LCD (17/03)

* Bisimilarity
  * equivalence
  * largest bisimulation

→ bisimulations are not equivalences

$$\begin{tikzcd} s \arrow[r, dashed] \arrow[d, "a"'] & t \arrow[r, dashed] \arrow[d, "a"'] & v \arrow[d, "a"] \\ s_1 \arrow[r, dashed] & t_1 \arrow[r, dashed] & v_1 \end{tikzcd}$$

$R = \{ (s,t), (t,v), (s_1,t_1), (t_1, v_1) \}$

bisimulation

- not reflexive $\quad s \not R s$
- not symmetric $\quad s R t$ but $t \not R s$
- not transitive $\quad s R t$ and $t R v$ but $s \not R v$

OBSERVATION : given P, Q

$P \sim Q$ iff
$$\boxed{\begin{array}{l} \text{if } P \xrightarrow{\alpha} P' \text{ then } Q \xrightarrow{\alpha} Q' \text{ and } P' \sim Q' \\ \text{if } Q \xrightarrow{\alpha} Q' \text{ then } P \xrightarrow{\alpha} P' \text{ and } P' \sim Q' \end{array}} \quad (*)$$

($\Rightarrow$) $\sim$ bisimulation

($\Leftarrow$) assume $(*)$ and show $P \sim Q$

we need to identify R bisimulation s.t. $P \, R \, Q$

define

$$R = \{ (P,Q) \} \cup \sim$$

and show that R is a bisimulation

given S, T processes st. $S \, R \, T$

if $S \xrightarrow{\alpha} S'$ then $T \xrightarrow{\alpha} T'$ and $S' \, R \, T'$

+ dual

two cases

(1) $S = P \ , \ T = Q$

if $P \xrightarrow{\alpha} P'$ by (*) $Q \xrightarrow{\alpha} Q'$ and $P' \sim Q'$

$\Downarrow$

$P' \, R \, Q'$

+ dual

(2) $S \sim T$

if $S \xrightarrow{\alpha} S'$ since $\sim$ is bisimulation $T \xrightarrow{\alpha} T'$

and $S' \sim T'$

$\Downarrow$

$S' \, R \, T'$

& dual

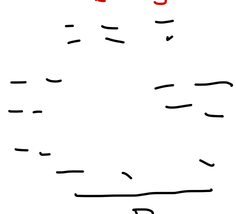
$\Rightarrow$ R is bisimulation $\Rightarrow$ $P \sim Q$ $\square$

We want $P \sim Q$

$R = \{(P,Q)\} \cup \sim$ bisimulation $\Rightarrow$ $R = \{(P,Q)\} \cup \sim \ \subseteq \ \sim$

coinductive logic

[P]



P

$P, Q ::= \alpha . P \mid P|Q \mid \ldots$

* <u>Strong Bisimilarity</u>

<u>strong bisimulation</u> is $R \subseteq Proc \times Proc$ s.t. give $P \, R \, Q$

if $P = P_0 \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} \ldots \xrightarrow{\alpha_m} P_m$

then $Q = Q_0 \xrightarrow{\alpha_1} Q_1 \xrightarrow{\alpha_2} \ldots \xrightarrow{\alpha_m} Q_m$ and $P_i \, R \, Q_i$ $(i = 0, .. m)$

<u>strong bisimilarity</u> $P \sim_{strong} Q$ if there is R strong bisim. s.t. $P \, R \, Q$

EXERCISE (exam): show $\sim \; = \; \sim_{string}$

OBSERVATION: Given P, Q, if $P \sim Q$ then $Traces(P) = Traces(Q)$

proof

assume $P \sim Q$. Then $P \sim_{string} Q$.

Then if $\alpha_1 \dots \alpha_n \in Traces(P)$

$$P = P_0 \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} \dots \xrightarrow{\alpha_n} P_n$$

thus

$$Q = Q_0 \xrightarrow{\alpha_1} Q_1 \dots \xrightarrow{\alpha_n} Q_n$$

hence $\alpha_1 \dots \alpha_n \in Traces(Q)$

Therefore $Traces(P) \subseteq Traces(Q)$

Since $\sim$ symmetric, we conclude $Traces(P) = Traces(Q)$.

* **EXERCISE**: $CTrace(P) = \{ \alpha_1 \dots \alpha_n \mid P \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_n} P' \not\rightarrow \}$

$P \sim Q \implies CTrace(P) = CTrace(Q)$

OBSERVATION: if $K \stackrel{def}{=} P$ then $K \sim P$

in fact

$R = \{ (K, P) \} \cup I$ bisimulation

$$\frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'}$$

OBSERVATION:

(i) $P \mid Q \sim Q \mid P$

(ii) $P \mid (Q \mid S) \sim (P \mid Q) \mid S$

(iii) $P \mid 0 \sim P$

proof

(i) $\mathcal{R} = \cancel{\{ (P|Q,\ Q|P) \} \cup \sim}$

$$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad \frac{P \xrightarrow{\alpha} P'}{Q|P \xrightarrow{\alpha} Q|P'}$$

$\in \mathcal{R}$

$$\mathcal{R} = \{ (P''|Q'',\ Q''|P'') \mid P'', Q'' \in Proc \}$$

$$P''|Q'' \ \mathcal{R} \ Q''|P''$$

if $P''|Q'' \xrightarrow{\alpha} \ldots$ several cases

(i) $$\frac{P'' \xrightarrow{\alpha} P_1}{P''|Q'' \xrightarrow{\alpha} P_1|Q''} \quad \text{then} \quad \frac{P'' \xrightarrow{\alpha} P_1}{Q''|P'' \xrightarrow{\alpha} Q''|P_1}$$

$\mathcal{R}$

same for other cases

(ii) $\mathcal{R} = \{ (P'|(Q'|R'),\ (P'|Q')|R') \mid P', Q', R' \in Proc \}$

(iii) $\mathcal{R} = \{ (P'|0,\ P') \mid P' \in Proc \}$

<u>EXERCISE</u>

(i) $P \mid (Q+S) \ \overset{?}{\sim} \ (P|Q) + (P|S)$

NO

$\overline{pub}.coffee.0 \mid (\overline{coffee}.0 + \overline{tea}.0) \not\sim (\overline{pub}.coffee.0 \mid \overline{coffee}.0) + (\overline{pub}.coffee.0 \mid \overline{tea}.0)$

$\downarrow \overline{pub}$ $\qquad \overline{pub} \swarrow \qquad \searrow \overline{pub}$

$coffee.0 \mid (\overline{coffee}.0 + \overline{tea}.0) \qquad (coffee.0 \mid \overline{coffee}.0) \qquad (coffee.0 \mid \overline{tea}.0)$

(ii) $(P|Q)\backslash L \overset{?}{\approx}_{N} P\backslash L \mid Q\backslash L$

$(coffee.0 \mid \overline{coffee}.0)\backslash coffee$ $\qquad$ $(coffee.0)\backslash coffee \mid (\overline{coffee}.0)\backslash coffee$

$\rightsquigarrow \tau$ $\qquad\qquad$ $\not\rightsquigarrow \tau$

(iii) $(P|Q)[f] \overset{?}{\approx}_{NO} P[f] \mid Q[f]$

$(coffee.0 \mid \overline{tea}.0)[^{bev}/_{coffee}, ^{bev}/_{tea}]$

$\xrightarrow{bev}$ $\qquad$ $\xrightarrow{\overline{bev}}$

$coffee.0[^{bev}/_{coffee}, ^{bev}/_{tea}] \mid \overline{tea}.0[^{bev}/_{coffee}, ^{bev}/_{tea}]$

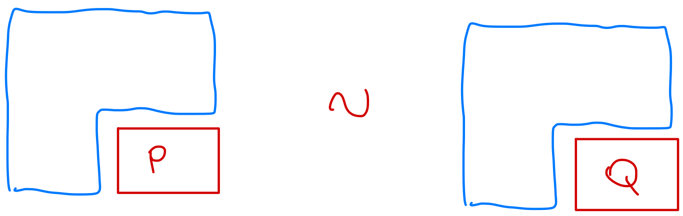
$\xrightarrow{bev}$ $\qquad$ $\xrightarrow{\tau}$ $\qquad$ $\xrightarrow{\overline{bev}}$

* Bisimilarity is compositional

We want that given $P \sim Q$ then $C[P] \sim C[Q]$

for all contexts $C[\;]$

$P \sim Q \Rightarrow$



Let P, Q, R processes with $P \sim Q$. Then

(i) $\alpha.P \sim \alpha.Q$

(ii) $P + R \sim Q + R$

(iii) $P \mid R \sim Q \mid R$

(iv) $P \setminus L \sim Q \setminus L$

(v) $P[f] \sim Q[f]$

proof

(i) We define a relation $\mathcal{R}$ including $(\alpha.P, \alpha.Q)$ and show that $\mathcal{R}$ is a bisimulation

$$\mathcal{R} = \{(\alpha.P, \alpha.Q)\} \cup \sim$$

given S, T s.t. $S \,\mathcal{R}\, T$ then

if $S \xrightarrow{\alpha} S'$ then $T \xrightarrow{\alpha} T'$ and $S' \,\mathcal{R}\, T'$

& dual

two cases

(i) $S = \alpha.P$, $T = \alpha.Q$

if $\alpha.P \xrightarrow{\alpha} P$ then $\alpha.Q \xrightarrow{\alpha} Q$ and $P \sim Q$

$\Downarrow$

$P \,\mathcal{R}\, Q$

& dual

(ii) $S \sim T$

as before

(ii) $P+R \sim Q+R$

we define

$$\mathcal{R} = \{ (P+R, Q+R) \} \cup \sim$$

$P+R \xrightarrow{\alpha} \_\_$ then $Q+R \xrightarrow{\alpha} \_\_$ $\in \mathcal{R}$

two possibilities

(i)
$$\frac{\boxed{P \xrightarrow{\alpha} P'}}{P+R \xrightarrow{\alpha} P'}$$

since $P \sim Q$

$$\frac{Q \xrightarrow{\alpha} Q'}{Q+R \xrightarrow{\alpha} Q'}$$

and $P' \sim Q'$
$\Downarrow$
$P' \,\mathcal{R}\, Q'$

(ii)
$$\frac{R \xrightarrow{\alpha} R'}{P+R \xrightarrow{\alpha} R'}$$

$\rightsquigarrow$
$$\frac{R \xrightarrow{\alpha} R'}{Q+R \xrightarrow{\alpha} R'}$$

$R' \,\mathcal{R}\, R'$ ok since $R' \sim R'$

(iii) $P \mid R \sim Q \mid R$

define

$$\mathcal{R} = \cancel{\{ (P \mid R, Q \mid R) \}} \cup \sim$$

$$\frac{P \xrightarrow{\alpha} P'}{P \mid R \xrightarrow{\alpha} P' \mid R}$$

$P \sim Q$ $\rightsquigarrow$

$$\frac{Q \xrightarrow{\alpha} Q'}{Q \mid R \xrightarrow{\alpha} Q' \mid R}$$

$P' \sim Q'$

$\in \mathcal{R}$?

we have to define R as

$$R = \{ (P' | R', Q' | R') \mid P' \sim Q' \text{ and } R' \text{ any process} \}$$

(iv) $P \setminus L \sim Q \setminus L$

$$R = \{ (P' \setminus L, Q' \setminus L) \mid P' \sim Q' \}$$ bisimulation

(v) $P[f] \sim Q[f]$

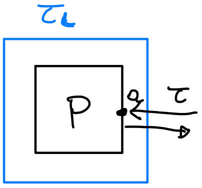
$$R = \{ (P'[f], Q'[f]) \mid P' \sim Q' \}$$ "

$\square$

EXERCISE

given P process

$L \subseteq \mathcal{A}$



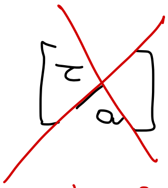
$$\frac{P \xrightarrow{\alpha} P'}{\tau_L(P) \xrightarrow{\tau} \tau_L(P')} \qquad \alpha \in L, \bar{L}$$

$$\frac{P \xrightarrow{\beta} P'}{\tau_L(P) \xrightarrow{\beta} \tau_L(P')} \qquad \beta \notin L, L'$$

This can be encoded as a context $C_L[\ ]$

s.t.

$$\tau_L(P) \sim C_L[P]$$



disallowed

EXERCISE: (Bisimulation up-to)

A bisimulation up-to bisimilarity is a relation $\mathcal{R} \subseteq Proc \times Proc$ such that when $P \,\mathcal{R}\, Q$

if $P \xrightarrow{\alpha} P'$ then $Q \xrightarrow{\alpha} Q'$ and $P' \sim P'' \,\mathcal{R}\, Q'' \sim Q'$

dual

> **SOUNDNESS**
>
> if $\mathcal{R}$ is a bisimulation up to $\sim$ and $P \,\mathcal{R}\, Q$ then $P \sim Q$

(EXAM)

If $\mathcal{R}$ is a ~~bisimulation up to~~ then $\mathcal{R}$ is a bisimulation NO