LCD (10/03)

* BEHAVIOURAL EQUIVALENCE

$CS = \overline{pub} . \overline{coin} . coffee . CS$

$CM = coin . \overline{coffee} . CM$ $\qquad\qquad$ $Spec = \overline{pub} . Spec$

$Off = (CS \mid CM) \setminus \{coin, coffee\}$

$$Office \overset{?}{\sim} Spec$$

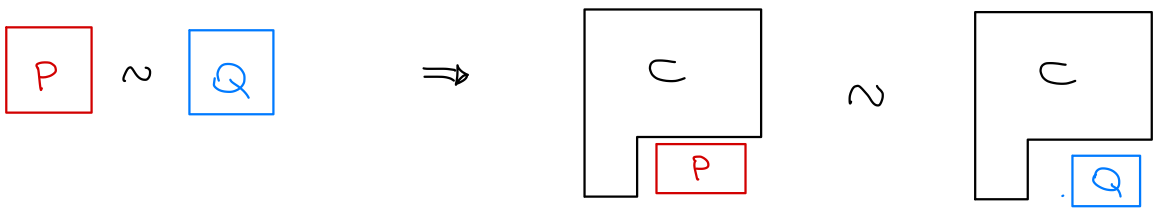
→ properties of $\sim$?

- reflexive $\quad P \sim P$
- symmetric $\quad P \sim Q \Rightarrow Q \sim P$
- transitive $\quad P \sim Q$ and $Q \sim R$ then $P \sim R$

$$\underbrace{Spec \sim Sys_1 \sim Sys_2 \sim \dots \sim Sys_n}_{\sim}$$

→ congruence / compositionality

if $P \sim Q$ then for every context $C[\ ]$ $\quad C[P] \sim C[Q]$



$Spec = Spec_1 \mid Spec_2$

$Sys_1 \sim Spec_1$

$Sys_2 \sim Spec_2$ $\quad \leadsto \quad Spec \overset{?}{\sim} Sys_1 \mid Sys_2$

need congruence

$\underline{Sys_1} \mid Sys_2 \sim Sys_1 \mid \underline{Spec_2} \sim Spec_1 \mid Spec_2$

Referential transparency

$P \; = \; \ldots\ldots \; exp \; \ldots\ldots$

$\uparrow$ replacing "exp" by its value this does not affect the behaviour

$$\begin{cases} 2x + y = 2 \\ x - y = 1 \end{cases} \Rightarrow \underline{y = x - 1} \rightsquigarrow 2x + x - 1 = 2$$

$$\Downarrow$$

$$3x = 3 \rightsquigarrow x = 1 \ \ldots$$

* the equivalence depends only on the observable behaviour

(OBSERVATIONAL EQUIVALENCE)

different observables lead to different equivalences

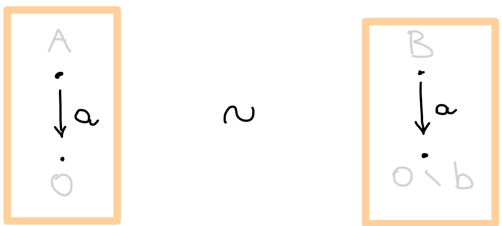
(Van Glabbeek

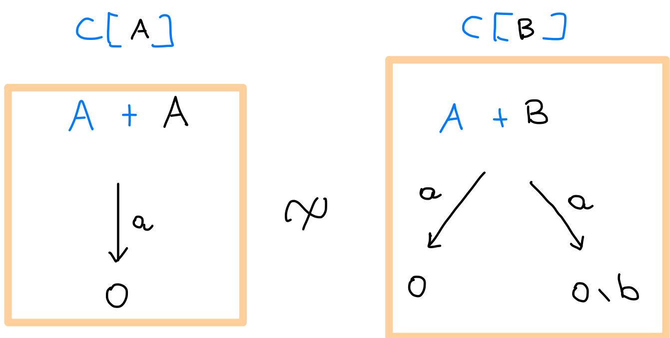
Linear Time / Branching time spectrum ...)

(0) same transitions

$A = a.0$

$B = (a.0)\backslash b$





$C[\ ] = A + \_\_$

$A \sim B$ but $C[A] \nsim C[B]$ NOT A CONGRUENCE!

(1) Trace equivalence

$$Tr(P) = \{ \alpha_1 \dots \alpha_n \mid P \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} P_2 \longrightarrow \dots \xrightarrow{\alpha_n} P_n \}$$

↖ traces

$$P \sim_T Q \quad \text{if} \quad Tr(P) = Tr(Q)$$

→ equivalence ? yes!

→ based on observable behaviour ? yes!

→ congruence ? yes! (not obvious)

[EXERCISE for the exam]

Example :

$$CTM = coin.(\overline{coffee}.CTM + \overline{tea}.CTM)$$

$$CTM' = coin.\overline{coffee}.CTM' + coin.\overline{tea}.CTM'$$

$$Tr(CTM) = Tr(CTM') = (coin.(\overline{coffee} + \overline{tea}))^* \, coin?$$

$$CTM \sim_T CTM'$$

$$CS = pub.\overline{coin}.coffee.CS$$

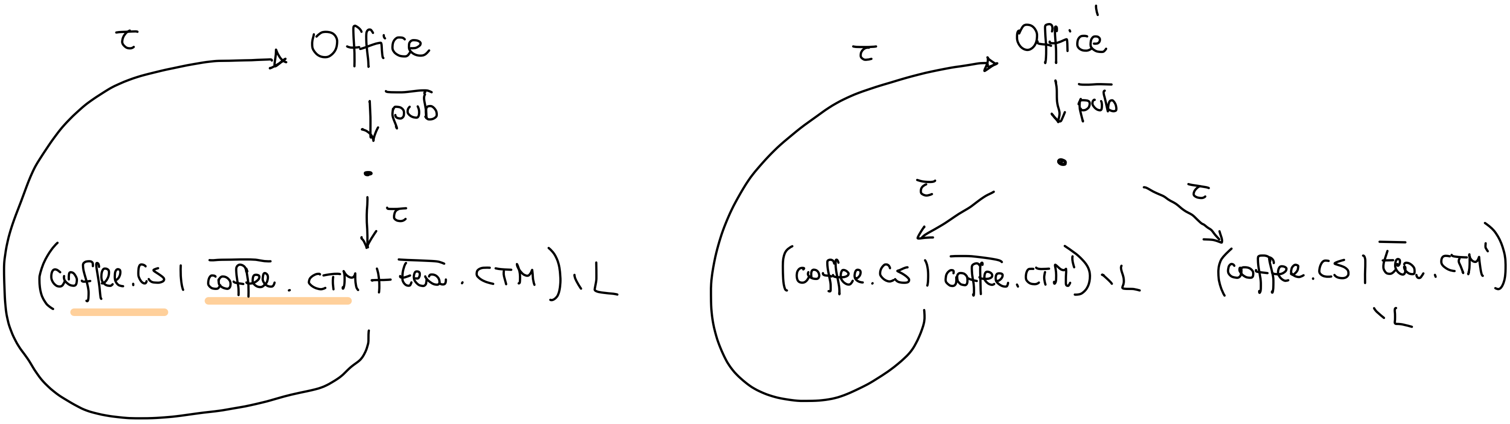
$$Office = (CS \mid CTM) \setminus L \qquad L = \{coin, coffee, tea\}$$

$$Office' = (CS \mid CTM') \setminus L$$

since $\sim_T$ is a congruence $Office \sim_T Office'$

$$Tr(Office) = Tr(Office') =$$

$$= (\overline{pub}.\tau\tau)^* (\varepsilon + \overline{pub} + \overline{pub}\,\tau)$$



same traces, but Office has no deadlock, while Office' has one.

(2) Completed trace

$$CTr(P) = \{ \alpha_1 \ldots \alpha_m \mid P \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_m} P' \not\rightarrow \}$$

and define

$$P \sim_{CT} Q \quad \text{if} \quad P \sim_T Q \quad \text{and} \quad CTr(P) = CTr(Q)$$

Note

$$CTr(Office) = \emptyset \qquad CTr(Office') \neq \emptyset$$

$$\boxed{Office \not\sim_{CT} Office'}$$

but...

$$CTr(CTM) = \emptyset = CTr(CTM') \quad \Rightarrow \quad \boxed{CTM \sim_{CT} CTM'}$$

$$Office = (CS \mid CTM) \setminus L$$

$$\not\sim_{CT}$$

$$Office' = (CS \mid CTM') \setminus L$$

not a congruence....

Problem with traces?

→ congruence

→ based on observable behaviour

→ disregard the properties of traversed states

$$CTM \xrightarrow{coin} \odot \xrightarrow{\overline{tea}} \bullet$$

$\overline{coffee}$ is possible

$$CTM' \xrightarrow{coin} \odot \xrightarrow{\overline{tea}} \bullet$$

$\overline{coffee}$ is not possible

<u>Idea</u> : $P \sim Q$ if

Def?

- when $P \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} \dots \xrightarrow{\alpha_n} P_n$

  then $Q \xrightarrow{\alpha_1} Q_1 \xrightarrow{\alpha_2} \dots \xrightarrow{\alpha_n} Q_n$ and $P_1 \sim Q_1 \dots P_n \sim Q_n$

- dual

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this is a property of $\sim$

satisfied by many relations

→ empty relation $\emptyset$

→ identity $P \sim Q$ if $P = Q$

* <u>Bisimilarity</u>

<u>Def</u> : A relation $R \subseteq Proc \times Proc$ is a bisimulation if

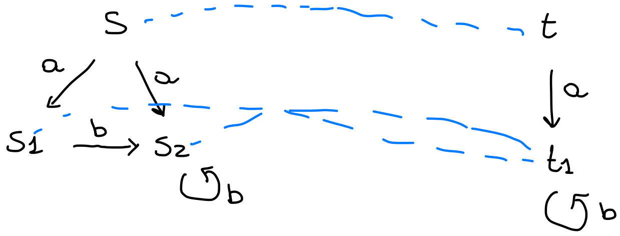
for all $P, Q \in Proc$ with $P \, R \, Q$

→ <u>for all</u> $P \xrightarrow{\alpha} P'$ <u>there is</u> $Q \xrightarrow{\alpha} Q'$ such that $P' \, R \, Q'$

→ for all $Q \xrightarrow{\alpha} Q'$ there is $P \xrightarrow{\alpha} P'$ such that $P' \, R \, Q'$

We say P, Q <u>bisimilar</u>, written $\boxed{P \sim Q}$ if there is R bisimulation such that $P \, R \, Q$ $\quad \left(\sim \; = \bigcup \{ R \mid R \text{ bisimulation} \} \right)$

Example



$R = \{ (s,t), (s_1, t_1), (s_2, t_1) \}$ bisimulation $\Rightarrow$ $s \sim t$

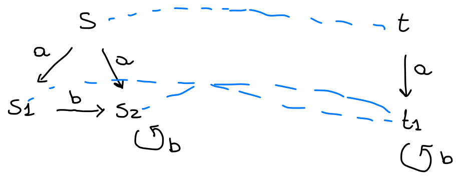
$\rightarrow$ $(s,t) \in R$

- all transitions of s

– $s \xrightarrow{a} s_1$ $\rightsquigarrow$ $t \xrightarrow{a} t_1$ and $s_1 \, R \, t_1$

– $s \xrightarrow{a} s_2$ $\rightsquigarrow$ $t \xrightarrow{a} t_1$ and $s_2 \, R \, t_1$

- all transitions of t



– $t \xrightarrow{a} t_1$

$\rightsquigarrow$ $s \xrightarrow{a} s_1$ and $s_1 \, R \, t_1$

$\rightarrow$ $(s_1, t_1) \in R$

– $s_1 \xrightarrow{b} s_2$ $\rightsquigarrow$ $t_1 \xrightarrow{b} t_1$ and $s_2 \, R \, t_1$

– dual: $t_1 \xrightarrow{b} t_1$ $\rightsquigarrow$ $s_1 \xrightarrow{b} s_2$ and $s_2 \, R \, t_1$

$\rightarrow$ $(s_2, t_1) \in R$

same

Example

$A = a\,A$ $B = (a.B) \setminus b$

$$\mathcal{R} = \{ (A, \underbrace{((B \setminus b) \setminus b) \dots \setminus b)}_{m \text{ restrictions}} \mid m \in \mathbb{N} \}$$ bisimulation

$\rightsquigarrow \quad A \sim B$

Non-Bisimilarity

$CTM = coin. \, (\overline{coffee}. \, CTM + \overline{tea}. \, CTM)$

$CTM' = coin. \, \overline{coffee}. \, CTM' + coin. \, \overline{tea}. \, CTM'$

$CTM \not\sim CTM'$

assume $CTM \sim CTM'$

hence there is $\mathcal{R}$ bisimulation s.t. $CTM \, \mathcal{R} \, CTM'$

observe $CTM \xrightarrow{coin} \underbrace{\overline{coffee}. \, CTM + \overline{tea}. \, CTM}_{CTM_1}$

by def of bisimulation

$CTM' \xrightarrow{coin} CTM'_1$ with $CTM_1 \, \mathcal{R} \, CTM'_1$

2 possibilities for CTM'_1

- $CTM'_1 = \overline{coffee}. \, CTM'$

  since $CTM_1 \xrightarrow{\overline{tea}} CTM$ there should be $CTM'_1 \xrightarrow{\overline{tea}}$ NO!

- $CTM'_1 = \overline{tea}. \, CTM'$

  since $CTM_1 \xrightarrow{\overline{coffe}} CTM$ there should be $CTM'_1 \xrightarrow{\overline{coffee}}$ NO!

contradiction! $\Rightarrow$ hence $CTM \not\sim CTM'$