Business Process Simulation

Lecture 6

Department of Industrial Engineering and Innovation Sciences, Information Systems Group

Overview on lecture modules

a) Simulation parameters for terminating simulations
b) Simulation parameters for non-terminating simulations
c) Analyzing warm up period in non-terminating simulations
d) Comparing simulation outcomes
e) Sensitivity analysis
f) Conclusion

Simulation Methodology (7 steps)

STEP 1: Project definition

- **Parameters for experiments: replication length, number of replications, warm-up period, etc.**
- **Output analysis: statistically reliable data, valid conclusions**

STEP 2: Design the simulation study

STEP 3: Conceptual model

STEP 4: Executable model and verification

STEP 5: Validation

STEP 6: Experiments and output analysis

STEP 7: Conclusion

TU/e

“Random in – Random out”

Input analysis:
Finding fitting distributions for: Inter arrival times, Service times, Chances, etc.

Real object system

Output analysis:
“ sample” of population, needs statistical analysis (use of appropriate methods)

Random input:
- Random numbers
- Random variates

Simulation model

Random output:
- Performance measures

Point estimate of μ

Confidence interval half width

$$\bar{Y} = \frac{\sum_{i=1}^{R} \bar{Y}_i}{R} = 7.50 \quad (\pm\ 0.15)$$

	Customer 1	Customer 2	Customer 3	…	Customer n-1	Customer n	Average
Replication 1	0.00	0.00	0.50	…	3.81	2.56	**7.47** (± 0.22)
Replication 2	0.00	0.03	1.67		7.25	8.44	**7.48** (± 0.29)
Replication 3	0.00	2.62	5.78	…	10.36	11.98	**7.63** (± 0.30)
…	…	…	…	…	…	…	…
Replication R	0.00	1.54	0.73	…	14.26	12.15	**7.66** (± 0.24)

Simulation experiment set-up

How to set up the experiment:

- Run length of a replication? (n / simulation time)
- Number of replications? (R)
- Independent replications?
- Start and end of measurement (warm up / cool down period)?

	Customer 1	Customer 2	Customer 3	…	Customer n-1	Customer n	Average
Replication 1	Y_{11}	Y_{12}	Y_{13}	…	$Y_{1(n-1)}$	Y_{1n}	$\mathbf{Y_1}$
Replication 2	Y_{21}	Y_{22}	Y_{23}		$Y_{2(n-1)}$	Y_{2n}	$\mathbf{Y_2}$
Replication 3	Y_{31}	Y_{32}	Y_{33}	…	$Y_{3(n-1)}$	Y_{3n}	$\mathbf{Y_3}$
…	…	…	…	…	…	…	**…**
Replication R	Y_{R1}	Y_{R2}	Y_{R3}	…	$Y_{R(n-1)}$	Y_{Rn}	$\mathbf{Y_R}$

Simulation experiment set-up

How to set up the experiment:

- Run length of a replication?
- Number of replications?
- Independent replications?
- Start and end of measurement (warm up / cool down period)?

First decide on:

- Terminating / steady state simulation?

Simulation Experiments

- Terminating simulation

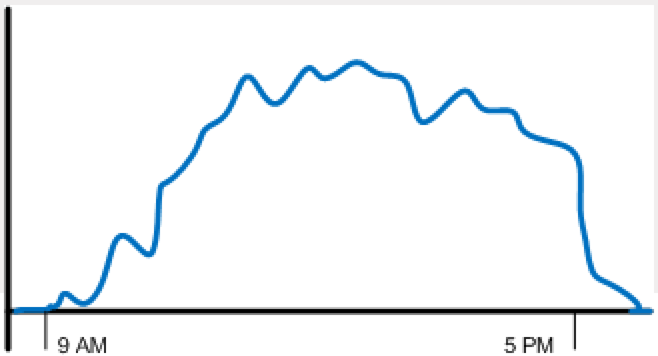
Only interested in the behavior of the system in a particular period

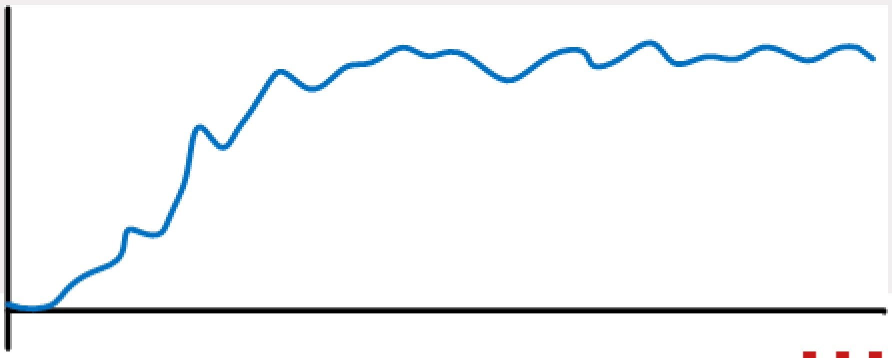
e.g. roadway system during rush hour, opening hours of a shop

- Nonterminating simulation

Interested in the steady-state behavior of a system

e.g. 24/7 production line





Which one is appropriate?

Depends on:

- goals of the study and
- nature of the system
- Statistical analysis for terminating simulations is a lot easier (however modeling the start and end conditions can be a lot more difficult)

Is steady-state relevant at all?

- 24 h/day, “lights-off” manufacturing, or office hours?
- Is system state conserved between days?

TU/e

Business Process Simulation

6a) Simulation parameters for terminating simulations

Department of Industrial Engineering and Innovation Sciences, Information Systems Group

Simulation Experiments

- **Terminating simulation**

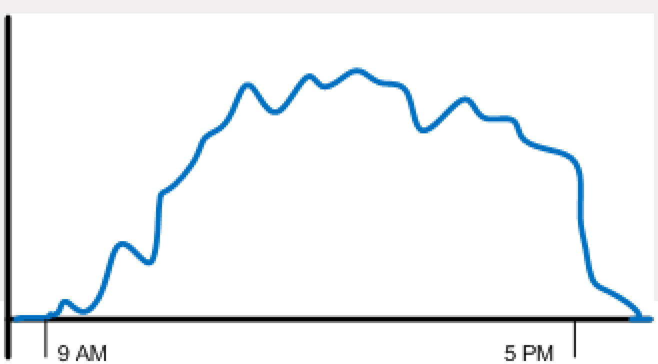
Only interested in the behavior of the system in a particular period

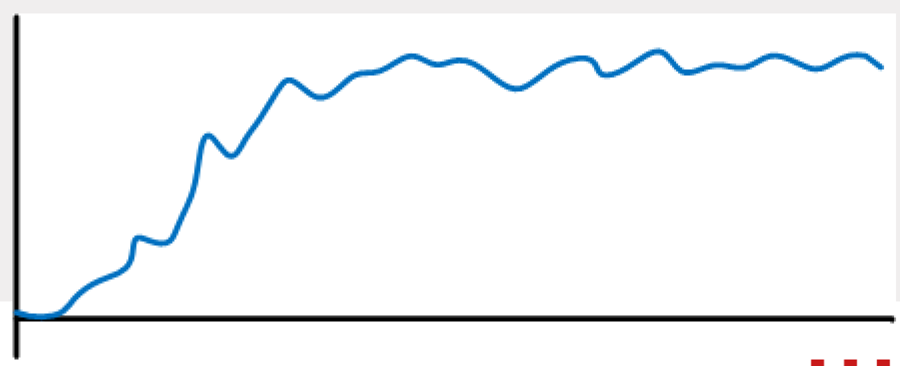
e.g. roadway system during rush hour, opening hours of a shop

- Nonterminating simulation

Interested in the steady-state behavior of a system

e.g. 24/7 production line



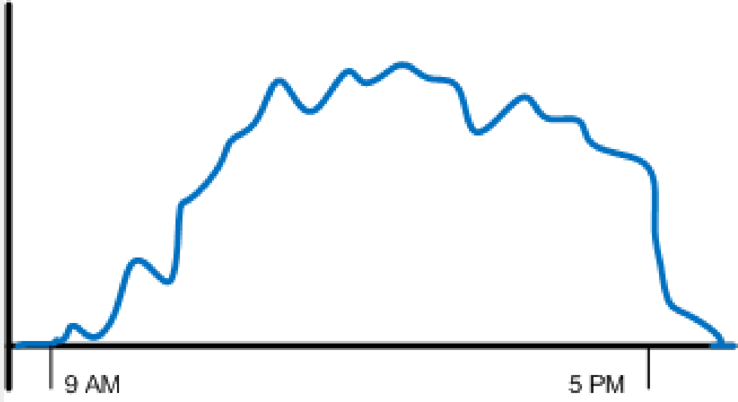


Simulation experiments

How to set up the experiment:

- Run length of a replication?
- Number of replications?
- Independent replications?
- Start and end of measurement (warm up / cool down period)?

Terminating Simulation



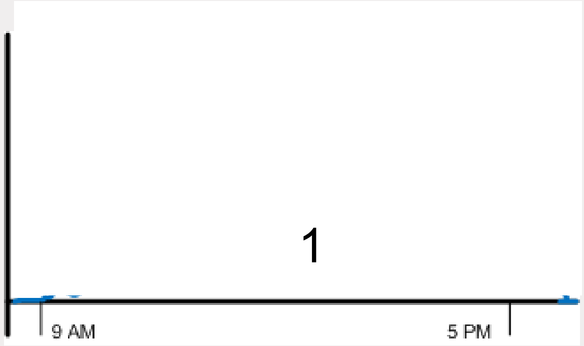
- Performance indicators to be estimated are defined relative to specific initial and termination conditions that are part of the model
- There is a "natural" and realistic way to model both the initial and stopping conditions
- Output performance measures generally depend on both the initial and stopping conditions
- Usually conducted by making several simulation replications over a period using different seed value

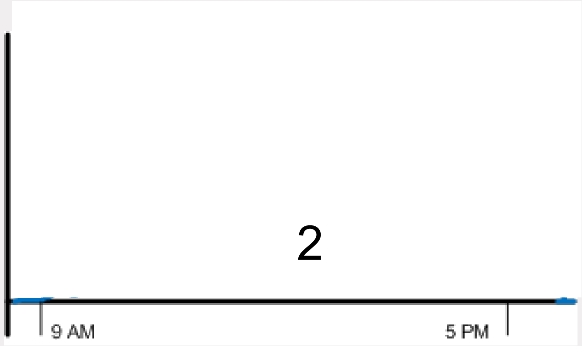
Statistical analysis for terminating simulations

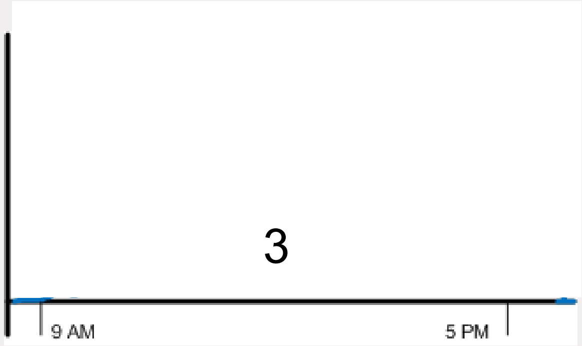
One simulation run is a sample of size one

- not worth much statistically - > multiple replications

Various runs (replications) with different seed value for random generator (different sequence of random numbers)







Terminating simulation experiments

How to set up the experiment:

✓ Start and end of measurement (warm up / cool down period)?

- Included in measurement of performance indicators

✓ Run length of a replication?

- Follows from start and termination conditions
- Defined by selecting the initial model state
- and by selecting a terminating event:
  - Time or item counts
  - Occurence of a certain event

Number of replications?

✓ Independent replications?

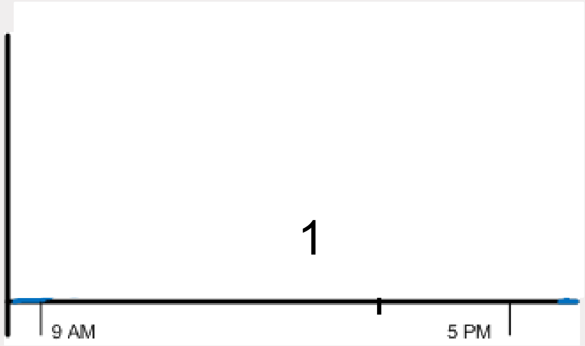
- Different simulation runs start with different seed values

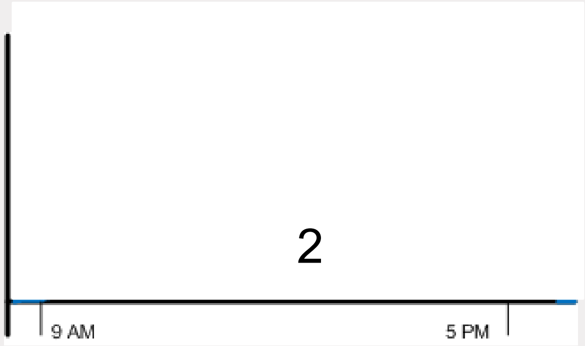
How to determine the number of replications?

One simulation run is a sample of size one

- not worth much statistically - > multiple replications

Various runs (replications) with different seed value for random generator (different sequence of random numbers)





… R = ?

How to determine the number of replications?

One simulation run is a sample of size one

- not worth much statistically - > multiple replications

Various runs (replications) with different seed value for random generator (different sequence of random numbers)

N>30

Number of replications not straightforward to determine

- Rely on central limit theorem to justify normality assumption even though it's generally not true
- A more accurate point estimate is likely to be achieved as the number of replications increase.
- However, there is a point where additional model replications will not significantly improve the exactness of a point estimate

Note that classical statistical methods don't work directly within a simulation run, due to autocorrelation usually present

How to determine the number of replications?

Approach based on a set of *m* pilot run(s)!

See also:
Banks et al section 11.4.2,
or
Law & Kelton, section 9,4,1

Determining the number of replications

- Depends on desired half-width (precision)
- Do a pilot run of m replications first (e.g. m = 5-10)
- Compute the required number of replications with a selected degree of error between $\overline{Y}$ and μ :

$$N = \left(\frac{S \cdot t_{m-1,1-\alpha/2}}{\overline{Y} \cdot \varepsilon} \right)^2$$

where:

N	=	The number of replications required, given m replications
m	=	Then number of replication made in the pilot-run
$\bar{X}(m)$	=	The estimate of the real mean m from m simulation runs (samples)
$S(m)$	=	The estimate of the real standard deviation s from m simulation runs
α	=	Level of significance
ε	=	Allowable percentage error of the estimate $\bar{X}(m)$; $\varepsilon = \frac{\lvert\bar{X}(m)-\mu\rvert}{\lvert\mu\rvert}$
$t_{m-1,1-\frac{\alpha}{2}}$	=	Critical value of the two-tailed t-distribution at a level α of significance, given $m-1$ degrees of freedom

Terminating simulation experiments

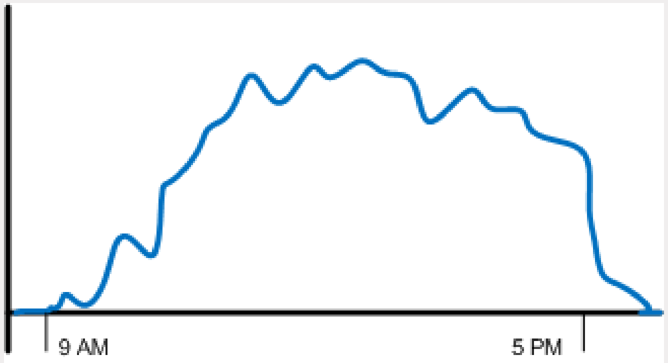
How to set up the experiment:

✓ Start and end of measurement (warm up / cool down period)?

- Included in measurement of performance indicators

✓ Run length of a replication?

- Follows from start and termination conditions
- Defined by selecting the initial model state
- and by selecting a terminating event:
  - Time or item counts
  - Occurence of a certain event



✓ Number of replications?

- Use central limit theorem to start with: 30 replications, or
- Determine number of replication needed for certain precision based on given *formula,* and,
- Increase number of replications if confidence intervals too big

✓ Independent replications?

- Different simulation runs start with different seed values

Point estimate of μ

$$\bar{Y} = \frac{\sum_{i=1}^{R} \bar{Y}_i}{R} = 7.50 \ (\pm\ 0.15)$$

Confidence interval half width

	Customer 1	Customer 2	Customer 3	…	Customer n-1	Customer n	Average
Replication 1	Y_{11}	Y_{12}	Y_{13}	…	$Y_{1(n-1)}$	Y_{1n}	$\mathbf{Y_1}$
Replication 2	Y_{21}	Y_{22}	Y_{23}		$Y_{2(n-1)}$	Y_{2n}	$\mathbf{Y_2}$
Replication 3	Y_{31}	Y_{32}	Y_{33}	…	$Y_{3(n-1)}$	Y_{3n}	$\mathbf{Y_3}$
…	…	…	…	…	…	…	…
Replication R	Y_{R1}	Y_{R2}	Y_{R3}	…	$Y_{R(n-1)}$	Y_{Rn}	$\mathbf{Y_R}$

Statistical analysis for terminating simulations

Make R IID replications of a terminating simulation

- Same initial conditions for each replication
- Same terminating event for each replication
- Different random numbers for each replication (by different seed value)

Let Yi be a summary measure of interest from the i-th replication

- e.g., Yi = average queue delay of all customers in i-th replication

then Y1, Y2, ..., YR are IID random variables, we can apply classical statistical analysis to them

- estimator for the mean,
- confidence intervals
- (hypothesis testing to conclude whether estimator for mean is significantly different from compared situation)

Business Process Simulation

6b) Simulation parameters for non-terminating (steady state) simulations

Department of Industrial Engineering and Innovation Sciences, Information Systems Group

Simulation Experiments

- Terminating simulation

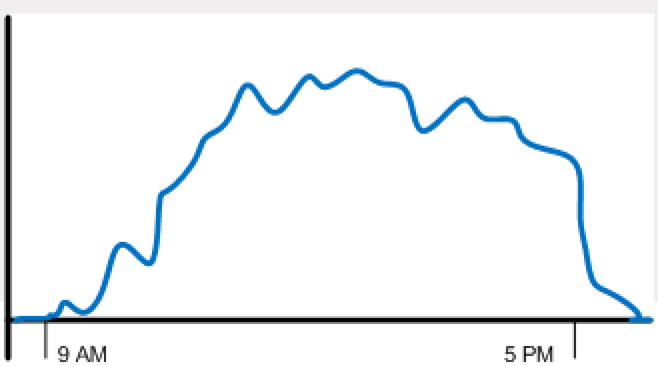
Only interested in the behavior of the system in a particular period

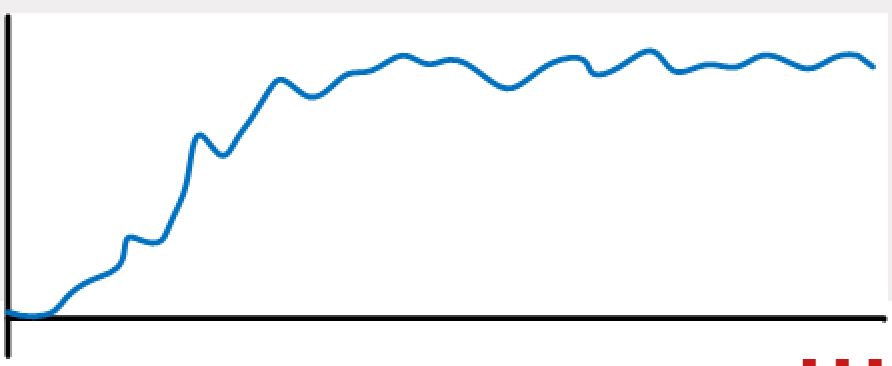
e.g. roadway system during rush hour, opening hours of a shop

- **Nonterminating simulation**

Interested in the steady-state behavior of a system

e.g. 24/7 production line

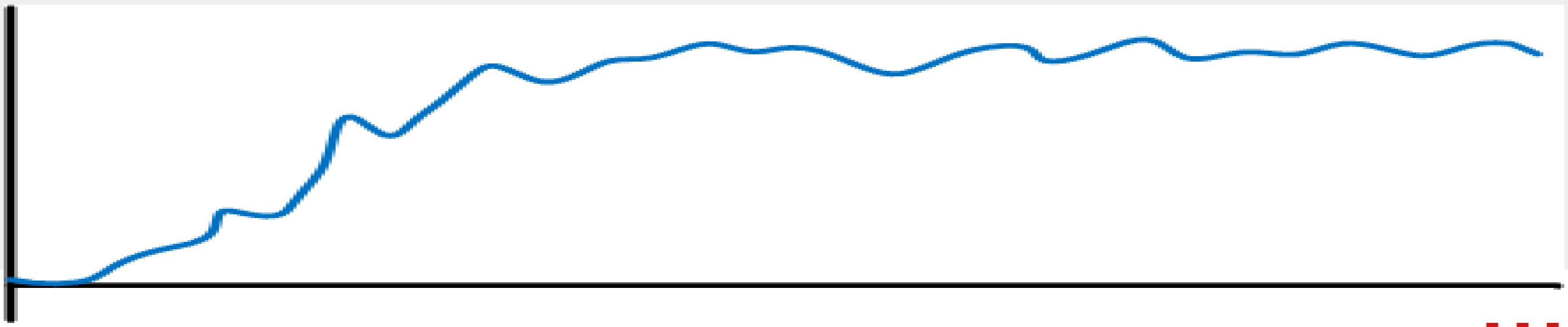




Non-terminating simulation experiments

How to set up the experiment:

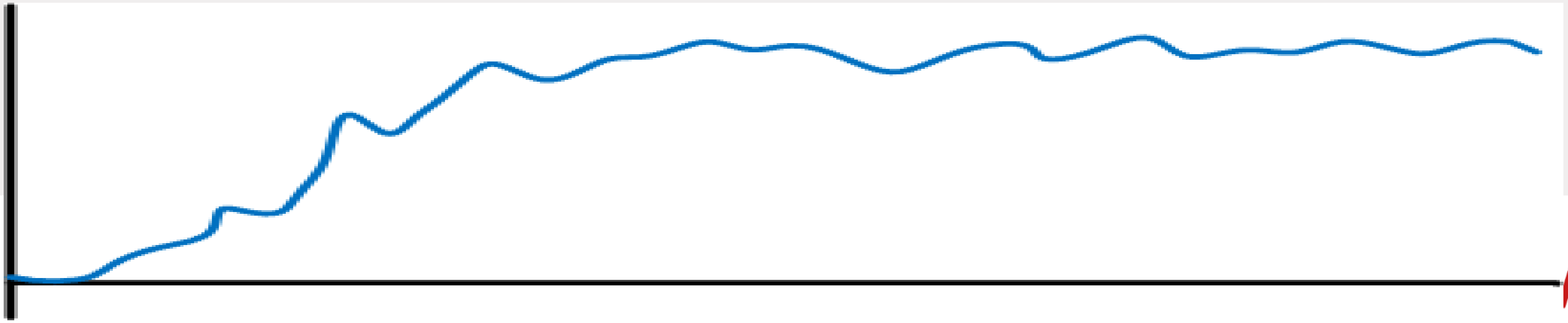
- Start and end of measurement (warm up / cool down period)?
- Run length of a replication?
- Independent replications?
- Number of replications?



Non-Terminating Simulations

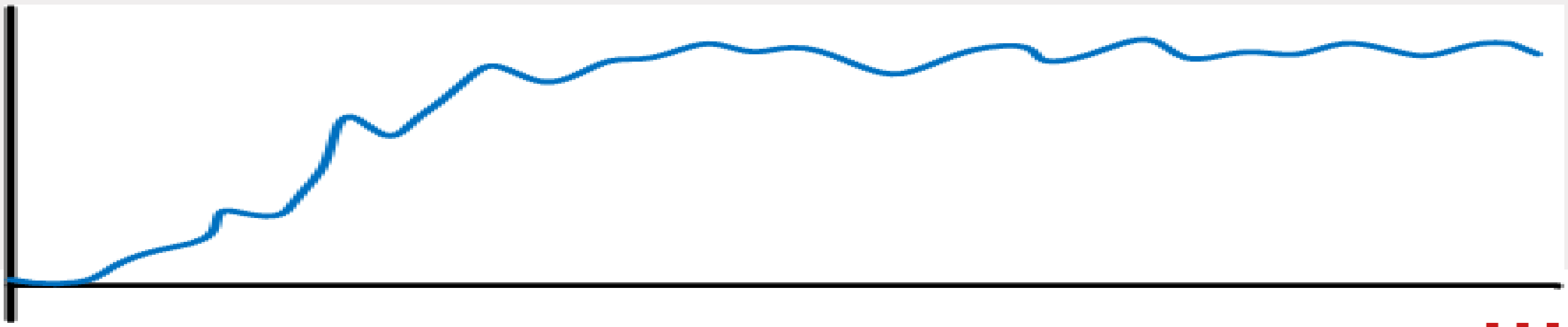
- There is no natural and realistic event that terminates the model
- Interested in “long-run” behavior characteristic of “normal” operation
- Steady-state parameter of the model: performance measure that is characteristic of a steady-state distribution of the process
- Theoretically, does not depend on initial conditions
- Practically, must ensure that run is long enough so that initial-condition effects have faded

- Not all nonterminating systems are steady-state!
  - There could be a periodic “cycle” in the long run, giving rise to steady-state cycle parameters



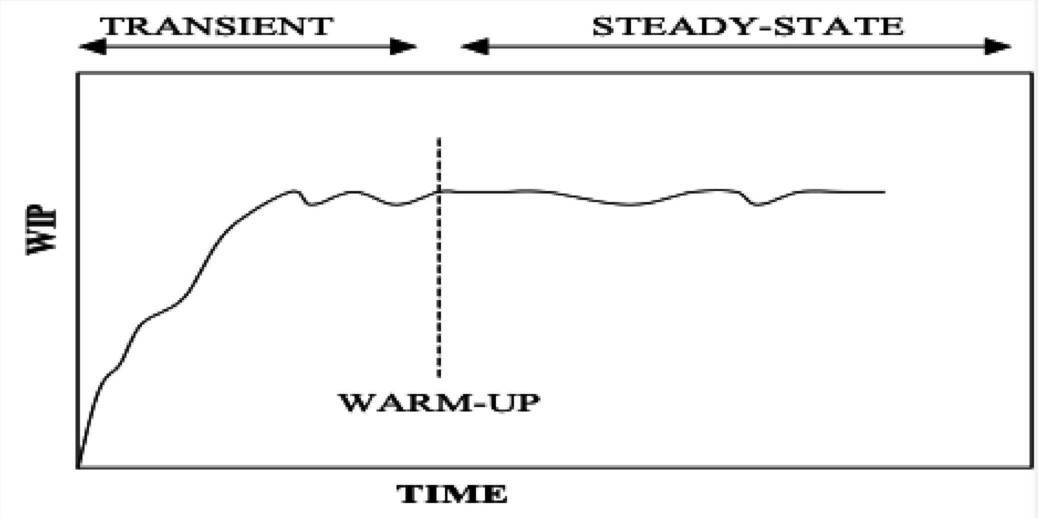
Steady state

- Does not mean that all the observations are all the same.
- Only means that all observations throughout the steady-state period will have approximately the same distribution.



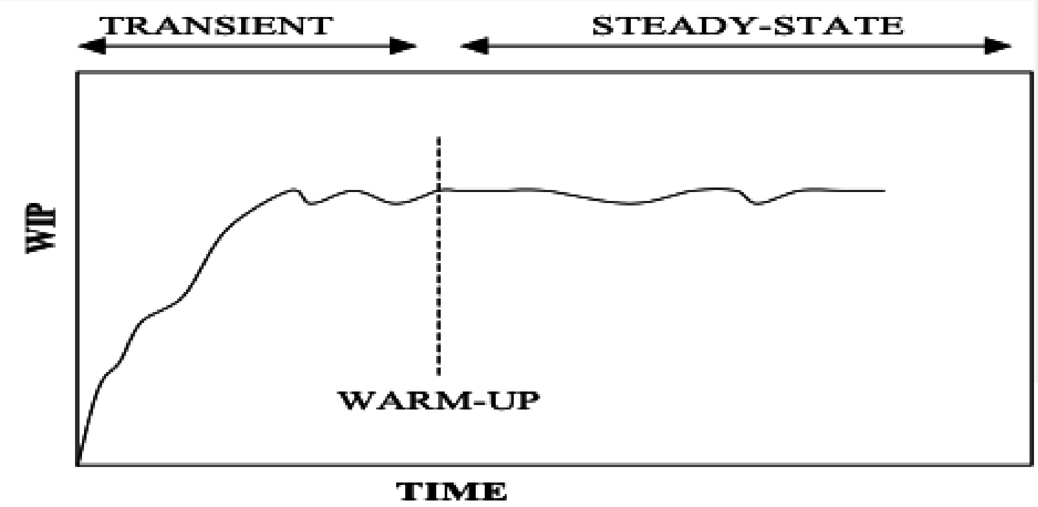
The Problem of the Initial Transient

- If steady-state is the goal, initial conditions will generally bias results of simulation for some initial period of time
- Most common technique to deal with this is to warm up the model, also called initial-data deletion



The Problem of the Initial Transient

- If steady-state is the goal, initial conditions will generally bias results of simulation for some initial period of time
- Most common technique to deal with this is to warm up the model, also called initial-data deletion
  - Identify index i (for discrete-time processes) or time t0 (for continuous-time processes) beyond which output appears not to be drifting any more
  - Data collection begins after a warm-up period is completed (delete observations until warm-up point)
  - After warmup period, observations will still have variance!

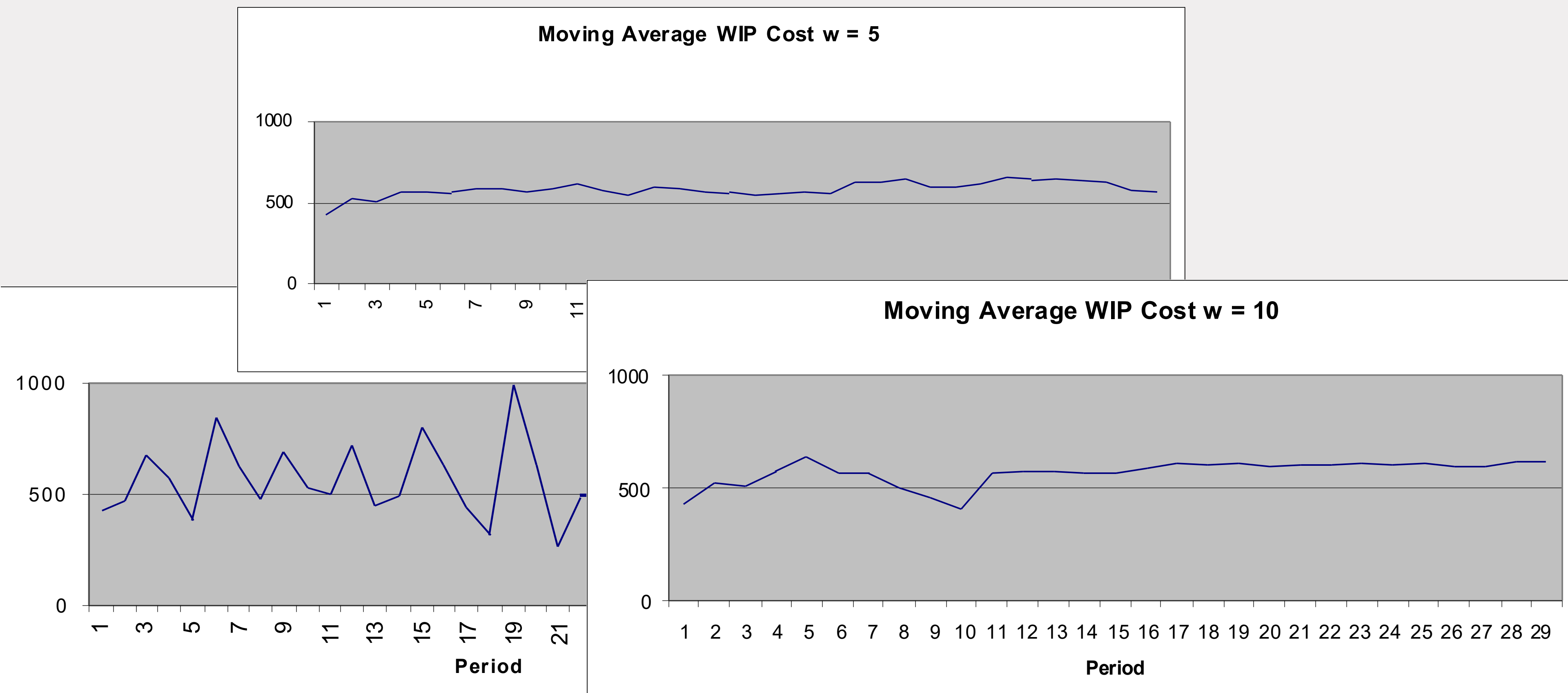


How to determine length of a warm-up period?

SEE BOOK
p.456-458

Welch graphical method

- Average the output values of several replications and
- Observe the diagram to determine warm-up period



Non-terminating simulation experiments

How to set up the experiment:

✓ Start and end of measurement (warm up / cool down period)?

- Not included in measurement
- Use *Welch graphical method* to determine warm-up period

Run length of a replication?

Independent replications?

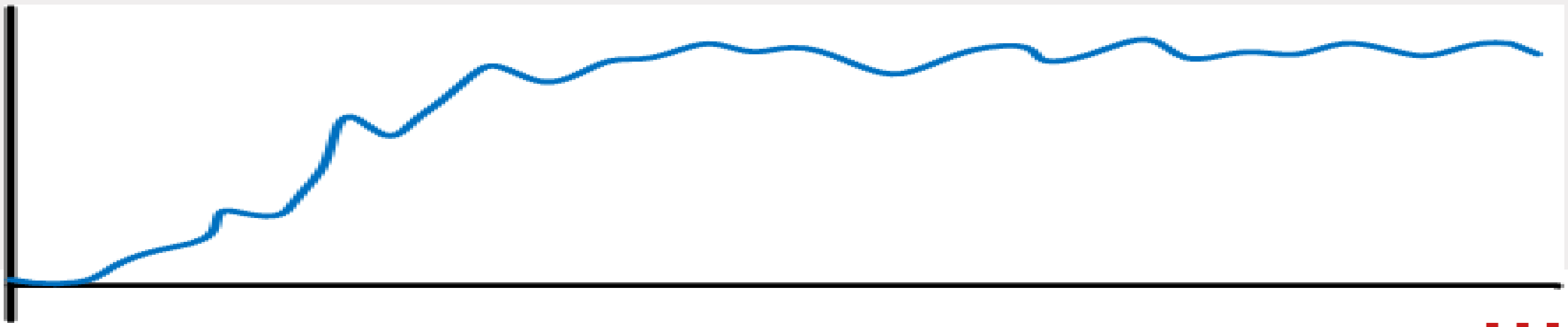
Number of replications?

Non-terminating simulation experiments

How to set up the experiment:

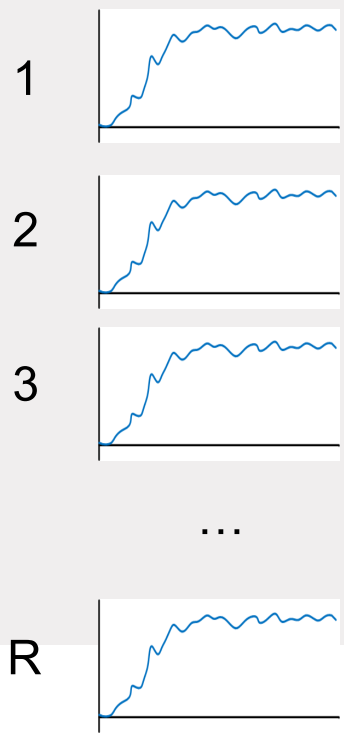
- Start and end of measurement (warm up / cool down period)?
- Run length of a replication?
- Independent replications?
- Number of replications?

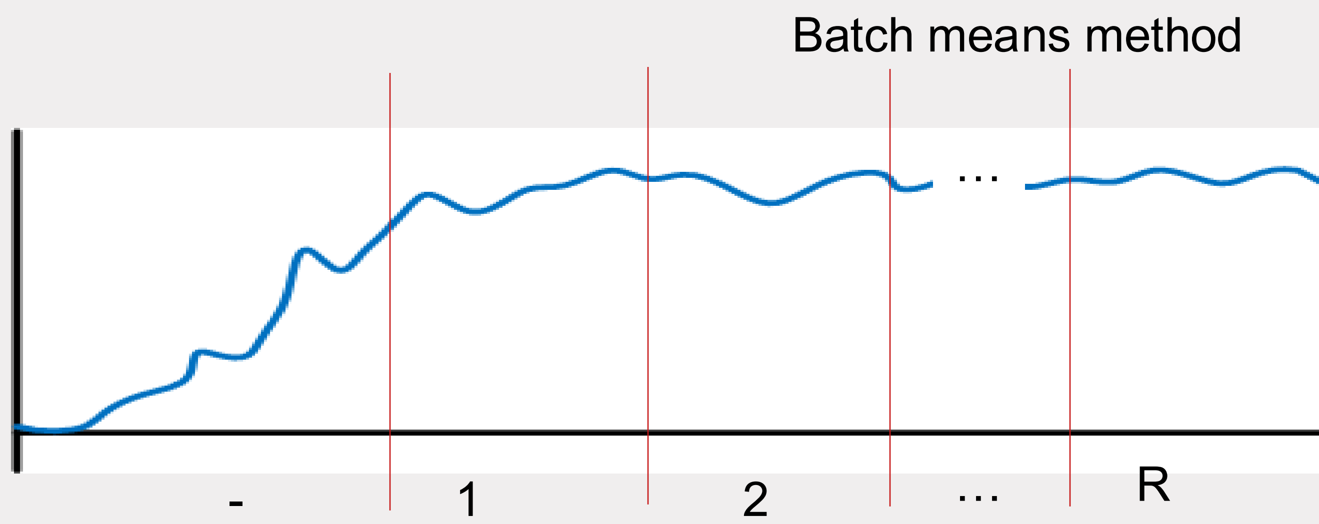
But first: how to obtain replications?



Obtaining sample observations

- Many ‘short’ simulation runs (one run = one replication)
- One very long run (split into several replications)





Obtaining Replications

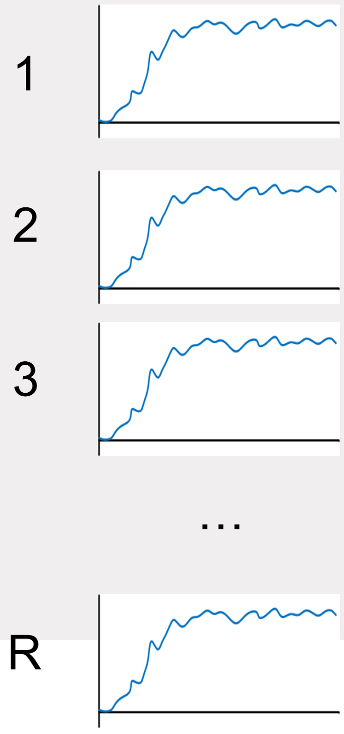
1) If the warm-up period is **short**:
   - *Separate replications* are preferred over batch intervals
   - Guarantees that observations are independent!
   - Still measurement only starts after warm-up period

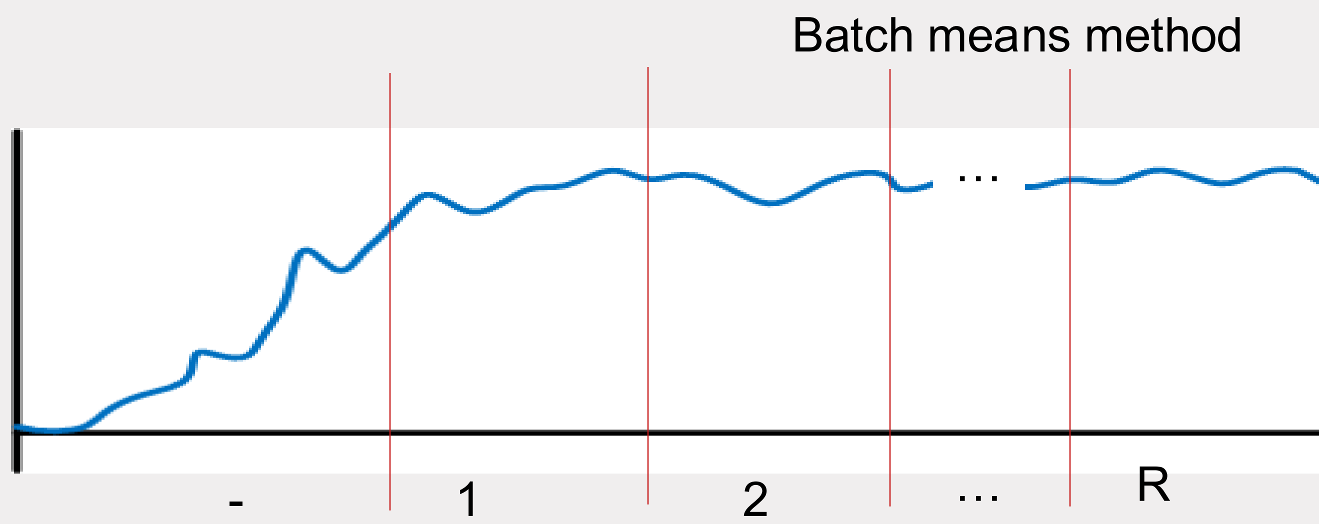
2) If the warm-up period is **long**:
   - *Batch means method* is preferred
   - Warm-up period is only incurred once
   - Reduces time to collect necessary observations
   - **But: possibility of autocorrelation!**

     => Check independence of simulation output!

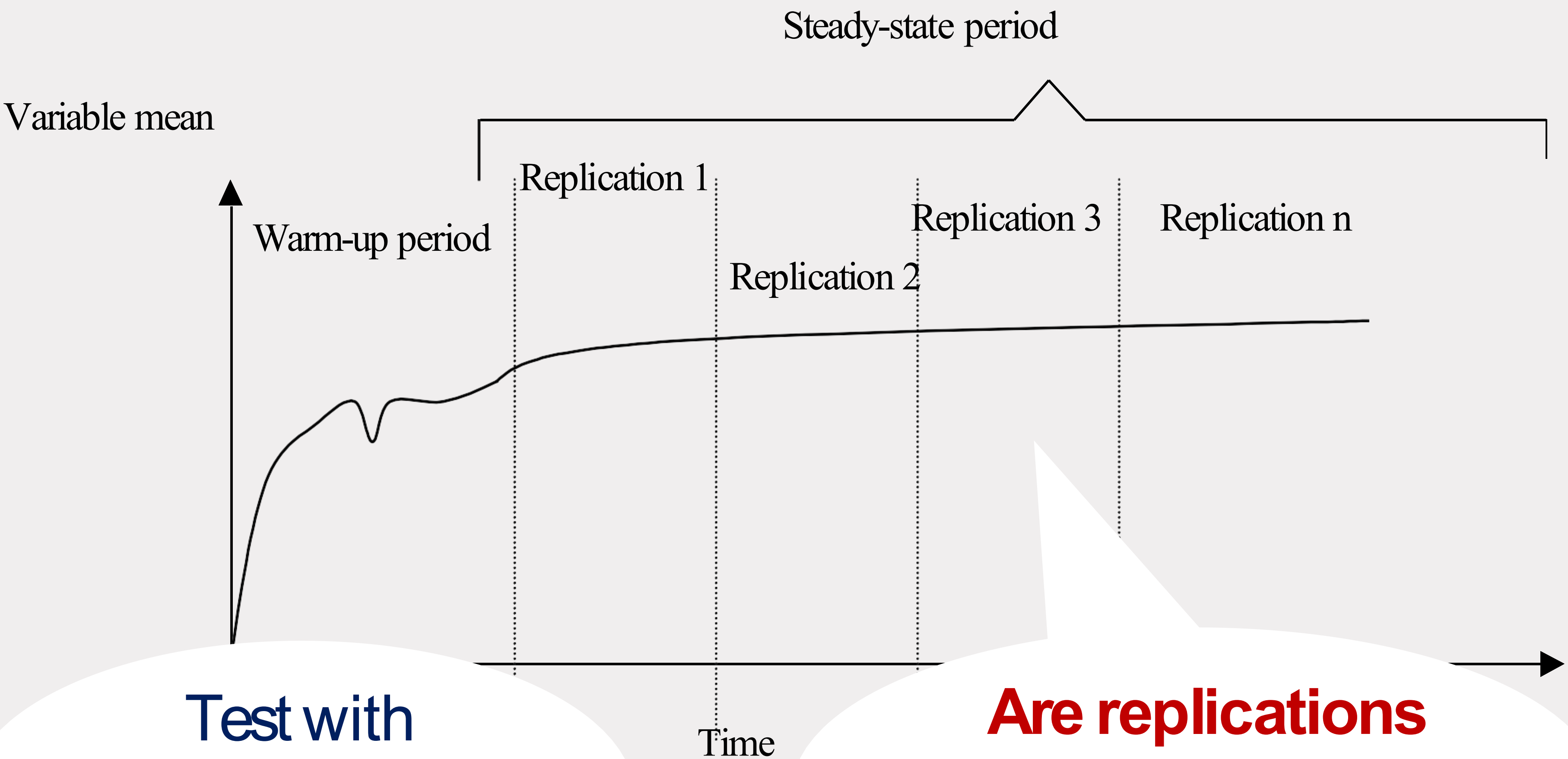
Obtaining sample observations

- Many ‘short’ simulation runs (one run = one replication)
- One very long run (split into several replications)





Batch means method



Test with Von Neumann Ratio!

Are replications long enough to be independent?

The Von Neumann ratio

$$Q = \frac{\sum_{1}^{n-1} (\bar{X}_i - \bar{X}_{i+1})^2}{\sum_{1}^{n} (\bar{X}_i - \bar{X})^2}$$

Use n (e.g. $n=5$) pilot replications (from one run) and calculate Q
The expected value of Q is 2
In practice, we may assume the data set to be independent if:

$$|Q-2| < 0,35$$

Non-terminating simulation experiments

How to set up the experiment:

✓Start and end of measurement (warm up / cool down period)?

- Not included in measurement
- Use *Welch graphical method* to determine warm-up period

- **Run length of a replication?**

✓Independent replications?

- Several runs with different seed value: OK
- Batch means method (one run split in several replications): check independence with *Von Neumann ratio*

- **Number of replications?**

TU/e

Determining the replication length

No method available ;-(

Determine an appropriate run length:

- Representative sample of steady-state response
- Based on interval between least frequent events
- Let every type of event happen many times
- (Plus initial warm-up time!)

Increase if confidence intervals simulation output too big

Non-terminating simulation experiments

How to set up the experiment:

✓ Start and end of measurement (warm up / cool down period)?
- Not included in measurement
- Use *Welch graphical method* to determine warm-up period

✓ Run length of a replication?
- No method available: reasoning needed for representative sample
  - Based on interval between least frequent events
  - Let every type of event happen many times
- Increase length if confidence intervals too big or if replications are not independent

✓ Independent replications?
- Several runs with different seed value: OK
- Batch means method (one run split in several replications): check independence with *Von Neumann ratio*

- **Number of replications?**

Determining the number of replications

- No method available
- Stochastic simulation? ⇒ Multiple model replications
- Rule of thumb: at least 3 to 5 (or 5-10) replications for each experiment.
- A more accurate point estimate is likely to be achieved as the number of replications increase (cf. smaller c.i.)
- However, there is a point where additional model replications will not significantly improve the exactness of a point estimate.
- In the non-terminating situation usually replications are longer!

Non-terminating simulation experiments

How to set up the experiment:

✓ Start and end of measurement (warm up / cool down period)?
- Not included in measurement
- Use *Welch graphical method* to determine warm-up period

✓ Run length of a replication?
- No method available: reasoning needed for representative sample
  - Based on interval between least frequent events
  - Let every type of event happen many times
- Increase length if confidence intervals too big or if replications are not independent

✓ Independent replications?
- Several runs with different seed value: OK
- Batch means method (one run split in several replications): check independence with *Von Neumann ratio*

✓ Number of replications?
- From literature: 3-5 or 5-10 replications
- Increase number of replications if confidence intervals to big, or
- Determine number of replication needed for certain precision with given *formula*

TU/e

How to make several replications (both terminating and non terminating) in SimPN?

```python
shop.store_checkpoint("initial state")

average_cycle_times = []

NR_REPLICATIONS = 50
SIMULATION_DURATION = 40000
WARMUP_TIME = 20000
for _ in range(NR_REPLICATIONS):
  reporter = ProcessReporter(WARMUP_TIME)
  shop.restore_checkpoint("initial state")
  shop.simulate(SIMULATION_DURATION, reporter)

  average_cycle_times.append(reporter.total_cycle_time /
                             reporter.nr_completed)
```

Statistical Analysis for Non-Terminating Simulations

- Assume that an appropriate warmup period, replication length, and number of replications, has been determined
- Make independent replications, each warmed up, or use Batch-Means method and check for independence
- Y_i = average of output measure on *i*-th replication

- Proceed with statistical analysis exactly as in terminating case
- Compute mean, variance estimates across replications, use confidence intervals

Non-terminating simulation experiments

How to set up the experiment:

✓Start and end of measurement (warm up / cool down period)?
- Not included in measurement
- Use *Welch graphical method* to determine warm-up period

✓Run length of a replication?
- No method available: reasoning needed for representative sample
  - Based on interval between least frequent events
  - Let every type of event happen many times
- Increase length if confidence intervals too big or if replications are not independent

✓Independent replications?
- Several runs with different seed value: OK
- Batch means method (one run split in several replications): check independence with *Von Neumann ratio*

✓Number of replications?
- From literature: 3-5 or 5-10 replications
- Increase number of replications if confidence intervals to big, or
- Determine number of replication needed for certain precision with given *formula*

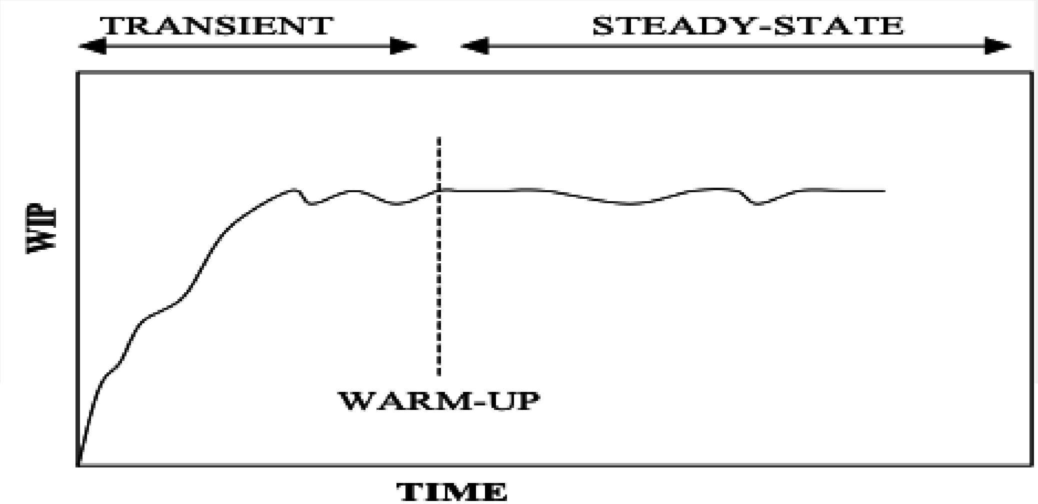
TU/e

Business Process Simulation

6c) Analyzing warm-up period (only in non-terminating simulations)

The Problem of the Initial Transient

- If steady-state is the goal, initial conditions will generally bias results of simulation for some initial period of time
- Most common technique to deal with this is to warm up the model, also called initial-data deletion
  - Identify index i (for discrete-time processes) or time t0 (for continuous-time processes) beyond which output appears not to be drifting any more
  - Data collection begins after a warm-up period is completed (delete observations until warm-up point)
  - After warmup period, observations will still have variance!

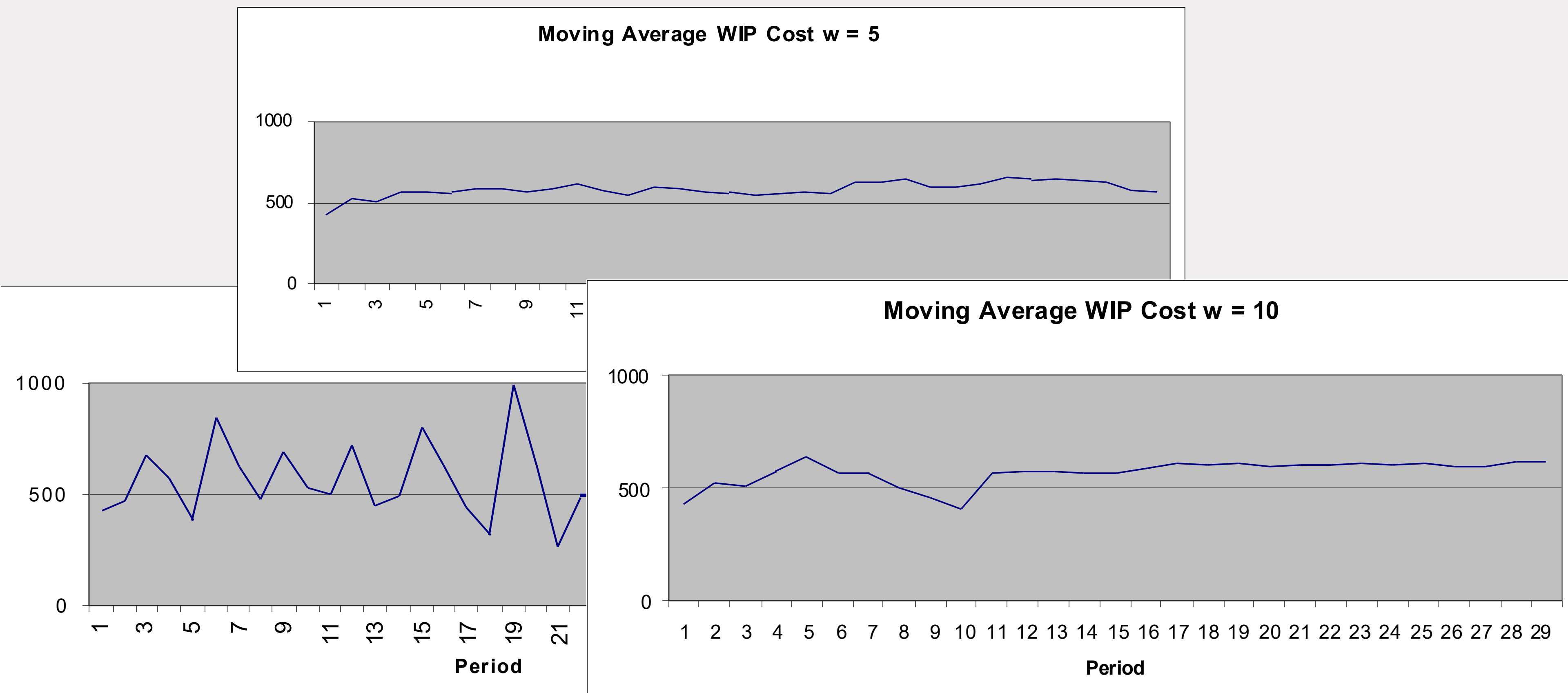


How to determine length of a warm-up period?

SEE READER,

Welch graphical method

- Average the output values of several replications and
- Observe the diagram to determine warm-up period



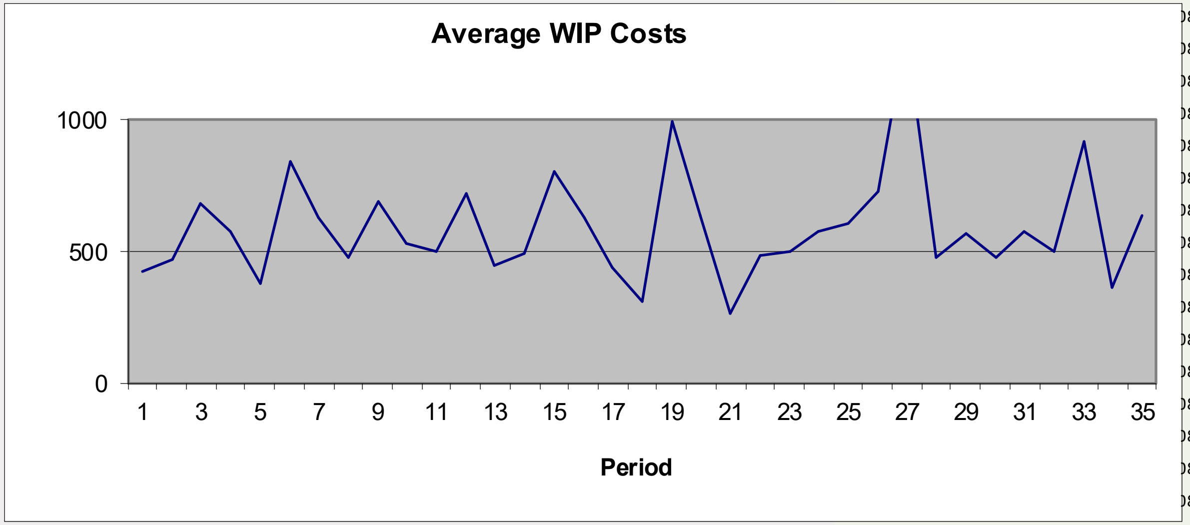
Welch Graphical Metho

i-th period	Yji = Observed costs during i-th period and j-th replication					Average Cost per period	Moving average cost during ith period various values of w	
Period (i)	Rep1	Rep2	Rep3	Rep4	Rep5		w=5	w=10
1	422	423,2667	426,2081	425,3725	427,632	424,8958	424,8958	424,8958
2	468	469,2667	472,2081	471,3725	473,632	470,8958	524,8958	524,8958
3	676	677,2667	680,2081	679,3725	681,632	678,8958	505,7131	505,7131
4	572	574,0623	576,2081	575,3725	577,632	575,055	571,4325	571,4325
5	375	376,2667	379,2081	378,3725	385,2687	378,8232	573,679	632,6281
6	842	843,2667	846,2081	845,3725	842,7548	843,9204	563,327	563,327
7	625	626,2667	629,2081	628,3725	628,861	627,5417	590,0191	566,2036
8	473	474,2667	477,2081	476,3725	476,861	475,5417	587,7141	495,0173
9	685	686,2667	689,2081	688,3725	688,861	687,5417	570,4971	456,2866
10	528	529,2667	532,2081	531,3725	531,861	530,5417	591,1747	405,815
11	500	501,2667	504,2081	503,3725	505,8749	502,9444	614,2369	568,137
12	716	717,2667	720,2081	719,3725	719,6942	718,5083	577,654	570,8989
13	443	444,2667	447,2081	446,3725	446,861	445,5417	549,0146	572,0893
14	487	488,2667	491,2081	490,3725	490,6942	489,5083	595,865	567,0417
15	800	801,2667	804,2081	803,3725	803,6942	802,5083	591,079	568,5103
16	630	631,2667	634,2081	633,3725	633,6942	632,5083	567,0203	584,99
17	439	440,2667	443,2081	442,3725	442,6942	441,5083	565,1977	607,9031
18	310	311,2667	314,2081	313,3725	313,6942	312,5083	544,9602	600,5485
19	988	989,2667	992,2081	991,3725	993,632	990,8958	556,5379	604,9464
20	632	633,2667	636,2081	635,3725	637,632	634,8958	567,1186	595,0585
21	263	264,2667	267,2081	266,3725	268,632	265,8958	560,0629	597,361
22	480	481,2667	484,2081	483,3725	485,632	482,8958	623,0254	597,1683
23	493	494,2667	497,2081	496,3725	498,632	495,8958	625,897	606,6629
24	570	571,2667	574,2081	573,3725	575,632	572,8958	649,114	602,5845
							2,6595	609,6506
							97,5686	592,4697
							8,7504	590,8886
							8,2958	610,4931
							5,9322	610,6398
							1,8413	
							6,9152	
							25,498	
							5987	
							2797	

	Customer 1	Customer 2	Customer 3	...	Customer n-1	Customer n	Average
Replication 1	Y_{11}	Y_{12}	Y_{13}	...	$Y_{1(n-1)}$	Y_{1n}	$\mathbf{Y}_1$
Replication 2	Y_{21}	Y_{22}	Y_{23}		$Y_{2(n-1)}$	Y_{2n}	$\mathbf{Y}_2$
Replication 3	Y_{31}	Y_{32}	Y_{33}	...	$Y_{3(n-1)}$	Y_{3n}	$\mathbf{Y}_3$
Replication R	Y_{R1}	Y_{R2}	Y_{R3}	...	$Y_{R(n-1)}$	Y_{Rn}	$\mathbf{Y}_R$

Welch Graphical Metho

i-th period	Yji = Observed costs during i-th period and j-th replication					Average Cost per period	Moving average cost during ith period various values of w	
Period (i)	Rep1	Rep2	Rep3	Rep4	Rep5		w=5	w=10
1	422	423,2667	426,2081	425,3725	427,632	424,8958	424,8958	424,8958
			081	471,3725	473,632	470,8958	524,8958	524,8958
			081	679,3725	681,632	678,8958	505,7131	505,7131
			081	575,3725	577,632	575,055	571,4325	571,4325
			081	378,3725	385,2687	378,8232	573,679	632,6281
			081	845,3725	842,7548	843,9204	563,327	563,327
			081	628,3725	628,861	627,5417	590,0191	566,2036
			081	476,3725	476,861	475,5417	587,7141	495,0173
			081	688,3725	688,861	687,5417	570,4971	456,2866
			081	531,3725	531,861	530,5417	591,1747	405,815
			081	503,3725	505,8749	502,9444	614,2369	568,137
			081	719,3725	719,6942	718,5083	577,654	570,8989
			081	446,3725	446,861	445,5417	549,0146	572,0893
			081	490,3725	490,6942	489,5083	595,865	567,0417
			081	803,3725	803,6942	802,5083	591,079	568,5103
			081	633,3725	633,6942	632,5083	567,0203	584,99
			081	442,3725	442,6942	441,5083	565,1977	607,9031
18	310	311,2667	314,2081	313,3725	313,6942	312,5083	544,9602	600,5485
19	988	989,2667	992,2081	991,3725	993,632	990,8958	556,5379	604,9464
20	632	633,2667	636,2081	635,3725	637,632	634,8958	567,1186	595,0585
21	263	264,2667	267,2081	266,3725	268,632	265,8958	560,0629	597,361
22	480	481,2667	484,2081	483,3725	485,632	482,8958	623,0254	597,1683
23	493	494,2667	497,2081	496,3725	498,632	495,8958	625,897	606,6629
24	570	571,2667	574,2081	573,3725	575,632	572,8958	649,114	602,5845
							2,6595	609,6506
							7,5686	592,4697
							8,7504	590,8886
							8,2958	610,4931
							5,9322	610,6398
							1,8413	
							6,9152	
							25,498	
							2,5987	
							8,2797	

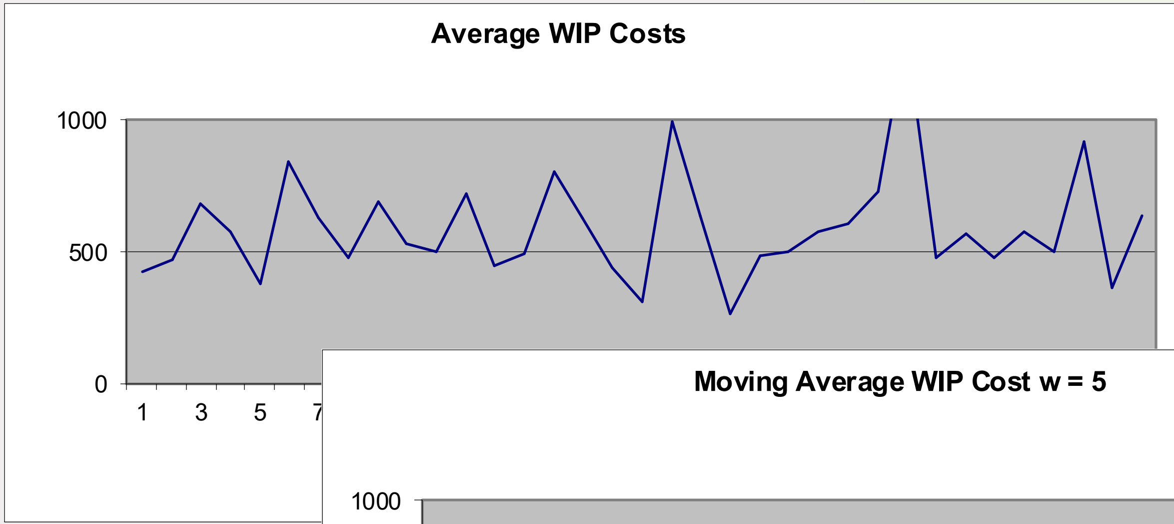


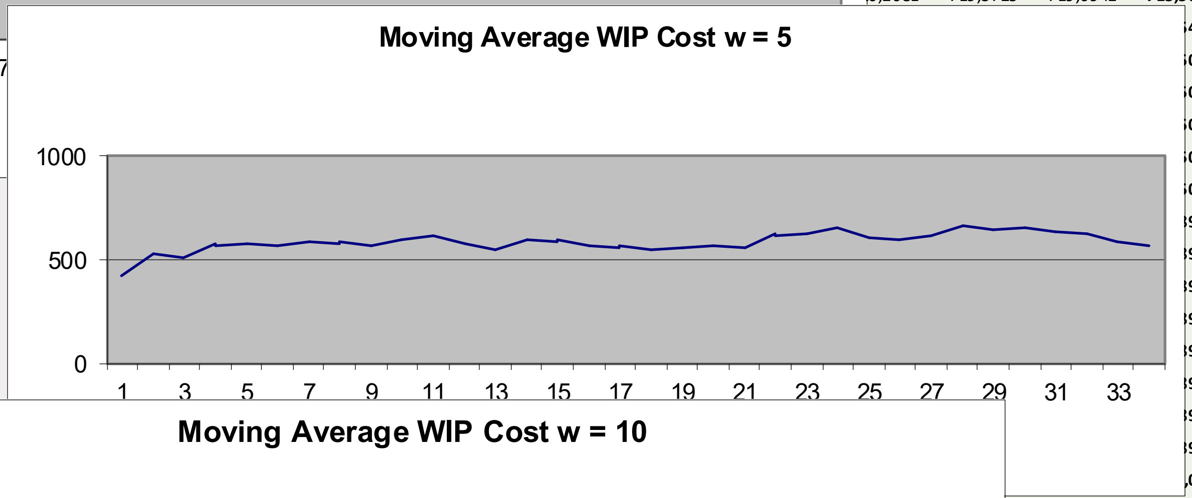
	Customer 1	Customer 2	Customer 3	...	Customer n-1	Customer n	Average
Replication 1	Y_{11}	Y_{12}	Y_{13}	...	$Y_{1(n-1)}$	Y_{1n}	Y_1
Replication 2	Y_{21}	Y_{22}	Y_{23}		$Y_{2(n-1)}$	Y_{2n}	Y_2
Replication 3	Y_{31}	Y_{32}	Y_{33}	...	$Y_{3(n-1)}$	Y_{3n}	Y_3
...	...	...	...	...	...	...	...
Replication R	Y_{R1}	Y_{R2}	Y_{R3}	...	$Y_{R(n-1)}$	Y_{Rn}	Y_R

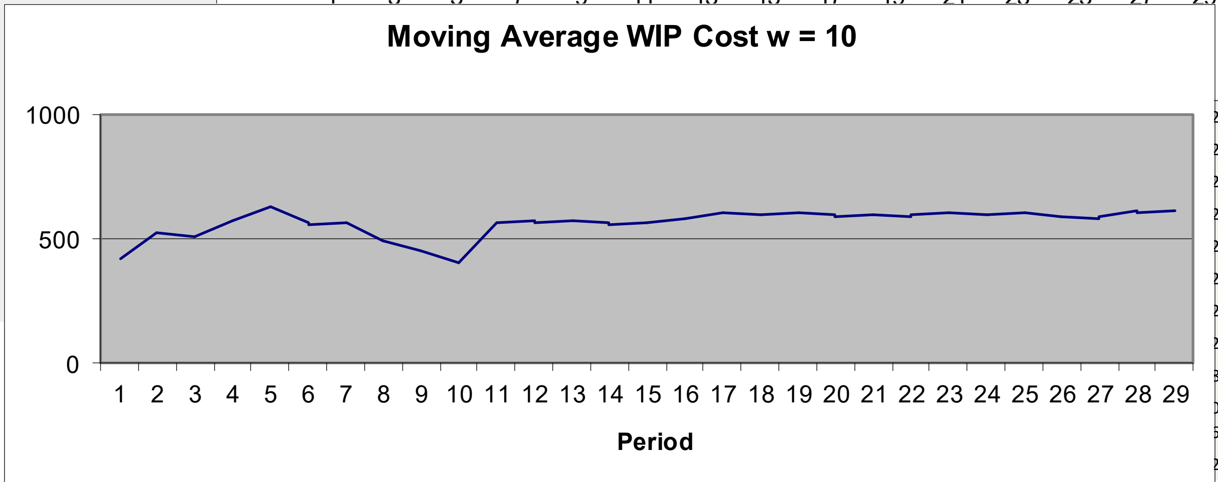
Welch Graphical Metho

i-th period	Yji = Observed costs during i-th period and j-th replication					Average Cost per period	Moving average cost during ith period for various values of w	
Period (i)	Rep1	Rep2	Rep3	Rep4	Rep5		w=5	w=10
1	422	423,2667	426,2081	425,3725	427,632	424,8958	424,8958	424,8958
			2,2081	471,3725	473,632	470,8958	524,8958	524,8958
			0,2081	679,3725	681,632	678,8958	505,7131	505,7131
			6,2081	575,3725	577,632	575,055	571,4325	571,4325
			9,2081	378,3725	385,2687	378,8232	573,679	632,6281
			6,2081	845,3725	842,7548	843,9204	563,327	563,327
			9,2081	628,3725	628,861	627,5417	590,0191	566,2036
			7,2081	476,3725	476,861	475,5417	587,7141	495,0173
			9,2081	688,3725	688,861	687,5417	570,4971	456,2866
			2,2081	531,3725	531,861	530,5417	591,1747	405,815
			4,2081	503,3725	505,8749	502,9444	614,2369	568,137
			0,2081	719,3725	719,6942	718,5083	577,654	570,8989
						417	549,0146	572,0893
						083	595,865	567,0417
						083	591,079	568,5103
						083	567,0203	584,99
						083	565,1977	607,9031
						083	544,9602	600,5485
						958	556,5379	604,9464
						958	567,1186	595,0585
						958	560,0629	597,361
						958	623,0254	597,1683
						958	625,897	606,6629
						958	649,114	602,5845
						958	602,6595	609,6506
						958	597,5686	592,4697
						,096	618,7504	590,8886
				25	475,632	473,0958	658,2958	610,4931
				25	570,632	567,8958	645,9322	610,6398
				25	482,632	479,8958	651,8413	
				25	581,632	578,8958	636,9152	
				25	501,632	498,8958	625,498	
				25	920,632	917,8958	582,5987	
				25				
				25				
				32				
				02				
				53	[illegible]	[illegible]		
				25	311,7659	315,5873		

$$\bar{Y}_i(w) = \begin{cases} \dfrac{\sum_{s=-w}^{w} \bar{Y}_{i+s}}{2w+1}, & if\ i > m - w \\ \dfrac{\sum_{s=-(i-1)}^{i-1} \bar{Y}_{i+s}}{2i-1}, & if\ i \le w \end{cases}$$







If there are multiple output variables…

… and if they disagree about what the appropriate warm-up period is, a decision must be made whether to use:

- different warm-ups for each variables,
- the same warm-up for each variable
  - which would have to be the maximum of the individual warm-ups, to be safe, and so would be conservative for most of the output variables

Business Process Simulation

6d) Comparing simulation outcomes

Comparing simulation outcomes

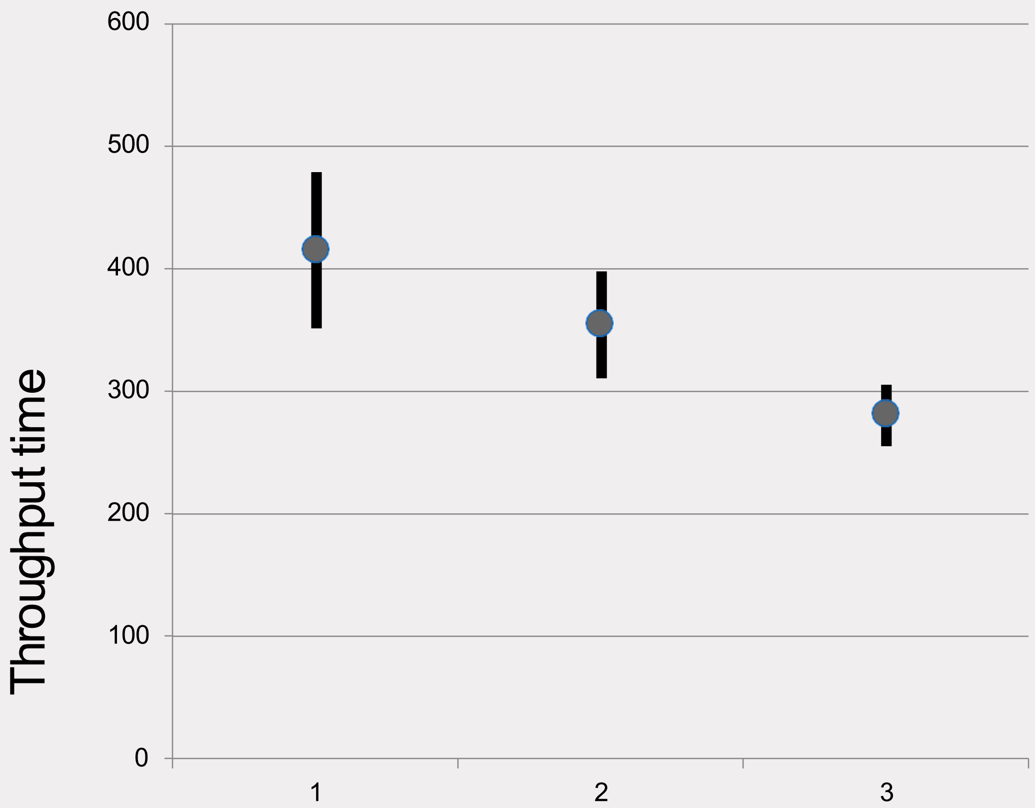
AS-IS vs TO-BE:
Many (probably most) simulation projects involve more than one system or configuration :

- Change the number of machines in some workcenters
- Alternative job-dispatch policies (FIFO, SPT, etc.)
- Alternative reorder-point, order quantities in inventory model

Compare those based on point estimates and confidence intervals obtained from simulation output analysis

Possibly use hypothesis testing to evaluate whether point estimates are significantly different

Confidence intervals



With 95% confidence:

- Situation 3 has a lower throughput time than 1
- Situation 3 has a lower throughput time than 2

We can <u>NOT</u> draw valid conclusions when comparing situation 1 and 2 !!!

TU/e

Comparing simulation outcomes

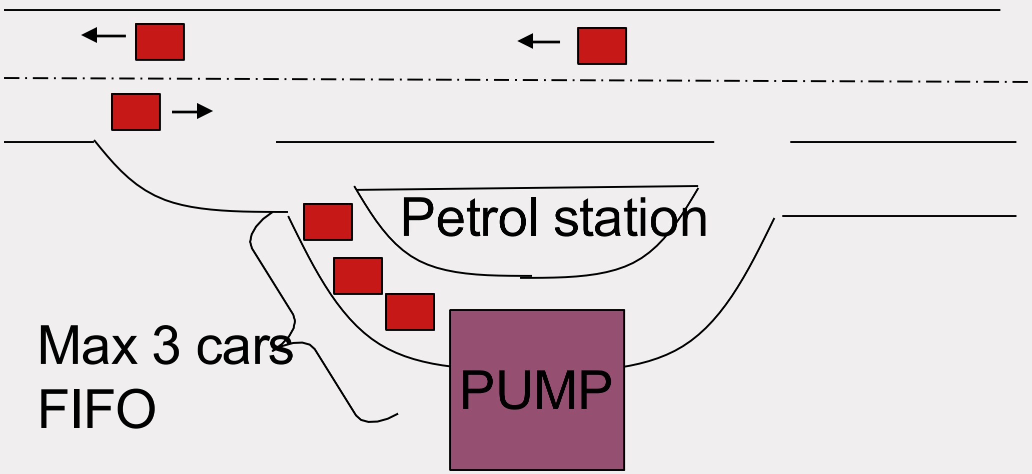
If confidence intervals overlap, real values for performance indicator may still be the same

=> no conclusions can be drawn

Try to make the confidence interval smaller by:

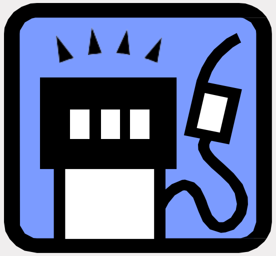
- Increasing number of replications
- Increasing replication length (only for non-terminating simulations)

EXAMPLE: The Petrol Station



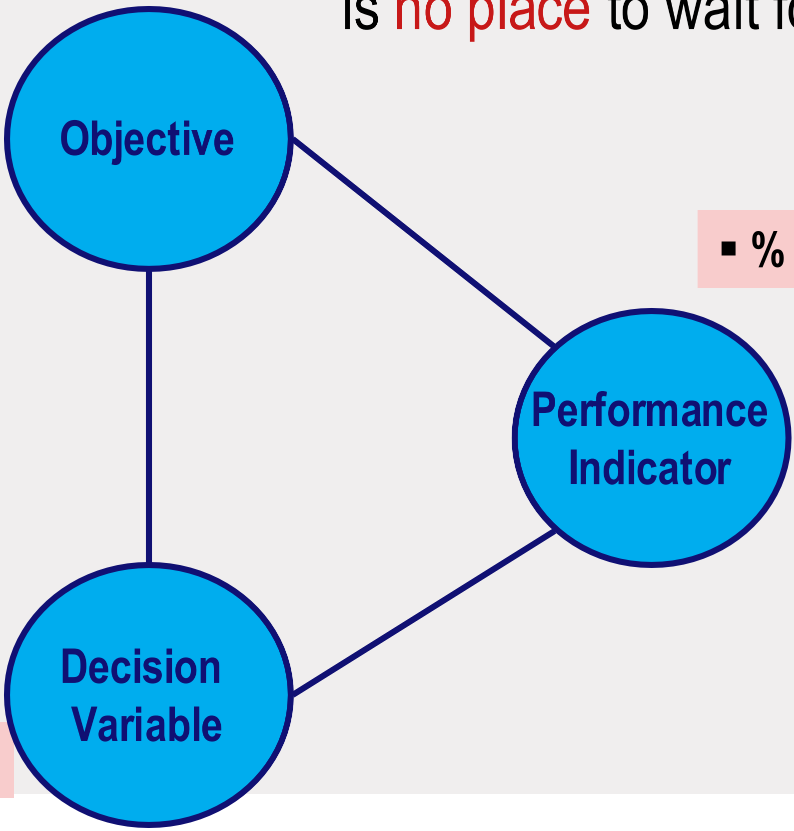
The owner of the petrol station has the feeling that some potential clients are leaving the station because there is no place to wait for service

Step 1.1: Decision Frame



…the feeling that some potential customers are leaving the station because there is no place to wait for service

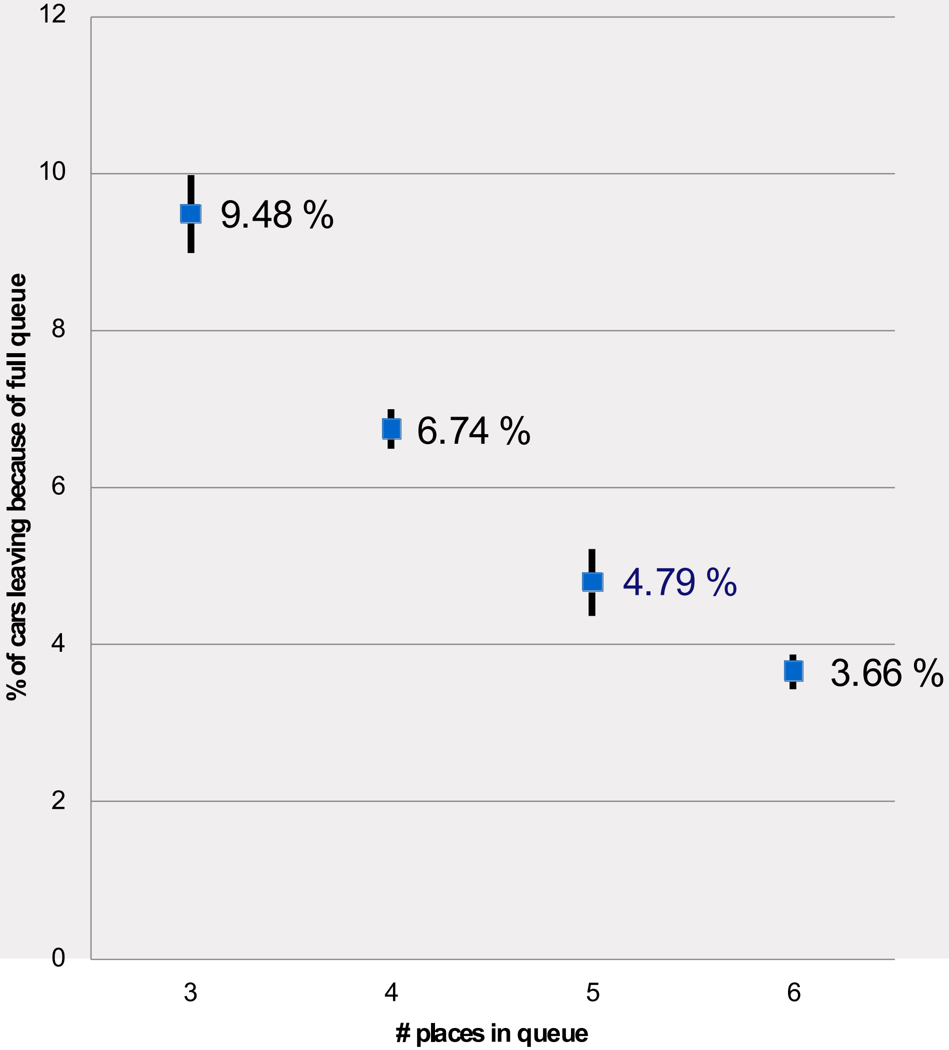
- Less customers driving on



- % of customers driving on

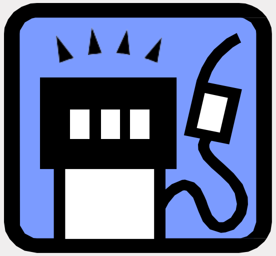
- size of queue area

Simulation results



- Which situation is best?
- Clearly the lowest percentage of leaving customers is with 6 places in the queue
- But is this desirable?

Alternative 1

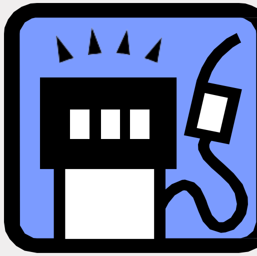


Suppose:
Customers get angry and unsatisfied when they have to wait too long; waiting time should not increase more than 50%

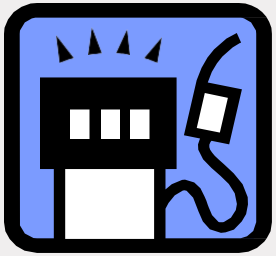
Decision Frame:

- Objective: optimize the number of places in the queue area such that a minimum % of customers drive on, but the average waiting time in the queue does not increase more than 50%
- Decision variables: capacity of the queue area
- Performance measures:
  - % of customers driving on
  - Average waiting time of customers in the queue

Simulation results



Alternative 2



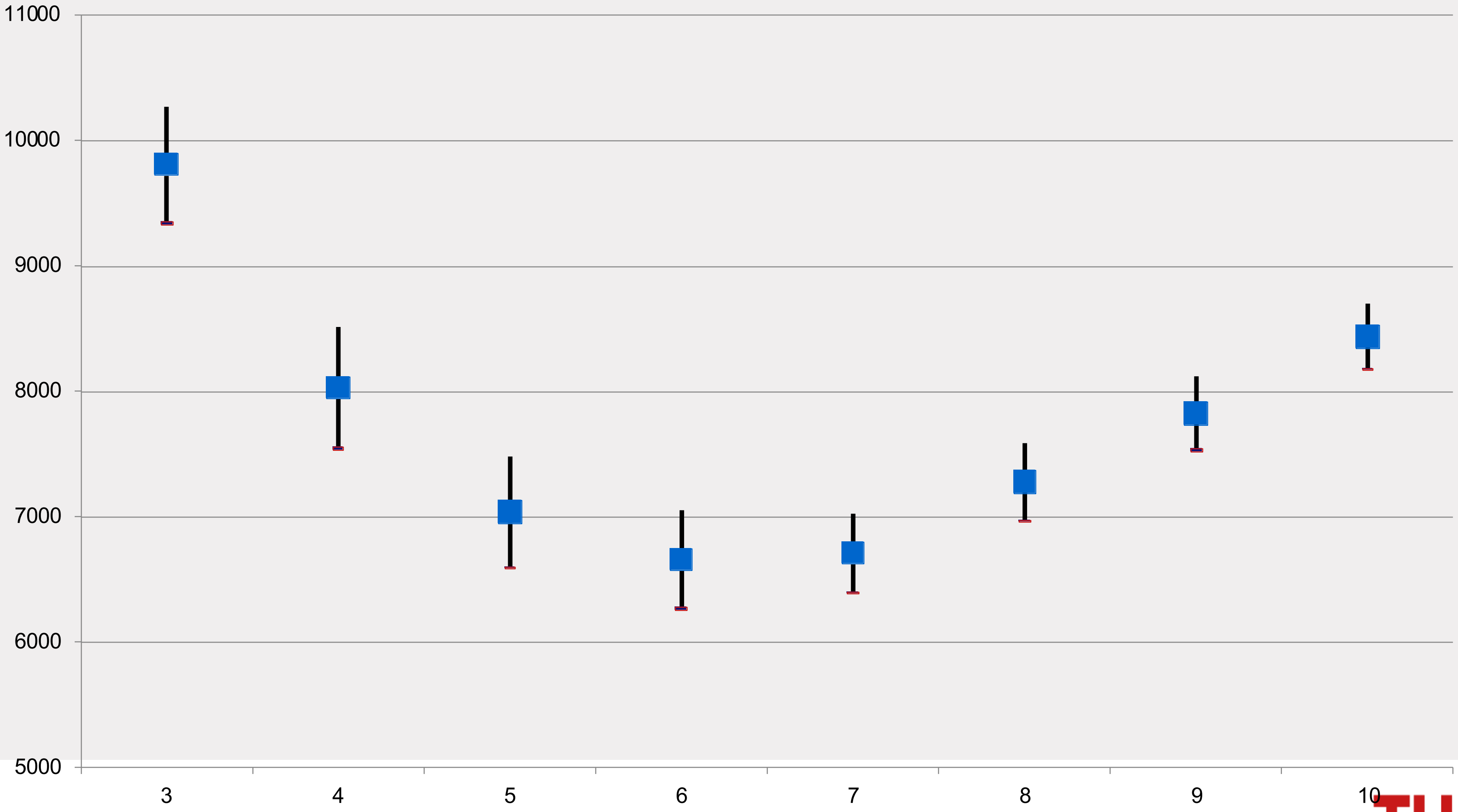
Suppose:

- One extra queueing place costs 1000 euros
- Each customer leaving ‘costs’ 100 euros
- Manager wants return on investment after 1000 customers

Decision Frame:

- Objective: earn more money by serving more customers (decrease the number of customers leaving because of full queue by extending the queue area)
- Decision variables: capacity of the queue area
- Performance measures: money lost by customers driving on because of a full queue:
  - number of customers lost * 100 euro’s +
  - number of additional queueing places * 1000 euro’s

Simulation results



TU/e

Simulation results

TU/e

TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Business Process Simulation

6e) Sensitivity analysis

Sensitivity analysis

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to different sources of uncertainty in its inputs

Goal: testing the robustness of the results of a model or system in the presence of uncertainty

Variation of environmental variables
=> select (and argue selection) the relevant one(s) yourselves

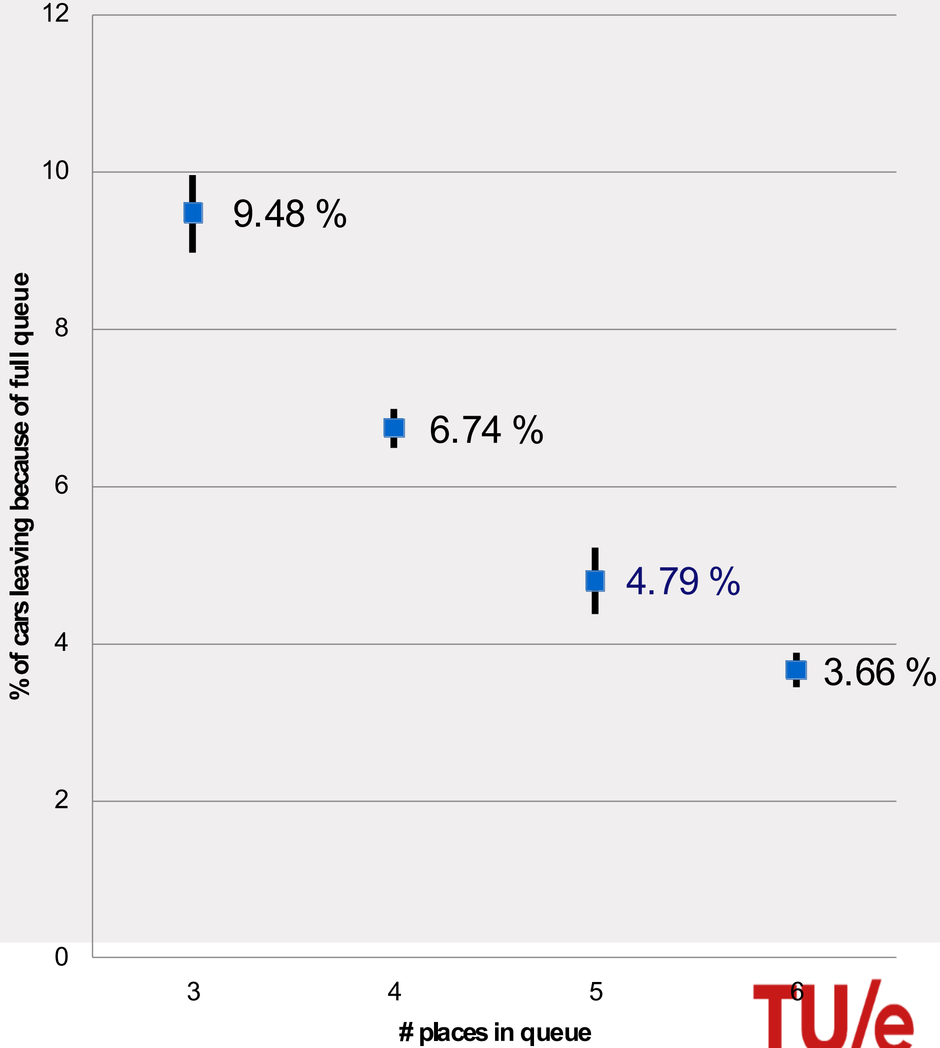
EXAMPLE: Sensitivity analysis

Effect of a change in the arrival process?

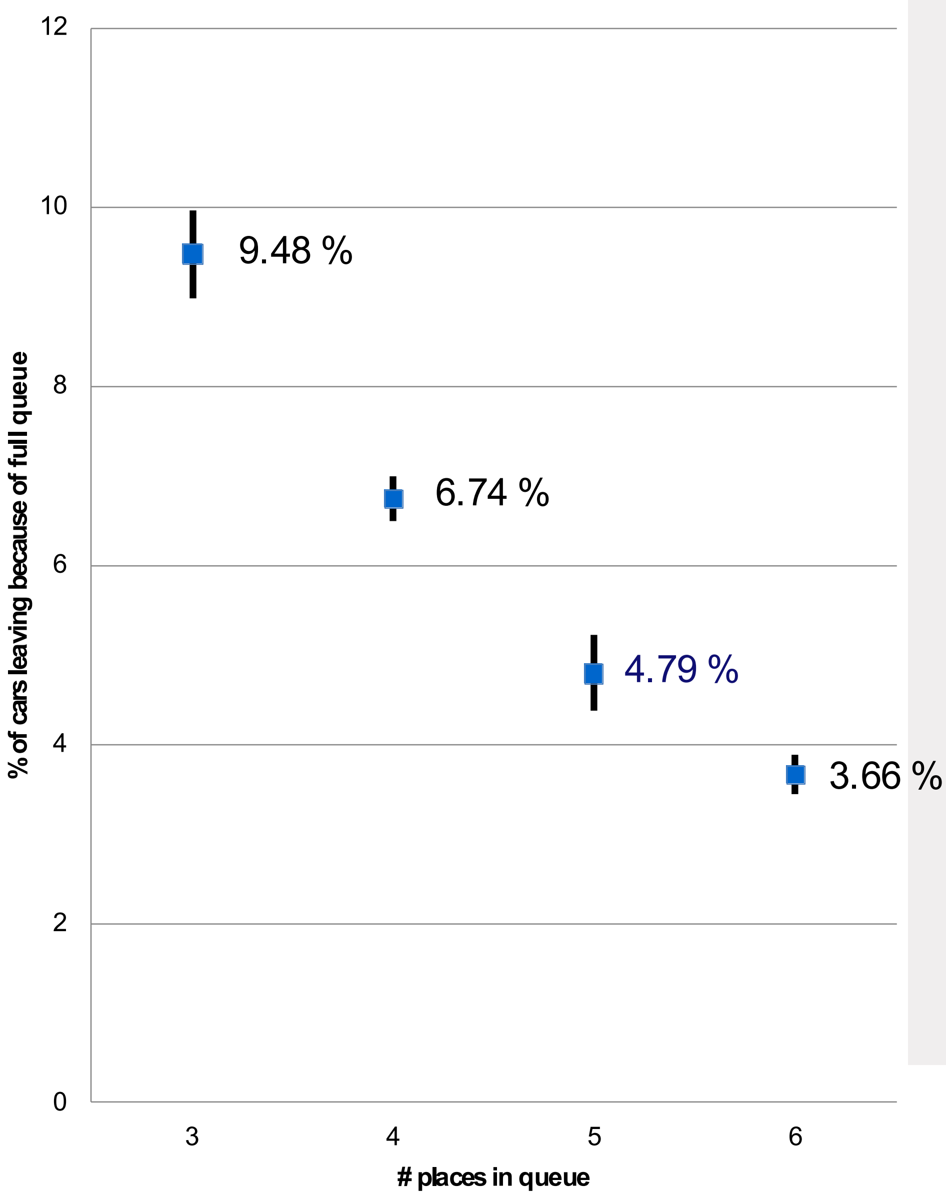
- 10% less customers
- 10% more customers

Effect of a change in the service times?

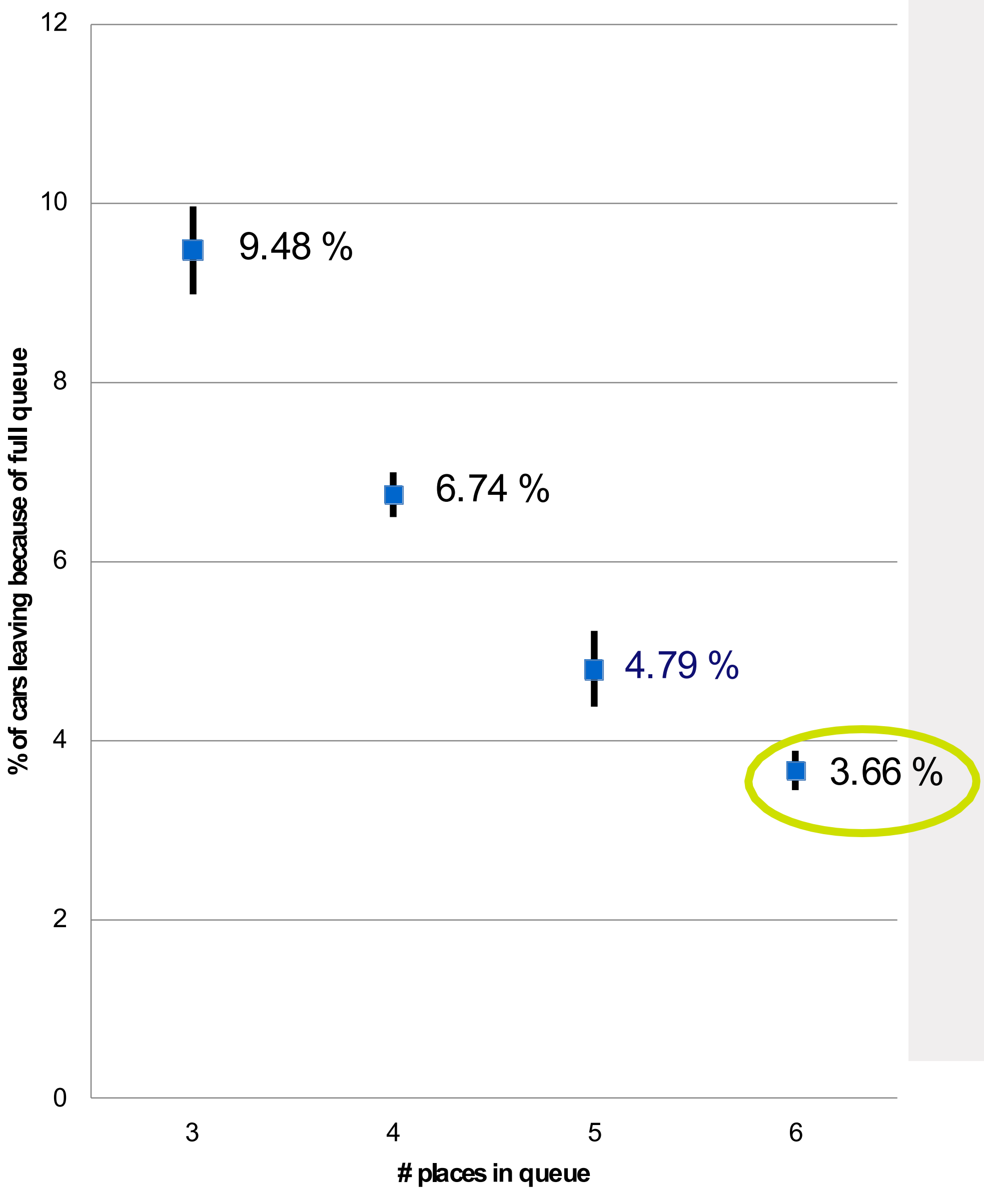
Effect of a change in …?



Simulation results



Simulation results



Business Process Simulation

6f) Conclusions

Simulation Methodology (7 steps)

STEP 1: Project definition

STEP 2: Design the simulation study

STEP 3: Conceptual model

STEP 4: Executable model and verification

- **Answer research questions**
- **Provide recommendations**

STEP 5: Validation

STEP 6: Experiments and output analysis

STEP 7: Conclusion

Conclusion of the project

- Based on output simulation experiments
  - statistical analysis !
- Compare alternatives
- Sensitivity analysis / extra redesign
- Link back to decision frame and problem context
  - costs?
  - practical relevance!
- Provide recommendations

Thank you for joining the course!

Interested in internship/master projects in Eindhoven?
I.Genga@tue.nl