

$x_{ij}$ : € investiti in  $i \in \{A, B, C\}$  nell'anno  $j \in \{1, 2, 3\}$  |  $y_{ij} = 1$  se investo in  $i$  l'anno  $j$ , 0 altrimenti  
 VAR |  $p$ : € spesi in prestito |  $r$ : rendita |  $t$ : tasse |  $z = 1$  M pagatore, 0 altrimenti  
 P // PAO A F NO PEELOO (H P)

$\max r - 20(y_{A1} + y_{A2} + y_{A3} + y_{B1} + y_{B2} + y_{B3} + y_{C1} + y_{C2} + y_{C3}) - 0,25p - t \quad (+30000 + p - p)$   
 s.t.  $r = 0,6x_{A1} + 0,2x_{A2} + 0,1x_{A3} + 0,5x_{B1} + 0,3x_{B2} + 0,2x_{B3} + 0,7x_{C1} + 0,4x_{C2} + 0,3x_{C3}$   
 $x_{B2} \leq 2500$   
 $x_{A1} + x_{B1} + x_{C1} \leq 10000$   
 $x_{A2} + x_{B2} + x_{C2} \leq 10000 + p + 10000 - (x_{A1} + x_{B1} + x_{C1}) \quad | \quad p \leq 7000$   
 $x_{A3} + x_{B3} + x_{C3} \leq 10000 + 10000 - (x_{A1} + x_{B1} + x_{C1}) + 10000 + p - (x_{A2} + x_{B2} + x_{C2})$   
 $x_{ij} \leq M y_{ij}, \forall i \in \{A, B, C\}, j \in \{1, 2, 3\}$

$3(x_{A1} + x_{B1} + x_{C1}) + 2(x_{A2} + x_{B2} + x_{C2}) + 1(x_{A3} + x_{B3} + x_{C3}) \geq 1,75 \cdot \sum_{i \in \{A, B, C\}} \sum_{j=1}^3 x_{ij}$   
 $\leq 2,25 \cdot \dots$   
 $3x_{A1} + 2x_{A2} + 1x_{A3} + 6x_{B1} + 4x_{B2} + 2x_{B3} + 12x_{C1} + 8x_{C2} + 4x_{C3} \leq 3 \cdot \dots$

$y_{B1} + y_{B3} \leq 1 \quad | \quad t \geq 0,15r - M(1-z) \quad | \quad r \leq 8000 + Mz$   
 $x_{ij} \in \mathbb{R}^+; \quad y_{ij} \in \{0,1\}; \quad p \in \mathbb{R}^+; \quad r \in \mathbb{R}^+; \quad t \in \mathbb{R}^+; \quad z \in \{0,1\}$

1. std.  $\min z = -2x_1 - x_2$   
 s.t.  $x_1 - x_2 - x_3 + x_4 = 4$   
 $x_1 - x_2 - 2x_3 + x_5 = 2$   
 $-x_1 + 2x_2 - x_3 + x_6 = 1$   
 $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	-2	5
-2	-1	0	0	0	0	-1	0
1	0	-1	1	0	0	0	4
1	-1	-2	0	1	0	0	2
-1	2	-1	0	0	1	0	1

4. ENUNCI... | AMM. PRM.  $\checkmark$  | sistema  
 duale  $\min u_1 + u_2 + u_3$   
 s.t.  $2u_1 + 3u_2 - 2u_3 \geq 2$   
 $2u_1 - 2u_2 - 2u_3 + u_4 = 1$   
 $u_1 - u_2 + 2u_3 + u_4 = 0$   
 $u_1 \geq 0, u_2 \leq 0, u_3 \text{ lib}, u_4 \leq 0$

cca  $x_1 > 0 \rightarrow [2u_1 + 3u_2 - 2u_3 = 2]$   
 $x_2 \text{ lib} \rightarrow$  nocera  $x_3 \text{ lib} \rightarrow$  nocera  
 $2x_1 = 1 \rightarrow //$ ;  $3x_1 > \rightarrow [u_2 = 0]$   
 vuole  $\rightarrow$  nocera;  $0 = 0 \rightarrow //$

sistema  
 $2u_1 + 3u_2 - 2u_3 = 2$   
 $u_2 = 0$   
 $2u_1 - 2u_3 + u_4 = 1$   
 $u_1 - u_2 + 2u_3 + u_4 = 0$   
 $u_1 = 1, u_2 = 0, u_3 = 0, u_4 = 1$   
 $u_3 = 0, u_4 = -1$   
 AMM. PRM.  $\checkmark$   
 P.C.  $\checkmark$   
 $\Rightarrow$  OTTIMA!

0	-3	-4	0	2	0	-1	4	+2R <sub>2</sub>
x <sub>4</sub>	0	1	1	-1	0	0	2	-R <sub>2</sub>
x <sub>1</sub>	1	-1	-2	0	1	0	2	=R <sub>2</sub>
x <sub>6</sub>	0	1	-3	0	1	0	3	+R <sub>2</sub>
0	0	-1	3	-1	0	-1	10	+3R <sub>1</sub>
x <sub>2</sub>	0	1	1	-1	0	0	2	=R <sub>1</sub>
x <sub>1</sub>	1	0	-1	1	0	0	4	+R <sub>1</sub>
x <sub>6</sub>	0	0	-4	-1	2	1	1	-R <sub>1</sub>
0	1	0	4	-2	0	-1	12	+R <sub>1</sub>
x <sub>3</sub>	0	1	1	-1	0	0	2	=R <sub>1</sub>
x <sub>1</sub>	1	1	0	2	-1	0	6	+R <sub>1</sub>
x <sub>6</sub>	0	4	0	3	-2	1	9	+4R <sub>1</sub>

5. a) F.C.  $\rightarrow x_B = [x_2, x_3, x_4]$ ; MIN SO (tutti non neg.)  
 b) no; parte  $x_7$  o  $x_8 < 0$  (non ammiss.)  
 c) entra  $x_1$ , esce  $x_4$   
 d)  $x_1: x_7; x_2: x_7; x_3: x_7; x_5: x_7$   
 e)  $[x_7, x_3, x_1]; [x_1, x_3, x_4]; [x_2, x_3, x_4]; [x_5, x_3, x_4]: \theta = 0$
6. a) [no max!];  $3,0 \leq b_2 \leq \max\{3,5,3,7\}$ , e.g.  $b_2 = 3,5$   
 b)  $[3,2; 3,5]$  (best bound; unimodal)  
 c)  $P_3, P_4, P_5, P_6$ ; chiuso  $P_5$  e  $P_6$  ( $LB \leq 3,5$ )  
 d)  $P_3$   
 e)  $LB = UB \in [3,2; 3,3]$

$\bar{c}_4 = -2 < 0$  e  $\bar{a}_{14} \leq 0 \Rightarrow$  ILLIMITATO  
 b) duale INAMMISSIBILE (dualità debole)

3. A B C D E F G Acca B Altero - Grafot...  

0	0	0 <sub>A</sub>	A						
1	0	4 <sub>A</sub>	2 <sub>A</sub>	5 <sub>A</sub>	0	0	0	0	BCD
2	0	3 <sub>C</sub>	2 <sub>A</sub>	3 <sub>C</sub>	3 <sub>D</sub>	1 <sub>D</sub>	6 <sub>D</sub>	0	BDEFA
3	0	3 <sub>C</sub>	2 <sub>A</sub>	3 <sub>C</sub>	1 <sub>D</sub>	0	4 <sub>D</sub>	0	EFG
4	0	3 <sub>C</sub>	2 <sub>A</sub>	3 <sub>C</sub>	1 <sub>D</sub>	3 <sub>E</sub>	1 <sub>D</sub>	0	F
5	0	3 <sub>C</sub>	2 <sub>A</sub>	3 <sub>C</sub>	1 <sub>D</sub>	3 <sub>E</sub>	2 <sub>E</sub>	0	G
6	0	3 <sub>C</sub>	2 <sub>A</sub>	3 <sub>C</sub>	1 <sub>D</sub>	3 <sub>E</sub>	2 <sub>F</sub>	0	

Grafot diagram showing nodes A, B, C, D, E, F, G and edges with weights. Path A-C-D-E-F-G is highlighted.

AMG: A-C-D-E-F-G  
 AMG  $\leq 3$  hop: A-C-D-G  
 AMG  $\leq 5$  hop: A-C-D-E-F-G