1

// x_{ij}: num pezzi tipo $i \in \{A,B,C\}$ realizzati in macchina $j \in \{1,2\}$
// p_k: " prodotti $k \in \{1,2\}$ assemblati
// y_j: 1 se uso macchina $j \in \{1,2\}$, 0 altrimenti
// z_{ij}: 1 se realizzo almeno 1pz un tipo $i \in \{A,B,C\}$ su macchina j, 0 altrimenti

$$\max\ 100p_1 + 80p_2 - (8x_{A1} + 5x_{B1} + 7x_{C1} + 7x_{A2} + 6x_{B2} + 8x_{C2} + 1500y_1 + 700y_2)$$

s.t. $x_{A1} \le 500$; $p_1 \ge 100$

$x_{A1} + x_{A2} \ge 1000 + 2p_1 + p_2$

$x_{B1} + x_{B2} \ge 2000 + p_1 + 3p_2$

$x_{C1} + x_{C2} \ge 3000 + 3p_1 + p_2$

$x_{ij} \le M z_{ij}$, $\forall i \in \{A,B,C\}, j \in \{1,2\}$

$p_1 - p_2 \le 50$ $\quad p_2 - p_1 \le 50$

$x_{A1} + x_{B1} + x_{C1} \le M y_1$

$x_{A2} + x_{B2} + x_{C2} \le M y_2$

// M costante grande

$z_{A1} + z_{B1} + z_{C1} \le 2$

$z_{A2} + z_{B2} + z_{C2} \le 2$

$z_{B1} + z_{B2} = 1$ (≤ 1)

$x_{ij}, p_k \in \mathbb{Z}_+$ $\quad y_j \in \{0,1\}$ $\quad z_{ij} \in \{0,1\}$ $\quad \forall i \in \{A,B,C\}, j \in \{1,2\}, k \in \{1,2\}$

$0{,}15(x_{A1} + x_{B1} + x_{C1}) + 0{,}1(x_{A2} + x_{B2} + x_{C2}) \le 2000$

$1 \cdot (x_{A1} + x_{B1} + x_{C1}) + 1{,}1(x_{A2} + x_{B2} + x_{C2}) \le 20 \cdot 1000$

2

f. std. $\min -2\hat{x}_1 - 4x_2 - x_3$
$(\hat{x}_1 = -x_1)$ s.t.
$2\hat{x}_1 + 2x_2 - x_3 + x_4 = 4$
$\hat{x}_1 - x_2 + 2x_3 + x_5 = 2$
$-4\hat{x}_1 + x_2 - 2x_3 + x_6 = 2$
$\hat{x}_1, x_2, x_3, x_4, x_5, x_6 \ge 0$

	$\hat{x}_1$	x_2	x_3	x_4	x_5	x_6	$-z$	$\bar{b}$	
	-2	-4	-1	0	0	0	-1	0	
x_4	2	2	-1	1	0	0	0	4	
x_5	1	-1	2	0	1	0	0	2	
x_6	-4	1	-2	0	0	1	0	2	
	0	-2	-2	1	0	0	-1	4	$(+R_1)$
x_1	1	1	-1/2	1/2	0	0	0	2	$(R_1/2)$
x_5	0	-2	5/2	-1/2	1	0	0	0	$(-R_1')$
x_6	0	5	-4	2	0	1	0	10	$(+2R_1)$
	2	0	-3	2	0	0	-1	8	$(+2R_1)$
x_2	1	1	-1/2	1/2	0	0	0	2	(R_1)
x_5	2	0	3/2	1/2	1	0	0	4	$(+2R_1)$
x_6	-5	0	-3/2	-1/2	0	1	0	0	$(-5R_1)$
	6	0	0	3	2	0	-1	16	$(+2R_2)$
x_2	5/3	1	0	2/3	1/3	0	0	10/3	$(+\frac{1}{2}R_2')$
x_3	4/3	0	1	1/3	2/3	0	0	8/3	$(\frac{2}{3}R_2)$
x_6	-3	0	0	0	1	1	0	4	$(+R_2)$

$z^* = -16$ $\quad x_1^* = -\hat{x}_1^* = 0$ $\quad x_2^* = 10/3$ $\quad x_3^* = 8/3$
$x_4^* = x_5^* = 0$ $\quad x_6^* = 4$ (3° v. lasco)

b) duale $w^* = -16$ (dualità forte)

4

enunciato...

1) AMM. PRIMALE $x_1 = 2 \ge 0$ ✓, $x_2 = 0$ ✓, $x_3 = 0$ ✓
$2 \ge 2$; $-2 \le 2$; $-2 = -2$; $2 \ge 1$ ✓

2) DUALE $\max\ 2u_1 + 2u_2 - 2u_3 + u_4$
s.t. $u_1 - u_2 - u_3 + u_4 \le 5$
$u_1 - 3u_2 - 2u_3 \ge -2$
$2u_1 + 2u_2 + 2u_3 + u_4 = -1$
$u_1 \ge 0, u_2 \le 0, u_3$ lib, $u_4 \ge 0$

3) UGUAL DA CCPD
$x_1 \ge 0, x_1 \ne 0 \Rightarrow u_1 - u_2 - u_3 + u_4 = 5$ (CCPD)
$x_2 \le 0, x_2 = 0 \Rightarrow$ //
x_3 lib $\Rightarrow$ // [vincolo duale =]
$x_1 + x_2 - 2x_3 = 2 \Rightarrow$ //
$-x_1 - 3x_2 + 2x_3 < 2 \Rightarrow u_2 = 0$
$-x_1 - 2x_2 + 2x_3 = -2$ per dim. prim. (in caso)
$x_1 + x_3 > 1 \Rightarrow u_4 = 0$

4) SISTEMA EQ DA CCPD + AMM. DUALE

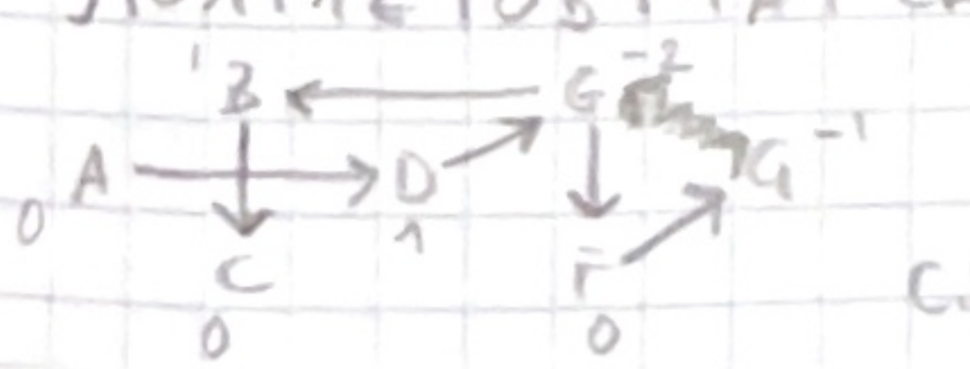
$$\begin{cases} u_1 - u_2 - u_3 + u_4 = 5 & \text{(CCPD)} \\ u_2 = 0 & \text{(CCPD)} \\ u_4 = 0 & \text{(CCPD)} \\ 2u_1 + 2u_2 + 2u_3 + u_4 = -1 & \text{(A.D.)} \end{cases} \Rightarrow \quad u_1 = 9/4,\ u_2 = 0,\ u_3 = -11/4,\ u_4 = 0$$

5) VERIFICA AMM. DUALE
$u_1 \ge 0$ ✓ $u_2 \le 0$ ✓, u_3 lib ✓, $u_4 \ge 0$
$u_1 - u_2 - u_3 + u_4 \le 5$ (per costr.)
$u_1 - 3u_2 - 2u_3 = 31/4 \ge -2$ ✓
$2u_1 + 2u_2 + 2u_3 + u_4 = -1$ (per costr.)

6) CONCLUSIONI
$\left.\begin{array}{l} x \text{ AMM PRIM}, u \text{ AMM DUALE} \\ x \text{ e } u \text{ in CCPD} \end{array}\right\} \Rightarrow x$ OTTIMA

3

h	A	B	C	D	E	F	G	Agg
0	0_A	∞_A	∞_A	∞_A	∞_A	∞_A	∞_A	A
1	0_A	2_A	2_A	1_A	∞_A	∞_A	∞_A	BCD
2	0_A	2_A	1_B	1_A	-2_D	4_C	∞_A	CEF
3	0_A	1_E	1_B	1_A	-2_D	~~3_C~~ 0_E	3_F	BFG
4	0_A	1_E	0_B	1_A	-2_D	0_E	-1_F	CG
5	0_A	1_E	0_B	1_A	-2_D	0_E	-1_F	∅



→ ALBERO (pred.)
no GRAFO (CCPD)

CM AG: A-D-E-F-G (CPROD.)

AG $\boxed{\le 3h}$: A-C-F-G $\quad$ (≤ 4): A-D-E-F-G (Albero)

5

a) F.C. per $x_B = [x_2, x_6, x_4]$, non ottima (vedi e)
b) no min ratio, x_6 DIVENTA < 0 (non amm)
c) entra x_3 (tra x_3, x_4, x_5), esce x_6 (no Bland)
d) $x_3 | x_6$; $x_4 | x_2$; $x_4 | x_1$
e) sì, problema illimitato (col. di x_5)

6

a) $UB_2 = 4{,}9$
b) $[4{,}1; 4{,}9]$
c) aperti P_3 P_4 P_5 P_6; chiudo P_3, P_4 e P_5
d) P_6
e) $LB = UB \in [4{,}1; 4{,}9]$