seem to decrease the tool life
    - small angles are more detrimental on the cutting tool
- Stronger nonlinearities are present in the interaction plot
  - the highest velocity and the intermediate angle seems to guarantee the longest life
  - the critical duration of the cutting tool is due to intermediate and low velocities and the lowers cutting angle

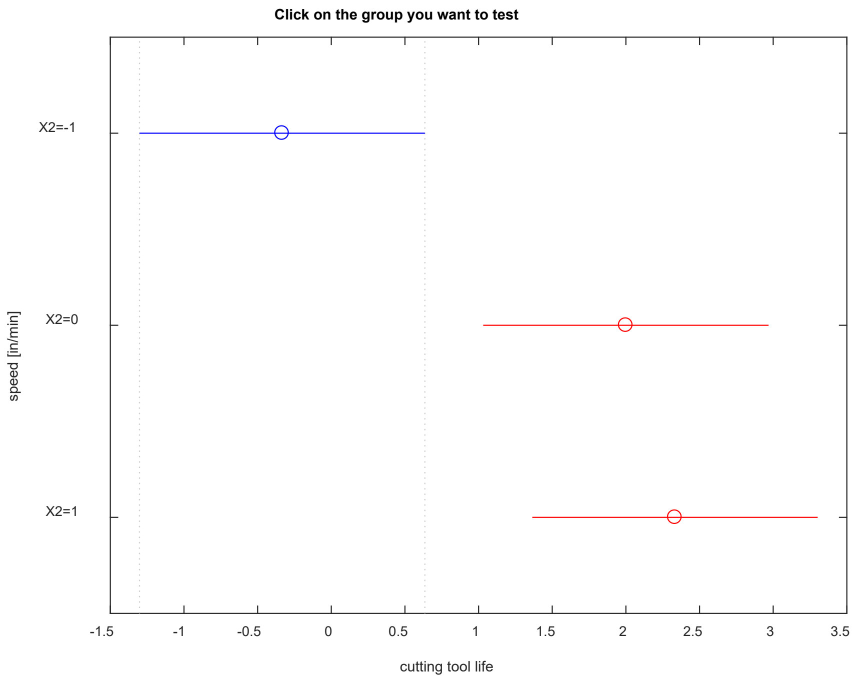




Main effect variability

- The effect of the higher cutting angle is not clearly distinguishable from the effects of the other ones
- The two highest speeds show that the main effects are not clearly distinguishable





Response surface modelling

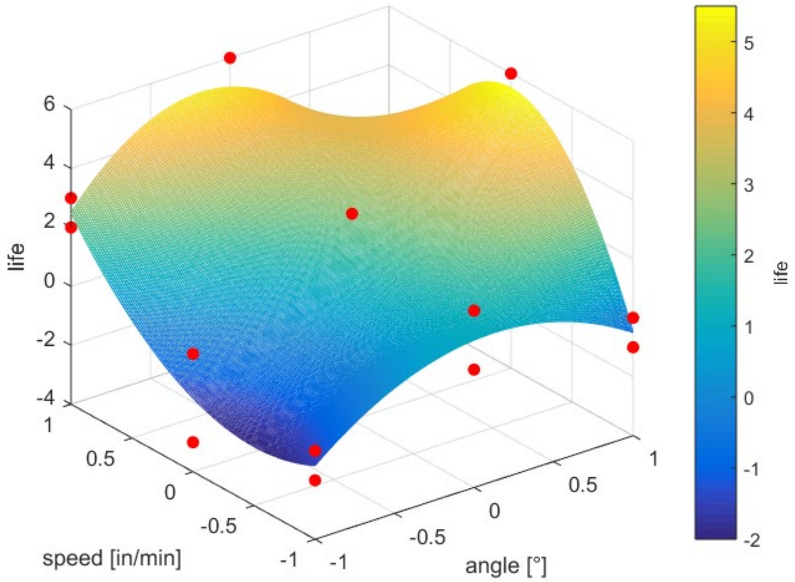
- We already know that:
  - strong non-linearity affect the system
  - we have to take into account interactions
- **Response surface**:

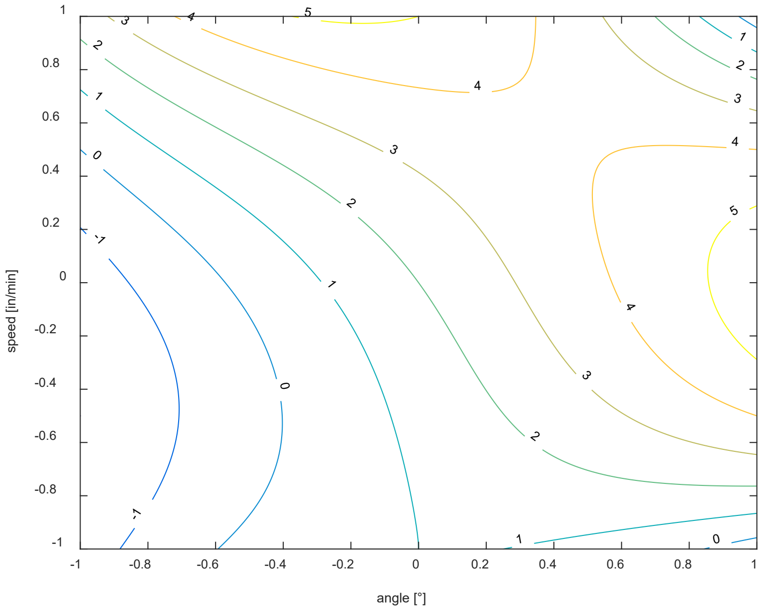
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 +$$
$$+\beta_{11} x_1^2 + \beta_{22} x_2^2 +$$
$$+\beta_{112} x_1^2 x_2 + \beta_{122} x_1 x_2^2 + \beta_{1122} x_1^2 x_2^2 + \varepsilon$$

  - several terms are included:
    - linear terms
    - interactions
    - quadratic terms
    - higher level interactions
- The model parameters are estimated through a least-square algorithm for the minimization of the fitting error

Response surface

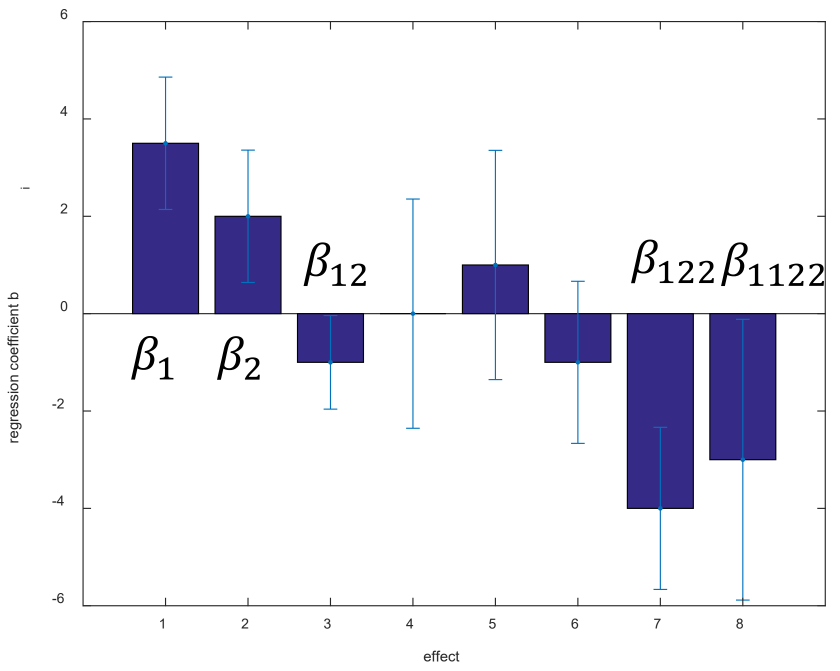
- The response surface shows good fitting performance ($R^2 = 0.8952$):
  - the surface is very close to the measured points (red dots)
  - the model identifies one zone which is critical for the cutting tool life: low angles and low speed
  - the most promising working domains are:
    - intermediate cutting speed with high angles
    - intermediate angles with high cutting velocity





Estimated model parameters

- The study of the estimated parameter confirms that:
  - there is a high and positive linear effect of the factors on the cutting tool life
  - the largest effect is related to the interaction term $x_1 x_2^2$, which is related to a strong decrease of the tool life
  - the quadratic interaction term $x_1^2 x_2^2$ is one of the most important, indicating once again a complex interaction between factors is the worst effect for the cutting tool life
    - the contribution of this interaction is affected by high levels of uncertainty
  - the effects of the other factors are affected by high uncertainty and are less relevant

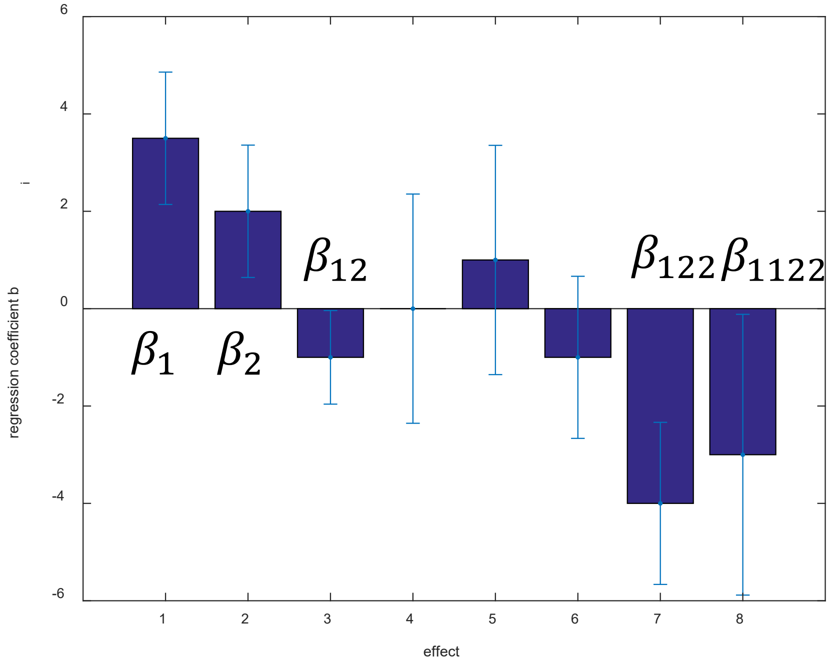


Model improvement?

- Determination coefficient:
  - $R^2 = 0.895$
  - $R^2_{\mathrm{adj}} = 0.802$

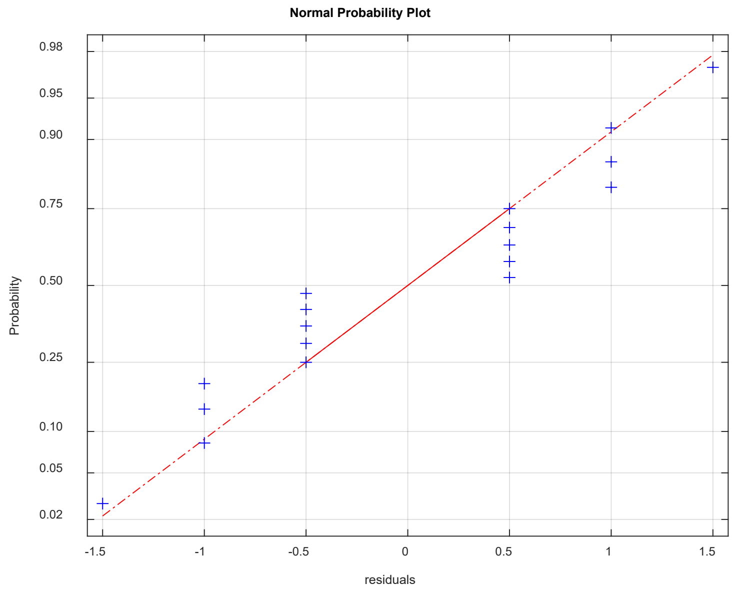
  the regression model can be improved

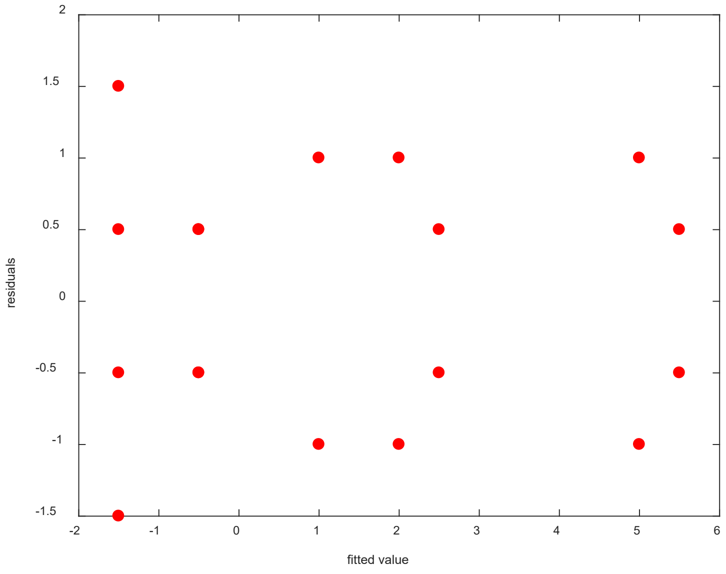
- What kind of model improvements can be provided to the regression model?
  - what predictors can be removed?
  - what enhancement of the model performance can be obtained?

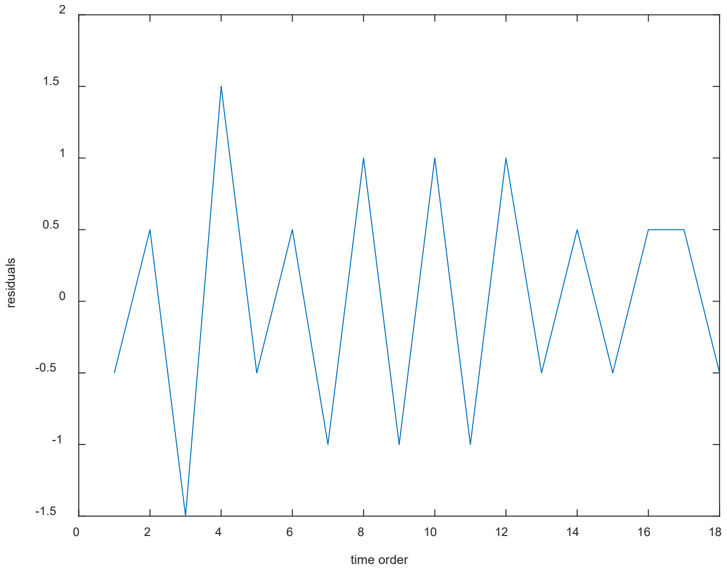


Analysis of the residuals

- The residual show sufficiently normal behavior in:
  - normal probability plot
  - against time
  - against the fitted values







… per sempre a fianco a me!