LCD (24/03)

Weak Bisimilarity

* Weak Bisimulation : $\mathcal{R} \subseteq Proc \times Proc$ s.t. whenever $P \,\mathcal{R}\, Q$

  (i) if $P \xrightarrow{\alpha} P'$ then $Q \stackrel{\alpha}{\Rightarrow} Q'$ and $P' \,\mathcal{R}\, Q'$

  (ii) if $Q \xrightarrow{\alpha} Q'$ then $P \stackrel{\alpha}{\Rightarrow} P'$ and $P' \,\mathcal{R}\, Q'$

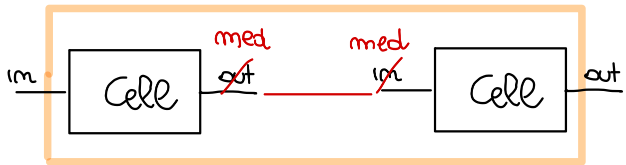
* Weak Bisimilarity : $P \approx Q$ if there exists $\mathcal{R}$ weak bisimulation such that $P \,\mathcal{R}\, Q$

Exercise : FIFO BUFFER (capacity 2)

$$Cell = in(x) . C(x)$$
$$C(x) = \overline{out}(x) . Cell$$

specification

$$F_2 = in(x) . F_1(x)$$
$$F_1(x) = \overline{out}(x) . F_2 + in(y) . F_0(x, y)$$
$$F_0(x,y) = \overline{out}(x) . F_1(y)$$



$$(\, Cell\,[med/out] \mid Cell\,[med/in]\,) \setminus med \approx F_2$$

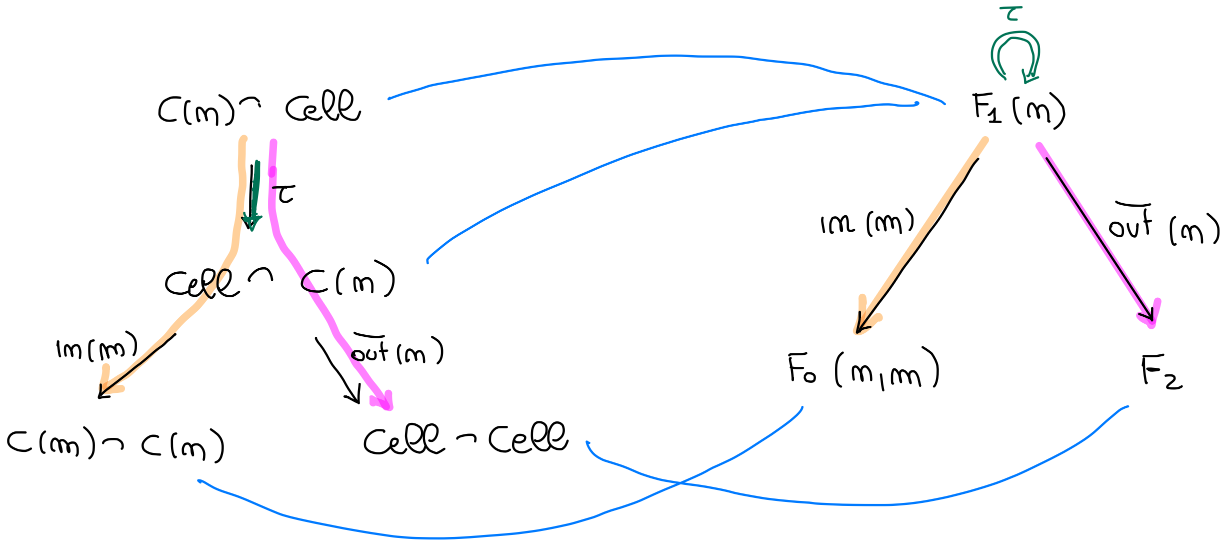
$$Cell \frown Cell$$

notation : A, B processes

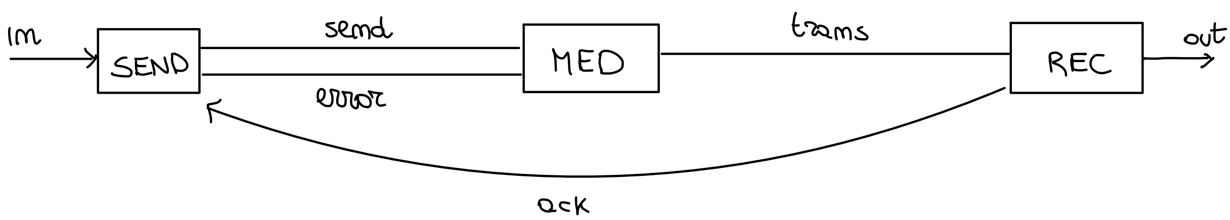
$$A \frown B = (A\,[med/in] \mid B\,[med/out]) \setminus med$$

weak bisimulation

$$R = \{ (Cell \frown Cell, F_2) \} \cup \left\{ \begin{array}{l} \boxed{(C(n) \frown Cell,\ F_1(n))} \\ (Cell \frown C(n),\ F_1(n)) \\ (C(m) \frown C(n),\ F_0(n,m)) \end{array} \middle|\ m,n \in \mathbb{N} \right\}$$



EXAMPLE: Non-lossy channel over a lossy channel



$SEND = in(x).\ SENDING(x)$

$SENDING(x) = \overline{send}(x)\ \ WAIT(x)$

$WAIT(x) = error.\ SENDING(x) + ack.\ SEND$

$REC = trans(x).\ DEL(x)$

$DEL(x) = \overline{out}(x).\ ACK$

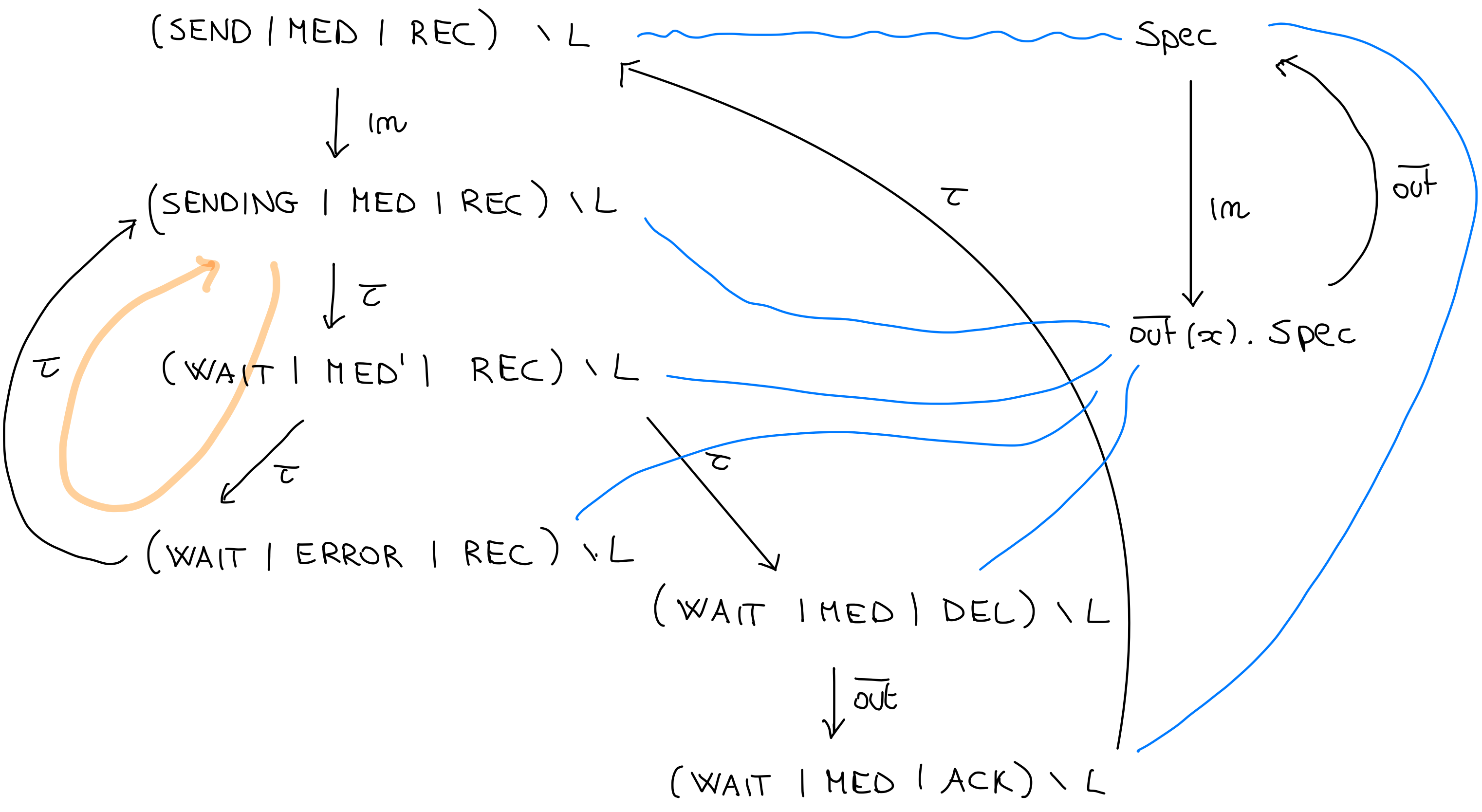
$ACK = \overline{ack}.\ REC$

$MED = send(x).\ MED'(x)$

$MED'(x) = \overline{trans}(x).\ MED + \tau.\ ERROR$

$ERROR = \overline{error}.\ MED$

$$\boxed{(SEND \mid MED \mid REC) \setminus \overbrace{\{send, trans, error, ack\}}^{L}} \approx \boxed{Spec = in(x).\ \overline{out}(x).\ Spec}$$



* <u>Weak Bisimilarity</u>

* equivalence
* (largest) weak bisimulation
* $\approx$ satisfies

$P \approx Q$ iff $\forall\, P \xrightarrow{\alpha} P'$ then $\exists\, Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \approx Q'$

& dual

<u>proof</u>

weak bisimilarity is bisimilarity on $\Rightarrow$.

<u>Compositional / Congruence</u>

if $P \approx Q$ then $C[P] \approx C[Q]$ $\forall$ context $C[\,]$?

$\boxed{\text{NO}}$ $\underset{P}{\underset{=}{0}} \approx \underset{Q}{\underset{=}{\tau.0}}$

$C[\,] = a.0 + \sqcup$

$$C[P] = a.0 + 0 \quad \not\approx \quad C[Q] = a.0 + \tau.0$$

$$C[P] \overset{\tau}{\Rightarrow} a.0 + 0 \xrightarrow{a} 0$$

$$C[Q] \xrightarrow{\tau} 0 \quad \cancel{\xrightarrow{a}}$$

OBSERVATION : If $P \approx Q$, R process

(i) $\alpha . P \approx \alpha . Q$

(ii) $P \mid R \approx Q \mid R$ EXERCISE

(iii) $P \setminus L \approx Q \setminus L$

(iv) $P[f] \approx Q[f]$

<u>proof</u>

(i) $\mathcal{R} = \{ (\alpha . P, \alpha . Q) \} \cup \approx$

(ii) $\mathcal{R} = \{ (P' \mid R', Q' \mid R') \mid P', Q', R' \text{ processes } P' \approx Q' \}$

you need weak bisim.

$$\frac{P \overset{\alpha}{\Rightarrow} P'}{P \mid Q \overset{\alpha}{\Rightarrow} P' \mid Q}$$

(iii) , (iv) same

Non-deterministic Choice

We would like if $P \approx Q$ then $P + R \approx Q + R$

let's try

$\mathcal{R} = \{ (P+R, Q+R) \} \cup \approx$ weak bisimulation

if $P + R \xrightarrow{\alpha} \underbrace{\_\_ \quad \text{then} \quad Q + R \overset{\alpha}{\Longrightarrow} \_\_}_{\mathcal{R}}$

two possibilities

(i) $\dfrac{R \xrightarrow{\alpha} R'}{P+R \xrightarrow{\alpha} R'}$ then $\dfrac{R \xrightarrow{\alpha} R'}{Q+R \xrightarrow{\alpha} R'}$ and $R' \approx R'$

$\Downarrow$

$R' \,\mathcal{R}\, R'$

(ii) $\dfrac{P \xrightarrow{\alpha} P'}{P+R \xrightarrow{\alpha} P'}$ then since $P \approx Q$

hence $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \approx Q'$

$$\boxed{\dfrac{Q \overset{\alpha}{\Rightarrow} Q'}{Q+R \overset{\alpha}{\Rightarrow} Q'}}$$ and $P' \,\mathcal{R}\, Q'$

No!!!

in the counterexample

$P = \tau.0 \approx Q = 0$

$R = a.0$

$\dfrac{P \xrightarrow{\tau} 0}{P+R \xrightarrow{\tau} 0} \rightsquigarrow Q \overset{\tau}{\Rightarrow} Q$ and $\cancel{\dfrac{Q \overset{\tau}{\Rightarrow} Q}{Q+R \overset{\tau}{\Rightarrow} Q}}$

Solutions to obtain a congruence

(1) Guarded sum

$$P, Q ::= K \mid \sum_{i \in I} \alpha_i . P_i \mid P|Q \mid P \backslash L \mid P[f]$$

can be τ

Exercise: show that in CCS with guarded sums weak bisimilarity is a congruence.

② Observational equivalence

$\approx$ not a congruence $\qquad 0 \cancel{\approx} \tau.0$

* consider the contextual closure $\hat{\approx}$

$P \hat{\approx} Q$ if for all $C[\ ]$ context $C[P] \approx C[Q]$

then

① $\hat{\approx}$ is an equivalence

② $\hat{\approx}$ is a congruence

③ $\hat{\approx} \subseteq \approx$ (if $P \hat{\approx} Q$ then $P \approx Q$)

④ $\hat{\approx}$ is the coarsest equivalence which refines $\approx$

(if $\equiv$ is a congruence and $\equiv \subseteq \approx$ then $\equiv \subseteq \hat{\approx}$)

Alternative characterisation

define $\Rightarrow_c$ (weak contextual transition)

$P \overset{\alpha}{\Rightarrow}_c P'$ if $P \xrightarrow{\tau}{}^* \xrightarrow{\alpha} \xrightarrow{\tau}{}^* P'$ $\leftarrow$ at least one step (also when $\alpha = \tau$)

$0 \overset{\tau}{\Rightarrow} 0$

$0 \cancel{\overset{\tau}{\Rightarrow}_c} 0$

observational congruence

$P \approx_c Q$ if when $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Rightarrow}_c Q'$ and $P' \approx Q'$

& dual

you can prove that (a) $\approx_c$ equivalence

(b) $\approx_c$ congruence

(c) $\approx_c$ refines $\approx$ (if $P \approx_c Q$ then $P \approx Q$)

(d) $\approx_c = \hat{\approx}$ $\quad \begin{cases} \approx_c \subseteq \hat{\approx} \\ \approx_c \supseteq \hat{\approx} \end{cases}$

GOOD EXERCISE

! DIFFICULT

simplifying assumption: for every process P there is a action

such that $P \overset{\alpha}{\longrightarrow} \not\xrightarrow{\alpha}$

$$\boxed{\text{EXERCISE} \quad 3.27}$$

P, Q $\qquad P \xrightarrow{\tau}{}^{*} Q$ and $Q \xrightarrow{\tau}{}^{*} P$ then $P \approx Q$

$$\boxed{\text{EXERCISE} \quad 3.29}$$

$P, \mathcal{A} \approx 0$