LCD (23/03)

* Bisimulation up-to bisimilarity

relation $R \subseteq Proc \times Proc$ such that when $P \mathrel{R} Q$

(i) if $P \xrightarrow{\alpha} P'$ then $Q \xrightarrow{\alpha} Q'$ and $P' \sim P'' \mathrel{R} Q'' \sim Q'$

(ii) dual

if R is a bisimulation up to $\sim$ and $P \mathrel{R} Q$ then $P \sim Q$

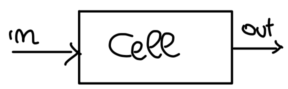
non solution: if R is a bisimulation up to $\sim$ then R is a bisimulation

(EXERCISE for the EXAM)

Exercise: Buffer with capacity 2 (unordered)

$Cell = in(x).\ C(x)$

$C(x) = \overline{out}(x).\ Cell$



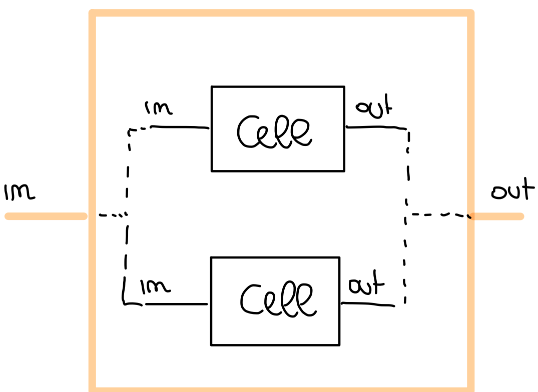
build a 2-places buffer by using Cell

$B_2 = in(x).\ B_1(x)$

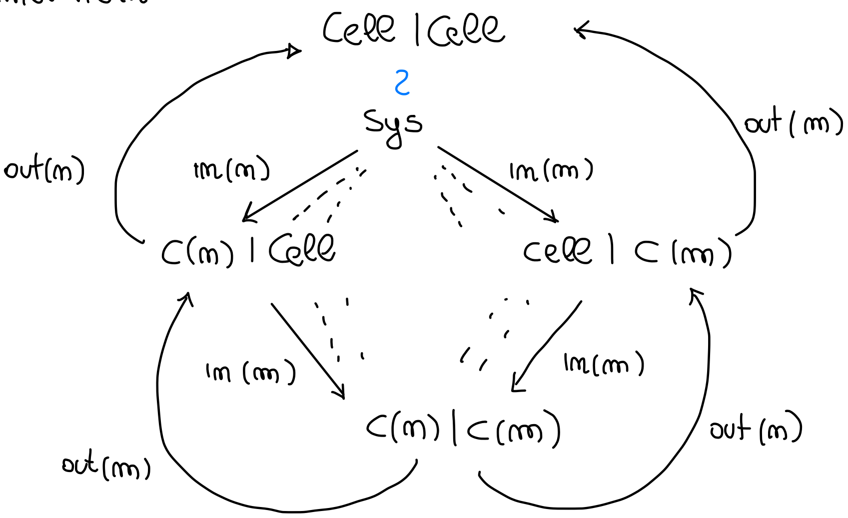
$B_1(x) = \overline{out}(x).\ B_2 + in(y).\ B_0(x,y)$

$B_0(x,y) = \overline{out}(x).\ B_1(y) + \overline{out}(y).\ B_1(x)$

solution: $Sys = Cell \mid Cell$



Bisimulation



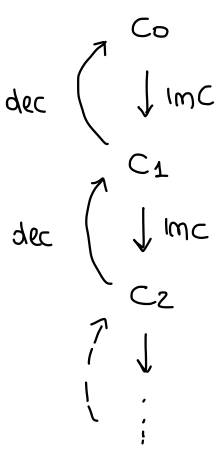
$$
\begin{aligned}
\mathcal{R} = \{(Sys, B_2)\} \;&\cup\; \{(C(n) \mid Cell,\ B_1(n)) \mid n \in \mathbb{N}\} \\
\{(Cell \mid \cancel{Cell},\ B_2)\} \;&\cup\; \{(Cell \mid \cancel{C(m)},\ B_1(m)) \mid m \in \mathbb{N}\} \\
&\cup\; \{(C(n) \mid C(m),\ B_0(n,m)) \mid n, m \in \mathbb{N}\} \\
&\cup\; \{(C(m) \mid \cancel{C(n)},\ B_0(n,m)) \mid n, m \in \mathbb{N}\}
\end{aligned}
$$

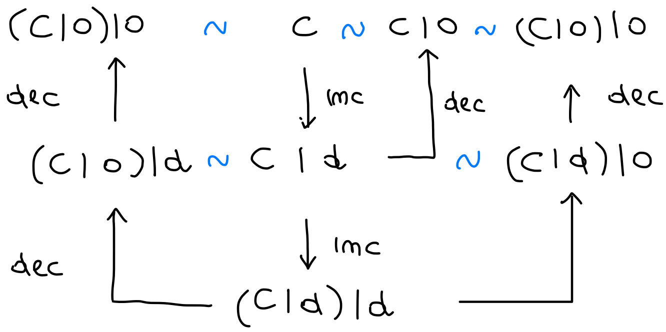
not a bisimulation without ×
only a bisimulation up to ∼

Example:

$$
\begin{cases}
C_0 = inc.\ C_1 \\
C_{n+1} = inc.\ C_{n+2} + dec.\ C_n
\end{cases}
$$

$$C = inc.\ (C \mid \underbrace{dec.0}_{d})$$





$$C_0 \sim C$$

bisimulation

$$R = \{ (C_n \,,\, \underbrace{(..(C|d)|d)\ldots)|d}_{n \text{ times}}) \mid n \in \mathbb{N} \}$$

$c \,|\, d^n$

consider

$$C_{n+1} \quad R \quad c \,|\, d^{n+1}$$

and

$$C_{n+1} \xrightarrow{dec} C_n \qquad \rightsquigarrow \qquad c \,|\, d^{n+1} \xrightarrow{dec} (\underbrace{..(C|d)|d) \ldots 0 \ldots d}_{\substack{n \text{ processes } d \\ 1 \text{ process } 0}})$$

$\wr$

$c \,|\, d^n$

$\notin R$

$\in R$

R is not a bisimulation but it is a bisimulation up to $\sim$

if counter is bounded

- bisimulation is infinite
- bisimulation up to $\sim$ is linear.

Weak Equivalences : WEAK BISIMILARITY

- equivalence
- compositional
- bisimulation (largest)
- expected equivalences hold ($P|0 \sim P$, $P|Q \sim Q|P$, ...)

special action τ (internal action)

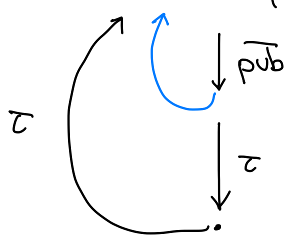
$$b.0 \not\sim b.\tau.0$$

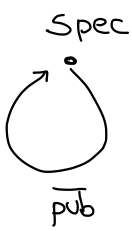
Example :

$CS = \overline{pub}.\overline{coin}.coffee.CS$ $\qquad\qquad$ $Spec = \overline{pub}.Spec$

$CM = coin.\overline{coffee}.CM$

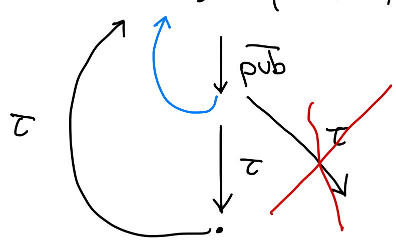
$(CM | CS) \setminus \{coin, coffee\}$ $\qquad \not\sim$





$CMB = coin.\overline{coffee}.CMB + coin.CMB$

$(CMB | CS) \setminus \{coin, coffee\}$



$(CMB \mid coffee.CS) \setminus \{coin, coffee\}$

$$\cdot \xrightarrow{\cancel{\tau}} \cdots \xrightarrow{\cancel{\tau}} \xrightarrow{a_1} \xrightarrow{\cancel{\tau}} - - - - \xrightarrow{\cancel{\tau}} \xrightarrow{a_n} \xrightarrow{\cancel{\tau}} - - - \xrightarrow{\cancel{\tau}} \cdot$$

$$\xrightarrow{a_1} \; \cdots\cdots \; \xrightarrow{a_n}$$

Weak transitions

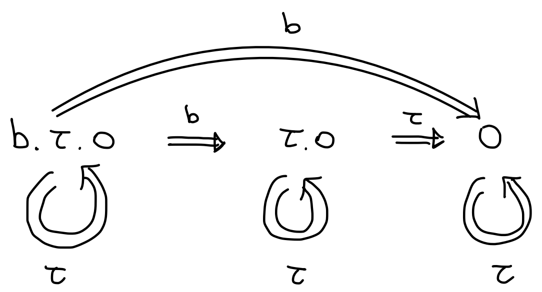
P, Q processes $\qquad P \overset{\alpha}{\Rightarrow} Q$ if

(i) $\alpha \neq \tau$ $\qquad P \xrightarrow{\tau}{}^{*} P' \xrightarrow{\alpha} Q' \xrightarrow{\tau}{}^{*} Q$

(ii) $\alpha = \tau$ $\qquad P \xrightarrow{\tau}{}^{*} Q$

any sequence, possibly empty $(P = Q)$

Example: $\qquad P = b.\tau.0$



Weak Bisimilarity

a relation $\mathcal{R} \subseteq Proc \times Proc$ is a weak bisimulation if when $P \mathcal{R} Q$

(i) if $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \mathcal{R} Q'$

(ii) if $Q \xrightarrow{\alpha} Q'$ then $P \overset{\alpha}{\Rightarrow} P'$ and $P' \mathcal{R} Q'$

We say P, Q weak bisimilar if there is $\mathcal{R}$ weak bisimulation s.t.

$P \mathcal{R} Q$ and $\qquad P \approx Q$

$$\approx \; = \; \bigcup \{ \mathcal{R} \mid \mathcal{R} \text{ weak bisimulation} \}$$

EXERCISE (EXAM)

a relation $\mathcal{R} \subseteq Proc \times Proc$ a weak string bisimulation if when $P \mathcal{R} Q$

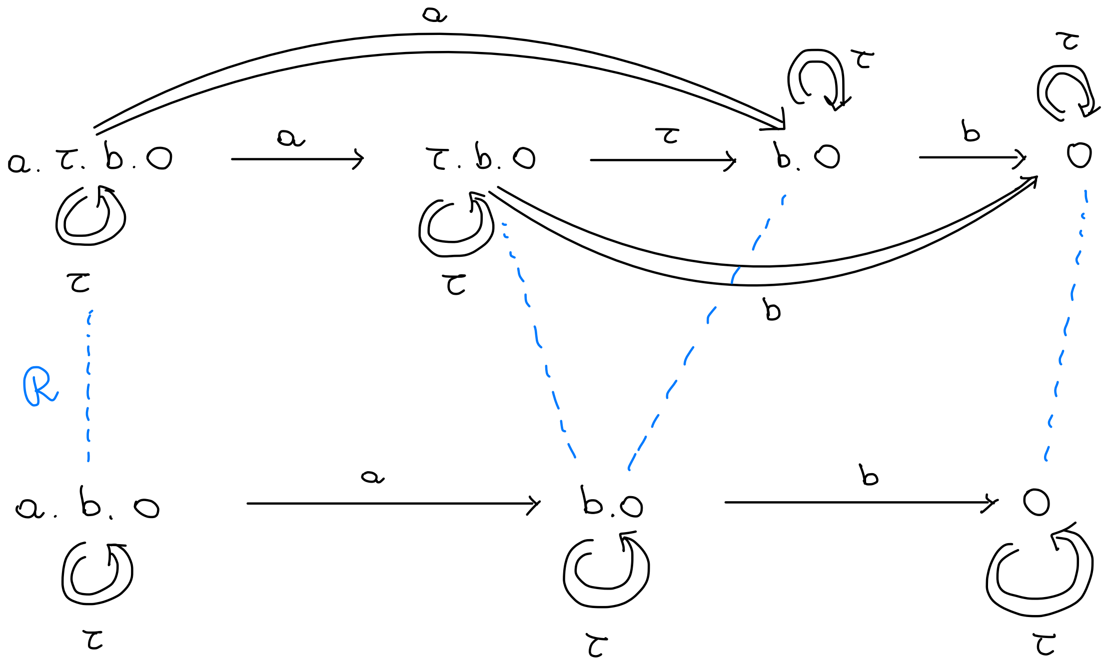
(i) if $P \overset{\alpha}{\Rightarrow} P'$ then $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \mathcal{R} Q'$

(ii) if $Q \overset{\alpha}{\Rightarrow} Q'$ then $P \overset{\alpha}{\Rightarrow} P'$ and $P' \mathcal{R} Q'$

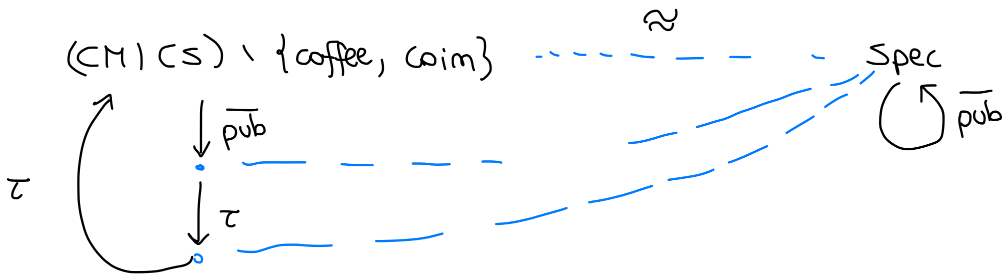
$\leadsto$ weak string bisimilarity $\approx_{string}$. Prove $\approx_{string} = \approx$

Example :

$P = a.\tau.b.0 \quad \approx \quad Q = a.b.0$



Example

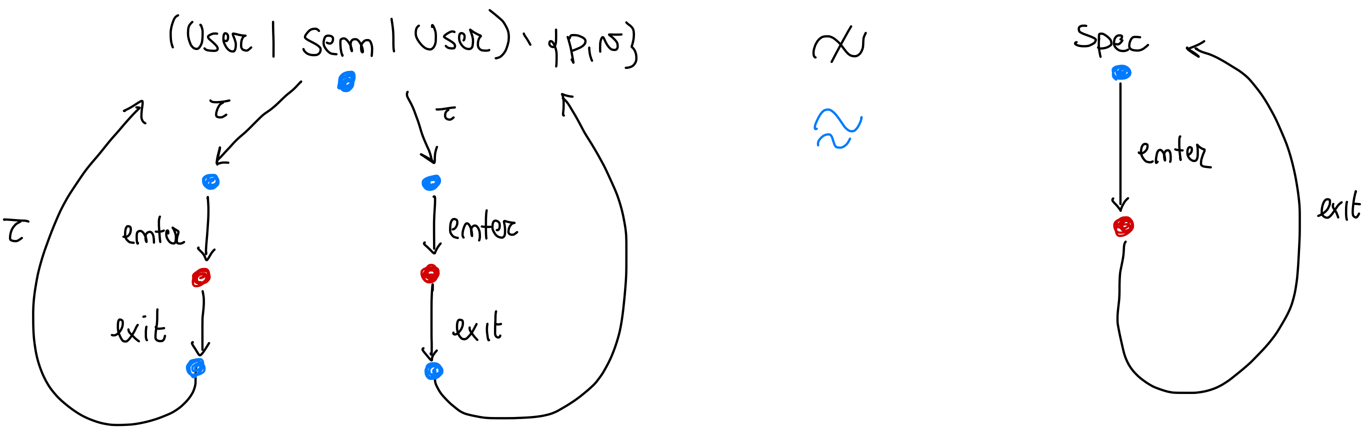


Example :

$Sem = p.v.Sem$

$Spec = enter.exit.Spec$

$User = \bar{p}.enter.exit.\bar{v}.User$

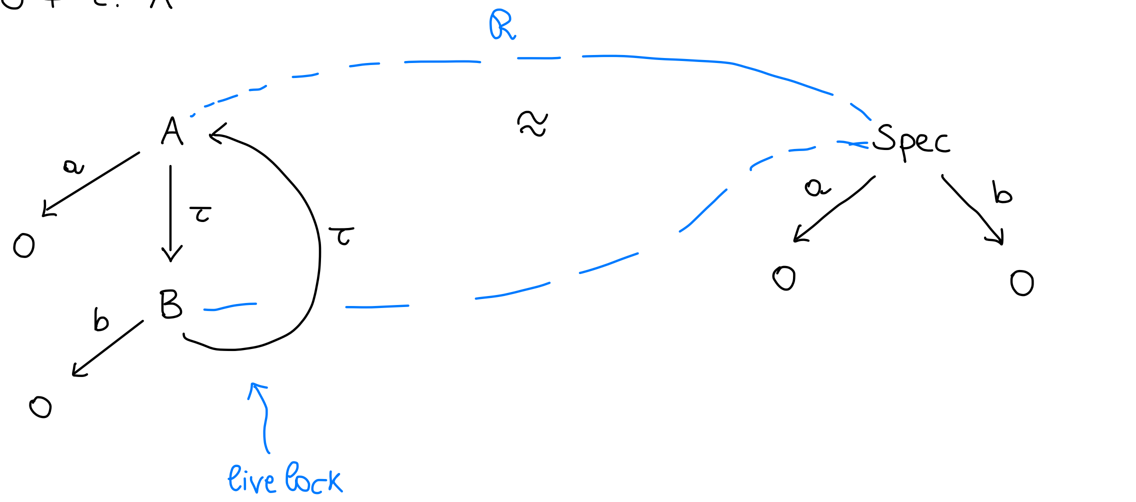


∗ Fair abstraction from Divergence

$A = a.0 + \tau.B$

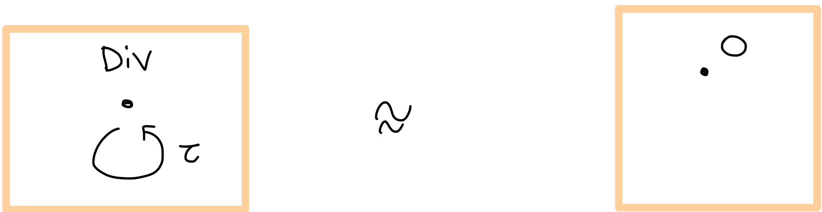
$B = b.0 + \tau.A$

$Spec = a.0 + b.0$



∗ Divergence is not observable

$Div = \tau.Div$



$(A_1 \mid A_2) \setminus \{a\}$

$A_1 = a.A_1 \qquad A_2 = \bar{a}.A_2$